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THE
STONE-MILLIS ARITHMETICS

INTERMEDIATE AND ADVANCED

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PREFACE

THE Stone-Millis Arithmetics have been prepared with a view to adapting the teaching of arithmetic more adequately to the interests and to the needs of children of the present day.

The adaptation to children's interests is secured through the use of the play instinct in the primary grades, and through the use, in all grades, of problem material relating to the various activities of children in and out of school, and to facts touching their experiences more or less directly. The adult point of view and the interests of adults have not been unduly obtruded into the work. In the second and third grades, especially, numerous games have been introduced that have been found, by actual use in the schoolroom, to be of intense interest to children, and to provide excellent drill work. Much problem material correlated with the hand work or industrial arts work of the modern school has been given. Many groups of problems relating to the common phases of community life with which the child comes into daily contact, have been used, such as collecting the mail, getting the coal supply, problems of the fruit vender, the parks, railroad traffic, etc.

Underlying this attempt more adequately to adapt the teaching of arithmetic to the interests of children, is the author's belief in the fundamental principles of education:

I. *All mental growth comes through the self-activity of the individual pupil in solving situations that to him are concrete and vital.*

II. *For school work to be educative there must be genuine legitimate motive or purpose underlying it; there must accompany it the feeling — the positive conviction — upon the part of the pupil, that the knowledge gained is going to further his present or future interests in some way.*

In adapting arithmetic to the needs of children, all topics that do not enter at some time or other into the everyday life of the average citizen, and all obsolete topics that represent business practices that have been abandoned, have been wholly eliminated.

The authors are thorough believers in adequate drill, and the abundant drill exercises and the plentiful provision for motivated drill throughout these books, should prove a strong feature of the series.

Book Two. This book covers the work recommended for the fifth, sixth, seventh, and eighth grades.

Part one: intermediate, for the fifth grade, is arranged in four divisions. The *first division* treats fractions orally within the limits of sixteenths. The relations used are those of the units of denominate numbers with which the pupil is already acquainted, and the applications are within the pupil's knowledge and experience in or out of school. The *second division* is devoted to whole numbers including three periods, and contains many tables of script for sight work to develop accuracy and rapidity. The *third division* is a second study of fractions, written work being given, but the results still limited to sixteenths. The *fourth division* presents mensuration, limited to linear and surface measures, drawing to scale, and simple work with right prisms.

Part two: intermediate, for the sixth grade, is arranged in four divisions. The *first division* takes up fractions and introduces a few larger fractions, but the old-time fractions with large denominators that are never encountered in everyday

life will not be found in this book. The Greatest Common Divisor is omitted entirely, and the Least Common Multiple is treated only briefly, with a suggestion that it, too, may be entirely omitted. The *second division* encourages rapid work with integers, and introduces short methods. The *third division* treats percentage, but limits its application to general problems, single trade discounts, and simple interest with integral rates and time in years and months. Only two kinds of problems in percentage are discussed in this book: (1) finding a per cent of a given number; (2) finding what per cent one number is of another. The *fourth division* extends mensuration to the areas of triangles, parallelograms, and trapezoids, and also includes the surfaces and volumes of right prisms, drawing to scale, and maps and plans.

Part one: advanced, for the seventh grade, is also arranged in four divisions. The *first division* reviews the denominate numbers treated in lower grades, and extends the subject to include angle and arc measure, etc. The *second division* reviews mensuration and extends it to include the circle and the right cylinder. The *third division* contains a brief treatment of proportion, with its applications to similar figures and simple machines. The *fourth division* reviews percentage and extends it to include profit and loss, commission, trade discount, simple interest (time in years, months, and days), and promissory notes.

Part two: advanced, for the eighth grade, is arranged in three divisions. The *first division* treats square root with its applications to the right triangle. The *second division* extends mensuration to include pyramids, cones, and spheres. The *third division* extends the applications of percentage to include property insurance, taxes and tariffs, successive discounts, bank discounts, and short methods of finding inter-

est, and discusses an excellent way to make the different methods of cancelling indebtedness, concrete and practical.

While many of the problems for the upper grades are of the kinds encountered in adult life, they are kept well within the comprehension of the pupil. Many suggestions are given for making the applications of arithmetic to business realistic to the pupil. These have been tried in the schoolroom under the direction of the authors, and have been found to work admirably. The problems are all real in the sense that they are problems that people have to solve in doing the world's work. While *exercises* have been given freely for drill work, the *problems* are not of the types usually found that are devised merely for mental gymnastics.

In the preparation of this series of arithmetics, the authors have had the constant and most efficient aid of Gordon A. Southworth, co-author of the popular *Southworth-Stone Arithmetics*. Every page of manuscript and proof has had his critical reading and intelligent criticism.

J. C. STONE.
J. F. MILLIS.

DECEMBER, 1910.

SUGGESTIONS TO TEACHERS

In using these books, the best results will be obtained by following the order of subjects when developing any topic. There need be no strict adherence, however, to the order of problems that follow any topic. Seek always to make the problem *concrete*. Ally it to the pupil's interests. Apply every topic to problems which the pupil meets in other school work and in everyday life, industrial art, science, domestic economy, etc.

The authors have not labeled problems "oral" or "written," an invariable custom with other books. They prefer, rather, that the pupil be encouraged to consider every problem carefully, study the relations of the numbers involved, and solve as far as possible without a pencil. It is to be believed that such a procedure will tend toward a more spontaneous method of analysis, and hence lead away from the mechanical forms so often seen in the schoolroom.

Neither have many forms of solution been given in the text. In most cases, the teacher should not require any particular form, but should encourage a pupil to study each problem and choose the method that seems to him to require the least figuring. Encourage short methods. Do not repress originality or individuality.

The applications of percentage are difficult only because the transactions are outside of the pupils' experiences. Seek to make the subject as vital and realistic as possible, as suggested in the notes of the text. Play "Going into Business." Secure blank forms of commercial papers — price

lists, invoices, receipts, checks, promissory notes, express and post-office money orders, drafts, insurance policies, tax bills, etc., and keep them as a part of the general equipment for the teaching of arithmetic.

Aim to secure a clear understanding of the value and use of every subject and a definite idea of what is given and what is required. Right thinking, accurate results, brief methods, rapid work, and a thorough grounding in what is fundamental and applicable to life in a work-a-day world are the essential things which the authors have tried to help teachers to secure. In the last analysis, however, all depends on the intelligence, enthusiasm, and skill which the teacher brings to her task, and the wisdom she displays in using the helps provided her.

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INTERMEDIATE ARITHMETIC

PART ONE: FIFTH YEAR

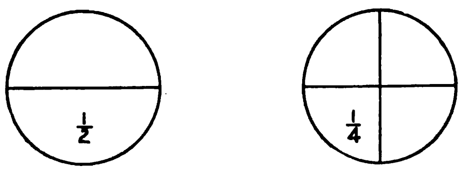
I. FRACTIONS

1. HALVES AND FOURTHS

1. How do we find $\frac{1}{2}$ of 4? $\frac{1}{2}$ of 6? $\frac{1}{2}$ of 8?

2. If I divide an apple into 2 equal parts, what do I call each part?

3. To find $\frac{1}{2}$ of a number or an object, as a circle or an oblong, we divide it into two equal parts. Then how many *halves* in a circle? In an oblong?



In a single thing there are two halves.

4. How do you find $\frac{1}{4}$ of 12? $\frac{1}{4}$ of 16? $\frac{1}{4}$ of 20?

5. How do you find $\frac{1}{4}$ of an apple? $\frac{1}{4}$ of a circle?

6. How many fourths in an apple? In a circle?

In a single thing there are four fourths.

2. FRACTIONS: COMPARING HALVES AND FOURTHS

1. Look at the circles on the preceding page and tell how many fourths in a half.

2. Draw a 1-inch square and a 2-inch square. Divide the 2-inch square into square inches.

3. What part of a 2-inch square is a 1-inch square?

4. How many square inches in $\frac{1}{2}$ of a 2-inch square?

5. How many cents in $\$\frac{1}{2}$? In $\$\frac{1}{4}$?

6. Compare $\$\frac{1}{2}$ with $\$\frac{1}{4}$.

7. $\frac{1}{2}$ gal. = —— qt.; $\frac{1}{4}$ gal. = —— qt.

8. $\frac{1}{2}$ ft. = —— in.; $\frac{1}{4}$ ft. = —— in.

9. $\frac{1}{2}$ doz. = ——; $\frac{1}{4}$ doz. = ——.

10. Which is the larger, $\frac{1}{2}$ or $\frac{1}{4}$? $\frac{1}{2}$ = —— $\times \frac{1}{4}$; $\frac{1}{4} = \frac{1}{2}$ of ——.

11. How many halves in an apple? How many fourths?

12. $1 = \frac{1}{2} + \frac{1}{2}$; $1 = \frac{1}{4} + \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$. $\frac{1}{2} + \frac{1}{4} = \frac{?}{4}$.

13. If I have a pie and eat $\frac{1}{4}$ of it, how much remains?

14. If $\frac{1}{4}$ of an orange is taken from $\frac{1}{2}$ of the orange, how much is left?

Practical Use of Halves and Fourths

1. How many pints in a quart? Then a pint is what part of a quart?

2. When milk is 10 cents a quart, how much is a pint worth?

3. How many quarts in a gallon? Then a quart is what part of a gallon?

4. When milk is 36 cents a gallon, how much is a quart worth?

5. How many pecks in a bushel? Then a peck is what part of a bushel?

6. When potatoes are 80 cents a bushel, how much is that for a peck?

7. How many ounces in a pound? In $\frac{1}{2}$ pound? In $\frac{1}{4}$ pound?

8. At 40 cents a pound, how much will 8 ounces of candy cost? How much will 4 ounces cost?

9. How much must your mother pay for 1 lb. 4 oz. of cheese at 20 ¢ a pound? 20 ¢ + $\frac{1}{4}$ of 20 ¢ = —— ¢.

10. John's mother sent him to buy a sirloin steak. It weighed 1 lb. 4 oz. How much will it cost at 28 ¢ a pound?

11. These girls have a sewing club. Alice is making a pillow for her doll. For the ruffle around it she bought 18 inches of ribbon. What part of a yard did she call for at the store? How much did it cost her at 24 ¢ a yard?

12. Gertrude is making a handkerchief for a Christmas present. She got a yard and a half of lace at the store for it. How much did it cost her at 10 cents a yard?

13. She got also a yard and a quarter of insertion for the handkerchief, at 8 cents a yard. How much did it cost her?

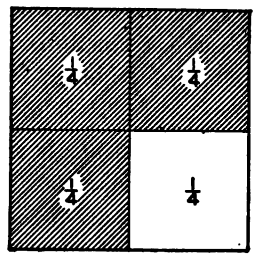
14. Elizabeth is making a tea apron. She bought 2 yards and 9 inches of lace for it at 12 ¢ a yard. How many yards did she call for at the store? How much did it cost?

15. Some of the girls are making skate bags of denim. If $\frac{1}{2}$ yard is required for each bag, how many yards does it take for 10 bags? How much does it cost at 22¢ a yard?

16. What part of a yard is 9 inches? If you need 45 inches of tape to bind a book bag, how much more than a yard is it?

17. In buying tape we usually ask for the length in yards and fractions of a yard. How much tape would you call for to bind the bag in Exercise 16? How much will it cost at 12¢ a yard?

3. WRITING MORE THAN ONE FRACTIONAL UNIT



1. Look at the drawing and tell how many fourths are shaded.

Three fourths is written $\frac{3}{4}$.

2. Three pecks are how many fourths of a bushel?

3. Three quarts are how many fourths of a gallon?

3 quarts = —— gallon.

4. Three quarters are how many fourths of a dollar?

5. To find $\frac{3}{4}$ of 12, we divide by 4 (find one fourth), and multiply the quotient by 3. How much is it?

Find $\frac{3}{4}$ of 8. Find $\frac{3}{4}$ of 16. Find $\frac{3}{4}$ of 24.

6. How many halves in a whole thing? Two halves is written $\frac{2}{2}$.

7. How many fourths in a whole? How is four fourths written?

8. How is two fourths written?

9. At 24¢ a pound, how much will $\frac{3}{4}$ of a pound of steak cost?

10. What part of a pound is 4 ounces? What part of a pound is 8 ounces? 12 ounces?

11. What part of a foot is 3 inches? 6 inches? 9 inches?

3 inches = —— foot. 9 inches = —— foot.

12. Write 2 quarts as a part of a peck. Write 6 quarts as a part of a peck.

13. How much will 3 quarts of vinegar cost at 28¢ a gallon?

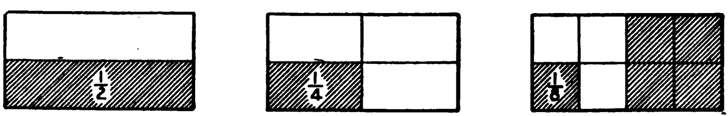
14. 6 hours = $\frac{1}{4}$ of a day. 12 hours = —— of a day. 18 hours = —— of a day.

15. John's father caught 12 fish, and John caught $\frac{3}{4}$ as many. How many did John catch?

16. Robert is 48 inches tall, and his sister is $\frac{3}{4}$ as tall. How many inches tall is she?

17. Our aquarium holds 28 gallons of water. It is $\frac{3}{4}$ full. How many gallons does it contain?

4. FRACTIONS INCLUDING EIGHTHS



1. How do you find $\frac{1}{8}$ of 16? $\frac{1}{8}$ of 24? $\frac{1}{8}$ of 32? $\frac{1}{8}$ of 40?

2. How do you find $\frac{1}{8}$ of an oblong?

3. How many eighths in a whole?

4. How many eighths are shaded?

5. *Five eighths* is written $\frac{5}{8}$. Write *seven eighths; four eighths; six eighths.*

6. Read: $\frac{3}{8}$, $\frac{4}{8}$, $\frac{2}{8}$, $\frac{7}{8}$.

7. Which figure shows how many? Which one shows what fractional part, or into how many parts the whole has been divided?

8. A quart is what part of a peck? Then 3 qt. are what part of a peck?

9. 1 qt. = $\frac{1}{8}$ pk.; 2 qt. = $\frac{2}{8}$ pk.; 3 qt. = —— pk.; 4 qt. = —— pk.

10. Write as a part of a peck, 5 qt.; 6 qt.; 7 qt.

At sight give the following:

11. $\frac{3}{4}$ of 8; of 12; of 16; of 20; of 24; of 28; of 32; of 36; of 40.

12. Give $\frac{3}{8}$ of each of the following: 16, 24, 32, 40, 48, 56, 64, 72, 80.

13. Give $\frac{5}{8}$ of each number in Exercise 12.

14. Give $\frac{7}{8}$ of each number in Exercise 12.

Practical Uses of Eighths

1. How many hours in $\frac{1}{8}$ of a day? In $\frac{3}{8}$ of a day? In $\frac{5}{8}$ of a day?

2. Of the 24 hours in a day, Roger sleeps $\frac{3}{8}$ of the time, he is in school $\frac{2}{8}$ of the time, and he studies at home $\frac{1}{8}$ of the time. How many hours does he spend in each way?

3. The grocer sells spices in boxes holding $\frac{1}{8}$ of a pound each. How many ounces of spice in a box?

4. Mary bought $\frac{7}{8}$ yard of ribbon for her dolls at 16¢ a yard. How much did it cost?

5. A class in sewing made skate bags of denim. Each bag took $\frac{5}{8}$ yard of cloth. How much did the denim for one

bag cost at 24¢ a yard? How many yards did it take for 16 bags? Find the total cost.

6. If silk costs $1.20 a yard, find the cost of $\frac{7}{8}$ yard.

7. In trimming a hat, a woman used $\frac{7}{8}$ yard of velvet for the under brim, and $2\frac{3}{8}$ yards for the upper brim and crown. How much did it cost at $2.40 a yard?

8. A class in sewing uses $\frac{3}{8}$ yard of lace for trimming one neckband. How many yards does it take for 2 dozen neckbands?

9. A merchant bought boys' suits for $4.80 apiece, and sold them at a gain of $\frac{3}{8}$ of what they cost him. How much did he gain on each suit? For how much did he sell each suit?

10. If a merchant marks toys for a special sale at $\frac{7}{8}$ of the regular price, what does he get for a dozen marbles the regular price of which is 32 cents?

11. For how much does he sell a gun of which the regular price is 80 cents?

12. For how much does he sell a doll carriage of which the regular price is 96 cents?

13. How much would you pay him for a pair of skates of which the regular price is $1.60?

14. Five out of every 8 people in a certain town live on the west side of the river. There are 12,000 people in all. How many live on the west side?

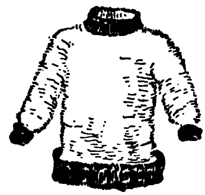
15. There are 32 pupils in a certain grade, of whom $\frac{3}{8}$ are boys. How many boys are there?

16. When meat is roasted, it loses about $\frac{3}{8}$ of its weight. A lady who keeps a delicatessen store bought 16 pounds of beef at 18 cents a pound and roasted it. She sold the roast

beef at 38 cents a pound. How much did she make on the 16 pounds?

17. She also bought hams weighing 32 pounds at 22 cents a pound. These she cooked, and sold at 40 cents a pound. If $\frac{3}{8}$ of the weight was lost in cooking, how much did she make on the hams?

18. If it takes $\frac{1}{8}$ of a skein of wool for the collar, cuffs, and bottom of a doll's sweater, and $\frac{1}{2}$ of a skein to make the main part, what part of a skein does it take for the whole sweater?



19. At 16¢ a skein, how much will the wool cost for 8 sweaters like the one in Problem 18?

20. To weave a holder a girl used $\frac{1}{2}$ yd. of striped material and $\frac{3}{4}$ yd. of plain. How much did she use in all?

21. Some girls made doll hats of raffia. In all, they used $\frac{5}{8}$ lb. of plain and $\frac{1}{4}$ lb. of colored. What part of a pound did they use? How many ounces was this? If they made 14 hats from the raffia, how much did each weigh?

22. If the girls mentioned in Problem 21 had 1 lb. of each kind of raffia when they began, how much of each was left?

23. Dorothy made a cooking apron. She used $\frac{3}{8}$ yd. for sleeves and bib, and $\frac{7}{8}$ yd. for the apron. How much material was needed in all? How much did it cost at 16¢ a yard?

24. How much material is needed to make such aprons as those in Problem 23 for a class of 24 girls? How much will it cost at $12\frac{1}{2}$¢ a yard?

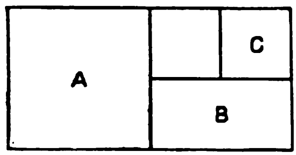
25. It was found that it takes $\frac{5}{8}$ of a ball of cotton to crochet a dishcloth. How many balls are needed by a class of 24 pupils if each is to make one cloth?

5. COMPARING HALVES, FOURTHS, AND EIGHTHS

NOTE. — In addition to the use of such devices as that shown below, the foot rule, divided into eighths of an inch, will be found an excellent means of making all kinds of comparisons of $\frac{1}{2}$, ^{1}, and $\frac{1}{8}$. Teachers should use it freely to supplement these exercises. If divided into sixteenths of an inch, the comparison may be extended.

1. How does an *oblong* resemble a square? Make an oblong 3 inches long and 2 inches wide.

2. Into how many equal parts is this oblong divided to get A? Then A is what part of the oblong?



3. If the oblong had been divided into parts like B, how many would there have been? Then what part of the oblong is B?

4. How many parts like C will make the whole oblong? What part of the oblong is C?

5. $\frac{1}{2}+\frac{1}{2}=$ ——; $\frac{1}{4}+\frac{1}{4}+\frac{1}{4}+\frac{1}{4}=$ ——; $\frac{1}{4}+\frac{1}{4}+\frac{1}{2}=$ ——.

6. Look at the figure, and tell what $\frac{1}{4}+\frac{1}{8}+\frac{1}{8}$ equals.

7. $\frac{2}{8}+\frac{1}{2}=\frac{?}{4}$; $\frac{1}{2}+\frac{1}{4}+\frac{2}{8}=$ ——; $1-\frac{1}{8}=$ ——.

8. Compare $\frac{1}{2}$ and $\frac{1}{4}$ thus: $\frac{1}{2}$ is 2 times $\frac{1}{4}$; $\frac{1}{4}$ is $\frac{1}{2}$ of $\frac{1}{2}$.

9. Compare $\frac{1}{4}$ and $\frac{1}{8}$; $\frac{1}{2}$ and $\frac{1}{8}$.

10. Cut an apple into 8 equal parts. What is each part called? How many of these eighths make one fourth of the apple?

11. Compare $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{2}$ and $\frac{1}{8}$; $\frac{1}{4}$ and $\frac{1}{8}$.

12. $\frac{1}{2}+\frac{1}{4}=$ —— fourths; $\frac{1}{2}+\frac{1}{4}+\frac{1}{8}=$ —— eighths.

13. $\frac{1}{2}-\frac{1}{8}=$ —— eighths; $\frac{1}{4}-\frac{1}{8}=$ —— eighths.

14. $1=$ —— halves, or —— fourths, or —— eighths.

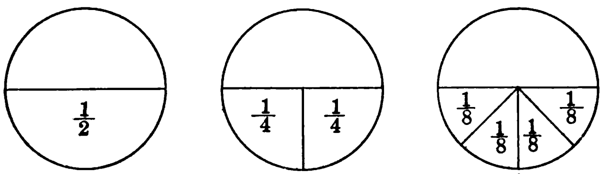
15. $1 - \frac{3}{8} =$ —— eighths; $1 - \frac{1}{4} = \frac{?}{4}$; $1 - \frac{3}{4} =$ ——.

16. When $\frac{1}{2}$ pound of tea costs 20 cents, how much shall I pay for $\frac{1}{4}$ lb.?

17. When $\frac{1}{2}$ dozen oranges cost 30 cents, how much will $\frac{1}{4}$ dozen cost?

18. When $\frac{1}{2}$ pound of pepper costs 40 cents, how much will $\frac{1}{8}$ of a pound cost?

19. When $\frac{1}{4}$ lb. of tea costs 25¢, how much is that per pound?



$$\frac{1}{2} = \frac{2}{4} = \frac{4}{8}$$

20. When $\frac{1}{8}$ bushel of potatoes costs 15¢, how much will $\frac{1}{4}$ of a bushel cost? How much will a bushel cost?

21. 8 pints equal 1 gallon. A pint is what part of a gallon? 2 pints equal what part of a gallon? 4 pints equal what part of a gallon?

22. 4 pecks = 1 bushel. 2 pecks = —— bushel. 3 pecks $= \frac{?}{4}$ bu.

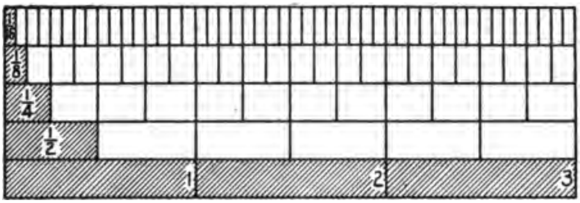
23. 6 = what part of a dozen? 3 = what part of a dozen?

24. How many hours longer is $\frac{1}{2}$ a day than $\frac{1}{8}$ of a day?

25. How many inches longer is $\frac{1}{2}$ a yard than $\frac{1}{4}$ of a yard?

26. $\frac{1}{2}$ a pound is how many ounces heavier than $\frac{1}{8}$ of a pound?

6. EXERCISES IN HALVES, FOURTHS, EIGHTHS, AND SIXTEENTHS



1. Look at the figure and tell how many 4ths in $\frac{1}{2}$.

2. How many 8ths in $\frac{1}{2}$? How many 8ths in $\frac{1}{4}$?

3. How many 16ths in $\frac{1}{2}$? In $\frac{1}{4}$? In $\frac{1}{8}$?

4. Look at the figure and see how to fill the blanks below:

one half = —— fourths
one half = —— eighths
one half = —— sixteenths
one fourth = —— eighths
one fourth = —— sixteenths
one eighth = —— sixteenths

5. $\frac{1}{2} = \frac{?}{4}$; $\frac{1}{2} = \frac{?}{8}$; $\frac{1}{2} = \frac{?}{16}$.

6. $\frac{1}{4} = \frac{?}{8}$; $\frac{1}{4} = \frac{?}{16}$; $\frac{1}{8} = \frac{?}{16}$.

7. How many ounces in a pound? Then an ounce is what part of a pound?

8. Write as parts of a pound: 2 oz.; 3 oz.; 5 oz.; 7 oz.; 9 oz.

9. At 32 cents a pound, find the cost of 1 lb. 2 oz. of butter. Find the cost of 2 lb. 4 oz.

10. Write in ounces: $\frac{5}{16}$ lb.; $\frac{3}{8}$ lb.; $\frac{7}{16}$ lb.; $\frac{3}{4}$ lb.; $\frac{7}{8}$ lb.

11. Write in pounds: 1 oz.; 3 oz.; 5 oz.; 7 oz.; 9 oz.

Practical Problems in Fractions

1. Pepper is sold in boxes containing $\frac{1}{8}$ lb. each. How many boxes does it take to make $\frac{1}{4}$ lb.? To make $\frac{1}{2}$ lb.?

2. A $\frac{1}{8}$-lb. box of caraway seed sells for 10 cents. What will $\frac{1}{4}$ lb. cost? $\frac{1}{2}$ lb.? 1 lb.?

3. One pound of cloves will fill how many boxes, each holding $\frac{1}{2}$ lb.? How many each holding $\frac{1}{4}$ lb.? How many each holding $\frac{1}{8}$ lb.?

4. A quart is what part of a peck? Then if you were a clerk selling a customer $\frac{1}{2}$ peck of potatoes, and measured them out with a quart measure, how many times would you fill the measure?

$$\tfrac{1}{2}\text{ pk.} = \tfrac{?}{8}\text{ pk.}$$

5. If you were selling a customer $\frac{1}{4}$ pk. of green beans, and measured them with a quart measure, how many times would you fill the measure?

$$\tfrac{1}{4}\text{ pk.} = \tfrac{?}{8}\text{ pk.}$$

6. A $\frac{1}{2}$-pint measuring cup used in cooking holds what part of a gallon? A quart is what part of a gallon? Then if a recipe called for a quart of material, and you measured it out with the $\frac{1}{2}$-pint measure, how many times would you fill the measure?

$$\tfrac{1}{4}\text{ gal.} = \tfrac{?}{16}\text{ gal.}$$

7. If a recipe called for $\frac{1}{2}$ gallon of material, and you measured it out with the $\frac{1}{2}$-pint measure ($\frac{1}{16}$ gallon), how many times would you fill the measure?

$$\tfrac{1}{2}\text{ gal.} = \tfrac{?}{16}\text{ gal.}$$

8. Some children have a lemonade stand at which they sell lemonade in glasses of two sizes. One glass holds $\frac{1}{16}$ gallon and the other $\frac{1}{8}$ gallon. How many of the smaller does it take to fill one of the larger?

9. If you were measuring with a ruler divided into 16ths of an inch, and wanted to measure $\frac{1}{4}$ inch, how many spaces on the ruler would you use? If you wanted to measure $\frac{1}{8}$ inch?

10. If a quarter-note in music has $\frac{1}{2}$ a beat, how much will a half-note have?

11. Henry picked up $\frac{1}{2}$ bushel of chestnuts and sold $\frac{5}{16}$ of a bushel. What part of a bushel did he have left?

12. What is the cost of $\frac{1}{2}$ lb. lard at 16¢ a pound? Of $\frac{1}{4}$ lb.? Of $\frac{1}{8}$ lb.? Of $\frac{1}{16}$ lb.? Of 5 ounces?

13. What is the cost of $\frac{3}{4}$ lb. boiled ham at 48¢ a pound? Of $\frac{3}{8}$ lb.? Of $\frac{5}{16}$ lb.?

14. Find the cost of $\frac{1}{4}$ yd. of silk at 96¢ a yard.

15. Find the cost of $\frac{1}{8}$ yd. of silk at $1.20 a yard.

16. Find the cost of $\frac{3}{4}$ yd. of ribbon at 32¢ a yard.

17. Find the cost of $\frac{1}{16}$ yd. of velvet at $3.20 a yard.

18. If a city block is $\frac{1}{8}$ mile long, what part of a mile are 2 blocks? 4 blocks?

19. In a race Henry finished in $14\frac{1}{8}$ sec. and John in $14\frac{1}{2}$ sec. By what part of a second did Henry win?

7. FRACTIONS GREATER OR LESS THAN ONE, OR EQUAL TO ONE

1. How many *halves* in a whole? This is written $\frac{2}{2} = 1$.

2. How many *fourths* in a whole? Write your answer in figures.

3. How many *eighths* in a whole? Write your answer in figures.

4. Is $\$\frac{5}{4}$ more or less than $1? How much?

5. Select the fractions equal to 1, those less than 1, and those greater than 1:

$$\frac{2}{2},\ \frac{3}{4},\ \frac{5}{4},\ \frac{3}{8},\ \frac{4}{4},\ \frac{5}{8},\ \frac{9}{8},\ \frac{11}{16},\ \frac{15}{16},\ \frac{8}{8},\ \frac{17}{16}.$$

6. Tell how much greater than 1 is each of the following:

$$\frac{5}{4},\ \frac{6}{4},\ \frac{7}{4},\ \frac{9}{8},\ \frac{11}{8},\ \frac{12}{8},\ \frac{17}{16},\ \frac{20}{16},\ \frac{21}{16}.$$

7. Tell how much less than 1:

$$\frac{3}{4},\ \frac{5}{8},\ \frac{7}{8},\ \frac{8}{8},\ \frac{15}{16},\ \frac{11}{16},\ \frac{9}{16}.$$

8. Which is larger, $\frac{1}{2}$ or $\frac{1}{4}$? $\frac{1}{4}$ or $\frac{1}{8}$? $\frac{1}{8}$ or $\frac{1}{16}$?

9. When $\frac{3}{4}$ of an apple is eaten, how much is left?

10. When you have gone $\frac{7}{8}$ of the distance to school, what part remains?

11. When you buy $\frac{15}{16}$ of a pound for a pound, how much do you lose?

12. When you have read $\frac{5}{8}$ of a book, what part have you yet to read?

13. When $\frac{3}{8}$ of your vacation is passed, what part of it have you left?

8. ADDING AND SUBTRACTING FRACTIONS

1. How many fourths in $\frac{1}{2}$? In $\frac{1}{2}+\frac{1}{4}$? In $\frac{1}{2}+\frac{3}{4}$?

2. How many eighths in $\frac{1}{4}$? In $\frac{1}{4}+\frac{1}{8}$?

3. $\frac{1}{2}+\frac{1}{4}=\frac{?}{4}$; $\frac{1}{4}+\frac{1}{8}=\frac{?}{8}$; $\frac{1}{8}+\frac{1}{16}=\frac{?}{16}$.

4. *Show by drawing and dividing rectangles that:*

$\frac{1}{2}=\frac{2}{4}$	$\frac{2}{4}=\frac{4}{8}$	$\frac{3}{8}=\frac{6}{16}$	$\frac{6}{8}=\frac{12}{16}$
$\frac{1}{4}=\frac{2}{8}$	$\frac{1}{8}=\frac{2}{16}$	$\frac{4}{8}=\frac{8}{16}$	$\frac{7}{8}=\frac{14}{16}$
$\frac{3}{4}=\frac{6}{8}$	$\frac{2}{8}=\frac{4}{16}$	$\frac{5}{8}=\frac{10}{16}$	$\frac{1}{2}=\frac{8}{16}$

5. *Read and add:*

$\frac{2}{4}+\frac{3}{4}=$	$\frac{2}{4}+\frac{3}{4}=$	$\frac{5}{8}+\frac{3}{8}=$	$\frac{3}{16}+\frac{5}{16}=$
$\frac{3}{4}+\frac{1}{4}=$	$\frac{1}{8}+\frac{5}{8}=$	$\frac{4}{8}+\frac{2}{8}=$	$\frac{4}{16}+\frac{3}{16}=$
$\frac{1}{8}+\frac{2}{8}=$	$\frac{2}{8}+\frac{3}{8}=$	$\frac{5}{8}+\frac{2}{8}=$	$\frac{5}{16}+\frac{11}{16}=$

6. *Read and subtract:*

$\frac{3}{4}-\frac{1}{4}=$	$\frac{3}{8}-\frac{1}{8}=$	$\frac{9}{16}-\frac{7}{16}=$	$\frac{15}{16}-\frac{3}{16}=$
$\frac{5}{8}-\frac{3}{8}=$	$\frac{7}{8}-\frac{6}{8}=$	$\frac{11}{16}-\frac{5}{16}=$	$\frac{9}{8}-\frac{1}{8}=$
$\frac{7}{8}-\frac{2}{8}=$	$\frac{5}{16}-\frac{1}{16}=$	$\frac{12}{16}-\frac{9}{16}=$	$\frac{5}{4}-\frac{1}{4}=$

Practical Problems in Addition and Subtraction

1. Mary bought $\frac{3}{4}$ yard of ribbon, and used all but $\frac{1}{4}$ yard. How much did she use?

2. Gertrude bought $\frac{7}{8}$ yard of lace for her dolls' clothes, and used $\frac{5}{8}$ yard. How much did she have left?

3. If she had used $\frac{3}{4}$ yard, how much would she have had left?

4. If she had used $\frac{1}{2}$ yard, how much would she have had left?

5. Alice bought $\frac{1}{2}$ yard of one kind of lace and $\frac{3}{4}$ yard of another kind. How much did she get in all?

6. A piece of crash for a towel is 30 inches long. If $\frac{3}{8}$ inch is taken up at each end in making a hem, how much is used in both hems? How long is the towel when finished?

7. The piece of crash for a towel is $28\frac{3}{4}$ inches long. At each end a $\frac{1}{4}$-inch hem is made. How much is taken up by the two hems? How long is the finished towel?

8. I am making a handkerchief out of a piece of linen $10\frac{7}{8}$ inches square. If I make a $\frac{1}{4}$-inch hem all around it, how long and wide will it be when finished?

9. The material for a pillowcase is 30 inches long. If I make a hem on one end $1\frac{5}{8}$ inches wide, how long will the pillow case be when finished?

10. If I buy $\frac{1}{2}$ yd. of lace in one piece, $\frac{3}{4}$ yd. in a second, and $\frac{7}{8}$ yd. in a third, how much do I buy all together?

11. I spent $\frac{1}{2}$ of my vacation at the lake, $\frac{1}{8}$ of it on a hunting trip, and the rest at home. How much of the vacation was spent at home?

12. If Raymond lives $\frac{1}{2}$ mile from school, and you live $\frac{3}{8}$ mile from school, how much farther does he have to go than you?

13. If you lost $\frac{1}{16}$ of your marbles, and gave away $\frac{1}{4}$ of them, what part of them would you have left?

14. Edward is $53\frac{3}{4}$ inches tall. Frederick is $51\frac{5}{8}$ inches tall. How much taller is Edward than Frederick?

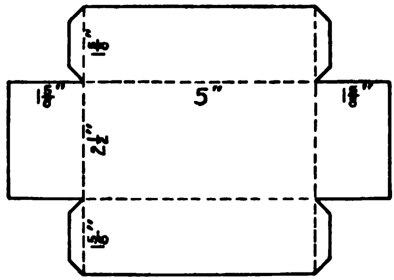
15. Charles is now $55\frac{9}{16}$ inches tall. When he was measured a year ago he was $52\frac{3}{8}$ inches tall. How much has he grown in the year?

16. Measure your height to the eighth of an inch. How much less than 5 feet tall are you?

17. Who is the tallest person in the class? How much taller is he or she than you?

18. John is nailing two boards together. One is $\frac{7}{8}$ inch thick and the other $\frac{3}{4}$ inch thick. How thick are the two together? If he is using nails $1\frac{1}{2}$ inches long, how much do they lack of going through?

19. This is the pattern of a cardboard box. It is to be 5 inches long, $2\frac{1}{2}$ inches wide, and $1\frac{5}{8}$ inches deep, when finished. Fold along the dotted lines, and paste. How long and how wide must the rectangular piece of cardboard be from which it is cut? Make the box.



20. A cardboard box $4\frac{1}{2}$ inches long, $3\frac{1}{4}$ inches wide, and $2\frac{3}{8}$ inches deep is to be made in the same way as that in Exercise 19. Find the length and the breadth of the rectangular piece of cardboard from which it is cut.

21. A printer is making a book in which the printed part of the page is $3\frac{3}{4}$ in. wide and $5\frac{1}{2}$ in. long. He wants to have a margin on all four sides $\frac{5}{8}$ in. wide. How long and how wide must the paper for the page be?

22. How long and how wide will two leaves of the book be? How many pages will there be on two leaves?

23. If the paper used in making the book will fold so as to make 8 leaves or 16 pages, what will the dimensions of the sheet of paper be?

24. Mr. Jones told his two boys that he could saw more wood in a day than both of them together. He sawed $1\frac{1}{8}$ cords. One boy sawed $\frac{3}{4}$ of a cord, and the other $\frac{1}{2}$ a cord. Which sawed more, the father or the boys, and how much?

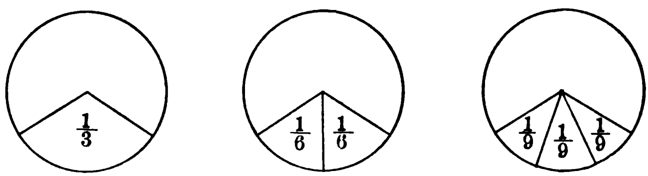
25. Theodore had a certain sum of money in the Savings Bank. Christmas week he drew out $\frac{5}{16}$ of all he had. He then had \$1.32 in the bank. How much did he have in the bank in the first place?

26. Daniel Fish makes a drawing of a tool box on a scale of $\frac{1}{8}$. In the drawing, the length of the tool box is 3 inches. What is the length of the box?

27. He made this box of boards $\frac{7}{8}$ of an inch in thickness. He planed them down $\frac{1}{16}$ of an inch on each side. How thick were the boards then?

9. THIRDS, SIXTHS, AND NINTHS

NOTE. — The following exercises should be varied and supplemented by use of the yardstick and oblongs.



$\frac{1}{3}$ $\frac{1}{3} = \frac{2}{6}$ $\frac{1}{3} = \frac{3}{9}$

1. In the whole of anything there are —— thirds, —— sixths, or —— ninths. $1 = \frac{3}{3} = \frac{6}{6} = \frac{9}{9}$.

2. Compare $\frac{1}{3}$ and $\frac{1}{6}$ thus: $\frac{1}{3} = 2 \times \frac{1}{6}$; $\frac{1}{6}$ is $\frac{1}{2}$ of $\frac{1}{3}$.

3. Compare $\frac{1}{3}$ and $\frac{1}{9}$.

4. If you give away $\frac{1}{3}$ of an apple, how much remains?

5. If an apple is cut into 6 equal parts, what is each part called? $1 - \frac{1}{6} =$ ——. $1 - \frac{5}{6} =$ ——.

6. How many feet in a yard? A foot is what part of a yard?

7. How many inches in $\frac{1}{3}$ yd.? In $\frac{1}{6}$ yd.? In $\frac{1}{9}$ yd.?

8. $\frac{1}{3} + \frac{1}{6} =$ —— sixths; $\frac{2}{3} - \frac{1}{6} =$ ——; $\frac{2}{3} + \frac{1}{9} =$ —— ninths.

9. $\frac{2}{3}+\frac{1}{6}=$ ——; $1-\frac{5}{9}=$ ——; $1-\frac{2}{9}=$ ——.

10. If $\frac{1}{3}$ of Henry's money is 8 cents, how much has he?

11. Lucy had some pet rabbits. 2 ran away. This was $\frac{1}{6}$ of all she had. How many had she?

12. John paid 5¢ for a pencil. This was $\frac{1}{3}$ as much as he paid for a book. How much did the book cost?

13. 3 is $\frac{1}{6}$ of what number?

14. An oil tank is $\frac{5}{6}$ full. 60 gallons are drawn out, and then the tank is found to be $\frac{1}{3}$ full. How many gallons will the tank hold?

Practical Problems

1. When you have read $\frac{1}{6}$ of a book, how much do you still have to read?

2. William has read $\frac{1}{3}$ of a storybook, and his sister has read $\frac{4}{9}$ of it. Which has read the more? How much more?

3. If you sleep $\frac{1}{3}$ of the day and play $\frac{1}{6}$ of it, how much of the day is left for other things?

4. How many is $\frac{1}{3}$ doz. eggs? $\frac{2}{3}$ doz.? $\frac{1}{6}$ doz.? $\frac{5}{6}$ doz.?

5. If you buy $\frac{2}{3}$ dozen bananas at 18¢ a dozen, how much do they cost?

6. If you buy $1\frac{1}{3}$ dozen oranges at 36¢ a dozen, how much do they cost?

7. If you buy $1\frac{5}{6}$ doz. eggs at 36¢ a dozen, how much do you pay?

8. Gertrude bought $1\frac{8}{9}$ yards of braid at 27¢ a yard. Find the cost.

9. How much will $4\frac{2}{9}$ yards of linen cost at 54¢ a yard?

10. Laura bought $\frac{8}{9}$ yards of lace, and used $\frac{2}{3}$ yards. How much did she have left?

11. A lady roasted meat to sell at a ladies' bazaar. It lost $\frac{4}{9}$ of its weight in cooking. What part of the original weight was the roasted meat?

12. If she bought 18 lb., how much did it weigh after it was roasted?

13. A merchant made a special sale, and marked his goods to sell at $\frac{2}{3}$ of the regular prices. The regular price of gingham was 27¢ a yard. At what price did he sell it?

14. For how much did he sell a ball in the toy department, of which the regular price was 15¢?

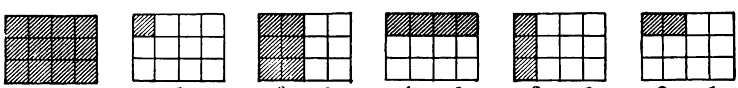
15. He reduced the price of some goods a second time $\frac{1}{6}$ of the original price. How much were they reduced all together?

10. NAMING THE TERMS OF A FRACTION

1. Draw a line and show $\frac{1}{2}$ of it. $\frac{3}{4}$ of it. $\frac{5}{8}$ of it. $\frac{5}{6}$ of it.

2. How would you show $\frac{5}{6}$ of a rectangle? How would you show $\frac{1}{12}$ of it?

3. Draw a circle. How many fourths in it? Erase $\frac{1}{4}$ of it. How many fourths remain? How do you write three fourths in figures?



1 or $\frac{12}{12}$ $\frac{1}{12}$ $\frac{6}{12}$ or $\frac{1}{2}$ $\frac{4}{12}$ or $\frac{1}{3}$ $\frac{3}{12}$ or $\frac{1}{4}$ $\frac{2}{12}$ or $\frac{1}{6}$

4. What parts of the whole oblongs are shaded?

5. Which is the largest part shaded? The smallest?

6. Compare $\frac{1}{3}$ of the oblong with $\frac{1}{2}$ of it. Which is the larger? How much larger? $\frac{1}{2} = \frac{6}{12}$; $\frac{1}{3} = \frac{4}{12}$; $\frac{6}{12} - \frac{4}{12}$ = ——.

7. Compare $\frac{1}{3}$ and $\frac{1}{4}$ in the same way. Compare $\frac{1}{3}$ and $\frac{1}{6}$.

8. How many numbers are required to express a fraction?

The two numbers needed to express a fraction are called its **terms.**

9. Into how many parts is each oblong divided in the illustration?

10. In the fraction $\frac{6}{12}$, which term shows this fact, 6 or 12? Which term shows the *name* of the part?

11. In $\frac{6}{12}$ how many 12ths? Does the upper or lower term show this?

The lower term of a fraction shows into **how many** *equal parts the whole thing has been divided. It names the equal parts, shows their* **size**, *and is the* **denominator.**

The upper term shows the **number** *of equal parts in a fraction, and is the* **numerator.**

11. LIKE FRACTIONS: ADDITION

1. Draw an oblong and divide it into halves.

2. Draw another and divide it into thirds.

3. In this way draw *fourths*, *sixths*, and *twelfths*.

4. In the last you have drawn, show your teacher $\frac{1}{2}$ of it; $\frac{1}{3}$ of it; $\frac{1}{4}$ of it; $\frac{1}{6}$ of it; $\frac{1}{12}$ of it.

5. How many of the small squares, or 12ths, make $\frac{1}{2}$? How many make $\frac{1}{3}$? How many make $\frac{1}{4}$? $\frac{1}{6}$?

6. $\frac{1}{2} = \frac{?}{12}$; $\frac{1}{3} = \frac{?}{12}$; $\frac{1}{4} = \frac{?}{12}$; $\frac{1}{6} = \frac{?}{12}$.

7. Compare $\frac{1}{3}$ and $\frac{1}{12}$; $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{1}{2}$ and $\frac{1}{12}$; $\frac{1}{6}$ and $\frac{1}{12}$.

8. Can you add *feet and inches* without changing to *like units;* that is, to units of the same size?

9. Before adding $\frac{1}{2}$ and $\frac{1}{4}$, we must think of the $\frac{1}{2}$ as —— fourths. $\frac{2}{4} + \frac{1}{4} = \frac{?}{4}$.

> *Fractions whose units are alike are* **like fractions.**

10. Are $\frac{5}{12}$ and $\frac{11}{12}$ like fractions? Are $\frac{1}{3}$ and $\frac{3}{4}$?

11. Add $\frac{1}{2}$ and $\frac{1}{12}$. **12.** Add $\frac{1}{4}$ and $\frac{1}{12}$. **13.** Add $\frac{1}{3}$ and $\frac{1}{12}$.

14. Which is larger, $\frac{1}{2}$ or $\frac{6}{12}$? Prove it.

15. $\frac{2}{3}$ of 12 is ——; $\frac{2}{3}$ of a foot = —— in.; $\frac{8}{12}$ of a foot is —— in. Compare $\frac{2}{3}$ and $\frac{8}{12}$.

> *When fractions are changed to like units, they have a* **common denominator.** *That is, their denominators are alike.*

Add:

16. $\frac{1}{4} + \frac{3}{8} + \frac{1}{2}$.	**19.** $\frac{3}{4} + \frac{7}{8} + \frac{1}{2}$.	**22.** $\frac{4}{5} + \frac{2}{5} + \frac{7}{10}$.
17. $\frac{1}{3} + \frac{5}{6} + \frac{7}{12}$.	**20.** $\frac{5}{8} + \frac{1}{2} + \frac{1}{4}$.	**23.** $\frac{3}{4} + \frac{7}{12} + \frac{1}{4}$.
18. $\frac{11}{12} + \frac{5}{6} + \frac{2}{3}$.	**21.** $\frac{1}{6} + \frac{1}{3} + \frac{11}{12}$.	**24.** $\frac{1}{4} + \frac{7}{8} + \frac{1}{2}$.

25. Elizabeth practices her violin lesson $\frac{1}{2}$ an hour before school, $\frac{1}{4}$ of an hour during the intermission, and $\frac{1}{3}$ of an hour after school. How much more than an hour does she practice in all?

26. If you cut $\frac{2}{3}$ of a yard of ribbon from a piece $\frac{3}{4}$ of a yard long, what part of a yard remains?

Exercises in Fractions

1. By looking at your 12-inch rule, fill the blanks in the following:

$\frac{1}{2}=\frac{?}{12}$; $\frac{1}{3}=\frac{?}{12}$; $\frac{1}{4}=\frac{?}{12}$; $\frac{1}{6}=\frac{?}{12}$; $\frac{5}{6}=\frac{?}{12}$;
$\frac{3}{4}=\frac{?}{12}$; $\frac{5}{6}=\frac{?}{12}$; $\frac{2}{3}=\frac{?}{12}$.

2. By looking at your rule, find:

9 in. – 7 in. = ——; $\frac{9}{12}-\frac{7}{12}=$ ——; $\frac{8}{12}-\frac{5}{12}=$ ——;
$\frac{11}{12}-\frac{6}{12}=$ ——; $\frac{7}{12}-\frac{1}{2}=$ ——.

3. By looking at your rule, find:

$\frac{5}{12}+\frac{1}{12}=$ ——; $\frac{3}{12}+\frac{7}{12}=$ ——; $\frac{5}{12}+\frac{4}{12}=$ ——;
$\frac{7}{12}-\frac{1}{3}=$ ——; $\frac{11}{12}-\frac{5}{6}=$ ——.

4. To get sixths from thirds, each third is divided into —— parts. Hence $\frac{2}{3}=\frac{?}{6}$.

5. $\frac{1}{2}=\frac{?}{4}=\frac{?}{6}=\frac{?}{8}=\frac{?}{12}$.

6. $\frac{1}{3}=\frac{?}{6}=\frac{?}{9}=\frac{?}{12}$.

7. $\frac{5}{6}=\frac{?}{12}$; $\frac{3}{4}=\frac{?}{8}$; $\frac{2}{3}=\frac{?}{12}=\frac{?}{9}=\frac{?}{6}$.

8. $\frac{1}{2}+\frac{1}{6}=\frac{?}{12}+\frac{?}{12}=$ ——; $\frac{2}{3}+\frac{1}{6}=\frac{?}{6}+\frac{1}{6}=$ ——.

9. $\frac{1}{2}+\frac{3}{16}=\frac{?}{16}+\frac{3}{16}=$ ——; $\frac{1}{4}+\frac{1}{2}+\frac{5}{16}=\frac{?}{16}+\frac{?}{16}+\frac{5}{16}=\frac{?}{16}$.

10. Compare $\frac{1}{2}$ and $\frac{1}{4}$; thus, $\frac{1}{2}$ is 2 times $\frac{1}{4}$; $\frac{1}{4}$ is —— of $\frac{1}{2}$.

In this way compare:

11.	12.	13.
$\frac{1}{2}$ and $\frac{1}{8}$.	$\frac{1}{3}$ and $\frac{1}{9}$.	$\frac{1}{6}$ and $\frac{1}{12}$.
$\frac{1}{4}$ and $\frac{1}{8}$.	$\frac{1}{3}$ and $\frac{1}{6}$.	$\frac{1}{2}$ and $\frac{1}{12}$.
$\frac{1}{4}$ and $\frac{1}{12}$.	$\frac{1}{3}$ and $\frac{1}{12}$.	$\frac{1}{2}$ and $\frac{1}{6}$.
$\frac{1}{4}$ and $\frac{1}{16}$.	$\frac{1}{8}$ and $\frac{1}{16}$.	$\frac{1}{2}$ and $\frac{1}{16}$.

Add:

14. $\frac{1}{4}+\frac{1}{2}+\frac{1}{8}$.
15. $\frac{3}{4}+\frac{1}{2}+\frac{3}{8}$.
16. $\frac{1}{2}+\frac{7}{8}+\frac{3}{4}$.
17. $\frac{3}{8}+\frac{1}{2}+\frac{1}{4}+\frac{5}{8}$.
18. $\frac{1}{2}+\frac{3}{4}+\frac{7}{8}+\frac{5}{8}$.

Subtract:

19. $\frac{3}{4}-\frac{1}{2}$.
20. $\frac{1}{2}-\frac{3}{8}$.
21. $\frac{3}{4}-\frac{5}{8}$.
22. $\frac{7}{8}-\frac{3}{4}$.
23. $\frac{3}{4}-\frac{1}{8}$.

Add:

24. $\frac{1}{6}+\frac{1}{3}$.
25. $\frac{2}{3}+\frac{5}{6}$.
26. $\frac{1}{12}+\frac{2}{3}$.
27. $\frac{3}{4}+\frac{5}{12}$.
28. $\frac{5}{6}+\frac{3}{4}$.

Add:

29. $\frac{1}{2}+\frac{1}{4}+\frac{1}{16}$.
30. $\frac{3}{4}+\frac{5}{16}+\frac{1}{2}$.
31. $\frac{3}{8}+\frac{3}{16}+\frac{1}{4}$.
32. $\frac{5}{8}+\frac{5}{16}+\frac{1}{2}$.
33. $\frac{3}{4}+\frac{3}{8}+\frac{7}{16}$.
34. $\frac{11}{16}+\frac{7}{8}+\frac{1}{2}$.

Practical Problems in Fractions

1. Mrs. Brown and her daughter went shopping. At the lace remnant counter they bought a piece of lace containing $\frac{7}{8}$ yd., another containing $\frac{3}{4}$ yd., and another containing $1\frac{1}{2}$ yd. How much did they get in all?

2. They paid 16¢ for one piece, 15¢ for one, and 27¢ for the other. Mrs. Brown gave the clerk a dollar bill. How much change did she receive?

3. Then they went to the ribbon remnant counter. There they got one piece of ribbon containing $1\frac{1}{4}$ yd., one piece containing $\frac{11}{12}$ yd., one containing $\frac{5}{6}$ yd., and one containing $\frac{2}{3}$ yd. How much ribbon did they get in all?

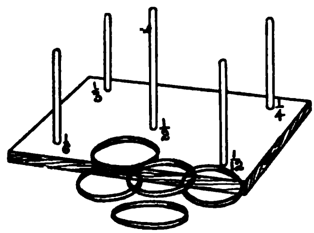
4. From there they went to the silk remnant counter, where they bought one piece containing $1\frac{1}{3}$ yd. But they

wanted $4\frac{1}{2}$ yd. of the one kind, so they got the rest at the regular silk counter. How much did they ask for there?

5. In the grocery department they ordered 6 bars of soap at $3\frac{1}{3}$ cents a bar, 9 cans of corn at $16\frac{2}{3}$ cents a can, $2\frac{1}{2}$ lb. of sweet potatoes at 6 cents a pound, and 8 pounds of sugar at $5\frac{1}{4}$ cents a pound. How much did they spend in the grocery department?

6. When they started to town, Mrs. Brown had $12 in her purse, and when they got home she had $1.85. How much did they spend that day?

7. In this ring-toss game there are five pins and five rings. Each player gets five throws. If the ring falls on the middle pin, it counts $\frac{1}{2}$, and on the other pins $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{6}$, and $\frac{1}{12}$, respectively. The rule is to add all of the counts that the player makes with the five rings. The one getting the highest score wins.



If you throw $\frac{1}{2}$, 0, $\frac{1}{6}$, $\frac{1}{12}$, and 0, what is your score?

8. If you throw $\frac{1}{2}$, $\frac{1}{2}$, $\frac{1}{3}$, 0, and $\frac{1}{6}$, what is your score?

9. If you throw $\frac{1}{12}$, 0, 0, $\frac{1}{6}$, and $\frac{1}{4}$, what is your score?

10. Robert and Harry make scores as follows. Who wins?

Robert, $\frac{1}{6}$, 0, $\frac{1}{2}$, $\frac{1}{3}$, 0
Harry, $\frac{1}{3}$, $\frac{1}{4}$, 0, $\frac{1}{12}$, $\frac{1}{6}$

11. Charles makes the largest score possible. How much does he score?

12. Give, without pencil, the scores as your teacher states the counts.

13. Paul mailed three Christmas packages. They weighed $\frac{1}{2}$ lb., $\frac{3}{8}$ lb., and $\frac{1}{4}$ lb. What was their total weight? How much was the postage on them at 16¢ a pound?

14. I mailed a package, weight $1\frac{1}{4}$ lb., at 32 ¢ a pound, and another weighing $1\frac{11}{16}$ lb. at 16¢ a pound. I could have sent them by express for 50¢. How much more did it cost me to mail them?

15. If the rainfall is $\frac{1}{4}$ inch Monday, $\frac{7}{8}$ inch Tuesday, $\frac{3}{4}$ inch Wednesday, $\frac{1}{2}$ inch Saturday, and $\frac{5}{8}$ inch Sunday, what is the total rainfall during the week?

16. How much greater is the rainfall Tuesday than Wednesday?

12. MIXED NUMBERS

1. How many 4ths in anything? How many 6ths? How many 8ths? How many 12ths?

2. How much more than 1 are $\frac{5}{4}$? How much more than 1 are $\frac{11}{8}$? How much more than 2 are $\frac{9}{4}$? How much more than 2 are $\frac{11}{5}$?

> *We see that dividing the numerator of the fraction by the denominator gives the number of whole things, and the remainder is the number of fractional units that are left.*

A number, such as $2\frac{1}{3}$, composed partly of a whole number and partly of a fraction, is a **mixed number.**

3. Change $\frac{12}{5}$ to a mixed number.

$\frac{12}{5} \div \frac{5}{5}$ or $12 \div 5 = 2\frac{2}{5}$. In practice, we say, $12 \div 5 = 2\frac{2}{5}$.

We prove our work by changing the mixed number back to a fraction. Thus

$$1 = \frac{5}{5};\ 2 = 2 \times \frac{5}{5},\ \text{or}\ \frac{10}{5};\ \frac{10}{5} + \frac{2}{5} = \frac{12}{5}.$$

Change to whole or mixed numbers:

4. $\frac{8}{5}$.	**9.** $\frac{21}{4}$.	**14.** $\frac{19}{6}$.	**19.** $\frac{43}{8}$.	**24.** $\frac{17}{6}$.
5. $\frac{9}{4}$.	**10.** $\frac{26}{8}$.	**15.** $\frac{34}{5}$.	**20.** $\frac{54}{9}$.	**25.** $\frac{49}{8}$.
6. $\frac{6}{2}$.	**11.** $\frac{34}{12}$.	**16.** $\frac{27}{9}$.	**21.** $\frac{61}{8}$.	**26.** $\frac{51}{9}$.
7. $\frac{13}{4}$.	**12.** $\frac{21}{6}$.	**17.** $\frac{32}{8}$.	**22.** $\frac{43}{6}$.	**27.** $\frac{84}{12}$.
8. $\frac{16}{8}$.	**13.** $\frac{17}{5}$.	**18.** $\frac{48}{12}$.	**23.** $\frac{28}{5}$.	**28.** $\frac{68}{9}$.

29. Give the sum of $\frac{1}{3}$, $\frac{3}{4}$, $\frac{5}{6}$, and $\frac{7}{12}$ as a mixed number.

13. FRACTIONAL PARTS OF WHOLE NUMBERS

1. What is $\frac{1}{4}$ of 16? Since $\frac{3}{4}$ is three times as much as $\frac{1}{4}$, $\frac{3}{4}$ of $16 = 3 \times$ ——, or ——.

2. We see, then, that to find a fractional part of a number we must both divide and multiply. By which term do we divide? By which, then, must we multiply?

Copy and fill blanks:

3. $\frac{3}{4}$ of $12 =$ ——.	**11.** $\frac{3}{4}$ of $36 =$ ——.
4. $\frac{2}{3}$ of $27 =$ ——.	**12.** $\frac{2}{3}$ of $21 =$ ——.
5. $\frac{3}{5}$ of $35 =$ ——.	**13.** $\frac{4}{5}$ of $25 =$ ——.
6. $\frac{4}{5}$ of $30 =$ ——.	**14.** $\frac{3}{4}$ of $20 =$ ——.
7. $\frac{2}{3}$ of $18 =$ ——.	**15.** $\frac{2}{3}$ of $12 =$ ——.
8. $\frac{2}{4}$ of $28 =$ ——.	**16.** $\frac{3}{4}$ of $24 =$ ——.
9. $\frac{3}{5}$ of $40 =$ ——.	**17.** $\frac{2}{5}$ of $40 =$ ——.
10. $\frac{4}{5}$ of $45 =$ ——.	**18.** $\frac{5}{6}$ of $48 =$ ——.

19. $2\frac{1}{2} \times \$8$ means $2 \times \$8 + \frac{1}{2}$ of $\$8$, or $\$16 + \$4$.

20. $3\frac{3}{4} \times 20$ ft. $= 3 \times 20$ ft. $+ \frac{3}{4}$ of 20 ft. $=$ ——.

Find:

21. $2\frac{1}{2} \times 10$ ft.

22. $1\frac{1}{8} \times 12$ yards.

23. $3\frac{1}{2} \times \$4$.

24. $2\frac{1}{4} \times 16$ bushels.

25. $1\frac{1}{8} \times 24$ men.

26. $4\frac{1}{2} \times \$10$.

27. $3\frac{1}{6} \times 12$ pounds.

28. $4\frac{1}{3} \times 9$ quarts.

29. $6\frac{1}{2} \times 8$ quarts.

30. $7\frac{1}{2} \times 10$ cents.

31. $6\frac{1}{2} \times 12$¢.

32. $5\frac{1}{3} \times 15$¢.

33. $4\frac{1}{2} \times 10$ quarts.

34. $6\frac{1}{3} \times 6$ feet.

35. $9\frac{1}{4} \times \$8$.

36. $7\frac{1}{5} \times 15$ yd.

14. MULTIPLYING BY MIXED NUMBERS

1. A farmer sold $8\frac{3}{4}$ acres of land at $196 an acre. How much did he receive for it?

We are to find $8\frac{3}{4} \times \$196$.

WORK		WORK
$\begin{array}{r} \$196 \\ 8\frac{3}{4} \\ \hline 49 \\ 3 \\ \hline 147 \\ 1568 \\ \hline \$1715 \end{array}$	EXPLANATION. — $\frac{1}{4}$ of $196 = 49$; $\frac{3}{4}$ of $196 = 3 \times 49 = 147$; $8 \times 196 = 1568$; $1568 + 147 = 1715$. So he receives $1715. Or we may multiply by 3 first and then divide by 4, as in the work at the right. The second method is generally the better.	$\begin{array}{r} \$196 \\ 8\frac{3}{4} \\ \hline 4\overline{)588} \\ \hline 147 \\ 1568 \\ \hline \$1715 \end{array}$

In this way find:

2. $7\frac{2}{5} \times 125$.

3. $9\frac{2}{3} \times 741$.

4. $5\frac{4}{5} \times 945$.

5. $8\frac{3}{4} \times 624$.

6. $6\frac{3}{8} \times 728$.

7. $12\frac{1}{2} \times \$894$.

8. $32\frac{3}{4} \times 532$ ft.

9. $16\frac{2}{3} \times 726$ yd.

10. $42\frac{3}{5} \times 795$.

11. There are $16\frac{1}{2}$ ft. in a rod. How many feet in a mile, or 320 rods?

12. How much will $17\frac{3}{4}$ yards of linen cost at 48¢ a yard?

Find the entire cost of the following:

13. $7\frac{1}{2}$ lb. sugar @ 6¢.
$\frac{3}{4}$ lb. tea @ 60¢.
$2\frac{1}{2}$ lb. coffee @ 48¢.

14. $2\frac{3}{4}$ yd. gingham @ 20¢.
$4\frac{1}{2}$ yd. braid @ 12¢.
$\frac{1}{2}$ doz. buttons @ 30¢.

15. The owner of a house sold it for $4816, and received $\frac{3}{4}$ of the money at the time of the sale. How much more was due?

15. BILLS

1. Check the following bill. That is, see whether it is correct.

CHICAGO, Jan. 26, 1911.

Mrs. Chas. Brenner,
513 East 47th St.,

To CLARK & THOMPSON, Dr.

DEALERS IN FANCY GROCERIES, 426 EAST 49TH ST.

Jan.	10	4 cans tomatoes	@ 12¢	48		
		2 1/2 lb. butter	@ 32¢	80		
		2 lb. coffee	@ 38¢	76	2	04
Jan.	12	3 cans soup	@ 20¢	60		
		2 1/4 lb. cheese	@ 24¢	54	1	14
					3	18
		Rec'd payment, Clark & Thompson.				

2. Look at the bill and give the *dates of the purchases*.

3. Who was the *buyer?*

4. Give the name of the *seller* or the *firm*.

Rule paper, make out and receipt the following bills, using different names for buyer and seller:

5. Feb. 5, $3\frac{3}{4}$ lb. meat at 16¢; Feb. 6, $8\frac{1}{4}$ lb. lard at 16¢, and $3\frac{1}{2}$ lb. steak at 22¢; Feb. 7, $4\frac{3}{4}$ lb. chicken at 24¢, and $2\frac{1}{4}$ lb. sausage at 12¢.

6. March 10, $3\frac{1}{4}$ lb. butter at 32¢, and $\frac{3}{4}$ lb. tea at 60¢; March 12, 2 heads lettuce at 6¢, $4\frac{1}{2}$ lb. raisins at 14¢, and $3\frac{1}{2}$ doz. eggs at 34¢.

7. Apr. 4, $\frac{3}{4}$ doz. grapefruit at 80¢, $2\frac{1}{2}$ doz. oranges at 40¢, and $1\frac{1}{2}$ doz. bananas at 20¢; Apr. 7, 2 pineapples at 18¢ each, $1\frac{3}{4}$ doz. oranges at 48¢, and $\frac{1}{2}$ doz. grapefruit at 90¢.

8. Dec. 10, $6\frac{3}{4}$ lb. roast beef at 16¢, $2\frac{1}{2}$ lb. sausage at 14¢; Dec. 12, $5\frac{1}{4}$ lb. chicken at 20¢, and $3\frac{1}{2}$ lb. steak at 22¢.

9. May 5, $8\frac{1}{2}$ yd. flannel at 96¢, and $4\frac{3}{4}$ yd. braid at 16¢; May 10, 12 yd. embroidery at $22\frac{1}{2}$¢, and 10 yd. lace at $27\frac{1}{2}$¢.

10. July 16, $1\frac{3}{4}$ doz. green corn at 20¢, 2 cantaloupes at $12\frac{1}{2}$¢, and $\frac{1}{2}$ doz. peaches at 40¢; July 24, 2 melons at 55¢, $1\frac{1}{2}$ doz. bananas at 30¢, and $2\frac{1}{2}$ doz. pears at 30¢.

Problems in the Dry Goods Business

1. What articles are bought and sold by the yard?

2. How many feet make a yard? How many inches?

3. If you want a piece of ribbon 18 inches long, what part of a yard would you call for at the counter?

4. How many inches make $\frac{1}{4}$ of a yard? $\frac{3}{4}$ of a yard?

5. Can you tell how many inches in $\frac{1}{8}$ of a yard?

6. If the ribbon I wish to buy costs 48¢ a yard, how much shall I pay for $1\frac{1}{4}$ yards?

7. Mrs. Edna Jones bought the following articles of Marshall Field & Co. Make and receipt a bill and find the whole cost.

$7\frac{1}{2}$ yd. Scotch gingham	@ 18¢
$2\frac{1}{4}$ yd. cambric	@ 12¢
6 yd. trimming	@ $4\frac{1}{2}$¢
$\frac{3}{4}$ doz. towels	@ $3

8. Edward Henderson bought the following articles of Mandel Bros. Make and receipt a bill of them.

$\frac{1}{2}$ doz. pillowcases 45″ by 33″	@ 25¢ pr.
4 hemstitched sheets 72″ by 88″	@ $1.75 pr.
2 pr. hemstitched sheets 80″ by 99″	@ $3.00 pr.
2 pr. Saxony blankets	@ $5.50 pr.
1 satin bedspread	@ $2.83

9. Mrs. Mills bought of "The Fair" the following articles. What was the amount of her bill?

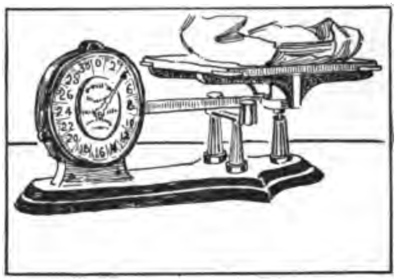
2 linen tablecloths	@ $3.00
18 napkins	@ $2.40 doz.
3 pr. Nottingham lace curtains	@ $1.50
3 window poles and trimmings	@ $0.25

10. How many skate bags can be made from 4 yards of denim if $\frac{3}{8}$ of a yard will make one bag? What will be the cost of the material for each bag if the denim is 24¢ a yard, and the tape costs 2¢?

11. How many yards of ribbon will be required to tie 48 diplomas if $\frac{3}{4}$ of a yard is used on each end of each diploma?

12. What will be the cost of $15\frac{3}{4}$ yards of tapestry carpet at 80¢ a yard?

Problems in a Grocer's Business



1. What articles are bought or sold by pounds and ounces?

2. What articles are bought and sold by the bushel? By the peck? By the quart? By the gallon? By the pint?

3. A grocer's counter scales show pounds and ounces. How many ounces make a pound?

4. 1 ounce is what part of a pound?

5. How many ounces in $\frac{1}{8}$ of a pound?

6. What part of a pound in 4 ounces?

7. 8 ounces are what part of a pound?

8. How many ounces in $\frac{3}{4}$ of a pound?

9. 14 ounces are how many eighths of a pound?

10. How much must you pay for $1\frac{1}{4}$ pounds of cheese at 20 ¢ a pound?

11. How much will the grocer charge you for 1 lb. 4 oz. of porterhouse steak at 32 ¢ a pound?

12. 5 pounds of spice are wrapped in packages containing 2 ounces each. How many packages are there?

13. Find the cost of the following articles:

$2\frac{3}{4}$ lb. print butter	@ 36 ¢
12 lb. granulated sugar	@ $5\frac{1}{2}$ ¢
4 cans peas	@ $12\frac{1}{2}$ ¢

14. Make and receipt a bill for the articles in Exercise 13.

15. Make a bill and find the cost of the following articles:

$4\frac{1}{2}$ lb. fowl	@ 20¢	per lb.
3 pk. potatoes	@ 80¢	per bu.
9 oranges	@ 40¢	per doz.

16. How much must Mrs. Wright pay for $1\frac{3}{4}$ lb. of tea at 64¢ a pound?

17. How much will $\frac{3}{8}$ of a bushel of apples cost at $1.20 a bushel?

18. There are 196 pounds in a barrel of flour. Which is cheaper, and how much, to buy a barrel for $7, or to buy the same quantity put up in bags at 90¢ a bag? A bag of flour weighs $24\frac{1}{2}$ lb.

Problems involving Comparison of Fractions

1. $\frac{1}{2}$ bbl. of flour weighs 98 pounds. What does $\frac{1}{4}$ of a barrel weigh? What do $1\frac{1}{2}$ barrels weigh?

2. When $\frac{1}{4}$ ton of hay is worth $5.25, how much must I pay for $\frac{1}{2}$ ton? How much for 1 ton?

3. When $\frac{1}{8}$ of a farm is worth $3200, how much is $\frac{1}{2}$ of it worth? ($\frac{1}{2}$ is how many times as large as $\frac{1}{8}$?)

4. A farmer sold 20 bushels of potatoes. This was $\frac{1}{4}$ of all he raised. How many did he keep?

5. When onions are 35¢ per peck, how much will 6 bushels cost?

6. When a 20-pound cheese is worth $1.90, how much will a 10-lb. cheese cost? Find from your answer the cost of a 30-lb. cheese. The cost of a 90-lb. cheese.

7. A man sold 54 acres, which was $\frac{1}{6}$ of his farm. How many acres remained?

HINT. — How many 6ths remained? Compare $\frac{5}{6}$ with $\frac{1}{6}$.

8. After walking $\frac{1}{2}$ the distance to school, a boy has 145 rods yet to go. How many rods from school does he live?

9. When $\frac{1}{3}$ of a ton of coal costs \$2.75, how much will a ton cost? How much will 15 tons cost?

10. When $\frac{1}{6}$ of a barrel of apples is worth 85¢, how much will 12 barrels of apples be worth?

11. A farmer sold 43 bushels of potatoes. If this was $\frac{1}{8}$ of all he raised, how many did he keep?

Review Problems involving Fractions

1. How many inches in $\frac{3}{4}$ of a foot?

2. What part of a bushel is a peck?

3. John jumps $5\frac{1}{4}$ feet. Edward's jump measures $\frac{1}{2}$ foot less. How far does Edward jump?

4. A child should sleep $\frac{1}{3}$ of his time. If he does this, how many hours in a day will he be awake?

5. A single jar holds $\frac{3}{4}$ of a pint. How much will 12 such jars hold?

6. What will $2\frac{1}{2}$ yards of ribbon cost at 16¢ a yard?

7. A sea captain, sixty years old, has spent $\frac{3}{4}$ of his life on the sea. How many years has he spent on the land?

8. Eliza has \$$\frac{1}{2}$, \$$\frac{1}{4}$, and \$$\frac{1}{10}$. How many cents has she in all?

9. What shall I pay for $\frac{3}{4}$ of a dozen eggs when two dozen are sold for \$0.72?

10. How much will $\frac{1}{4}$ of a pound of 60-cent tea and $\frac{1}{2}$ pound of 40-cent coffee cost?

11. What part of an hour is 40 minutes?

12. How many inches in $\frac{1}{4}$ of a yard?

13. Find $\frac{7}{8}$ of 56.

14. How much will $\frac{5}{8}$ of a pound cost when one pound costs 64¢?

15. Take $\frac{2}{3}$ of 24 from $\frac{5}{6}$ of 30.

16. Gladys has read all but $\frac{1}{8}$ of a book containing 72 pages. How many pages has she still to read?

17. What does a man earn in a week (6 days) at \$$2\frac{1}{2}$ a day?

18. A cow gives $3\frac{1}{2}$ quarts of milk in a day. How much will she give in a week?

19. Add:

$\frac{1}{2}+\frac{1}{4}=$; $\frac{2}{3}+\frac{1}{6}=$; $\frac{3}{4}+\frac{1}{8}=$; $\frac{2}{3}+\frac{1}{12}=$; $\frac{2}{3}+\frac{5}{9}=$.

20. Subtract:

$\frac{7}{12}-\frac{1}{4}=$; $\frac{5}{9}-\frac{1}{3}=$; $\frac{5}{6}-\frac{1}{2}=$; $\frac{11}{12}-\frac{2}{3}=$; $\frac{1}{2}-\frac{3}{8}=$.

21. A man went to China when he was 18 years old, and returned at the age of 45. What part of his life was spent in China?

22. Mr. Lane was to have $\frac{1}{8}$ of all he dug to pay him for digging my crop of potatoes. If his share was 8 bushels, what was mine?

23. Jennie is $\frac{1}{7}$ as old as her mother. If Jennie is $7\frac{1}{2}$ years old, how old is her mother?

24. I leave home at half-past seven and travel $6\frac{3}{4}$ hours. At what hour do I reach the end of my journey?

25. A farmer paid \$$3\frac{3}{5}$ for 4 bushels of seed wheat. How much did he pay for a bushel?

26. What part of a day is 15 hours?

II. WHOLE NUMBERS

16. NOTATION AND NUMERATION

1. Name the ten figures used in writing numbers. How do we write numbers larger than 9?

> *The nine* **figures,** 1, 2, 3, 4, 5, 6, 7, 8, 9, *are sometimes called* **digits.** 0 *is called* **zero, cipher,** *or* **naught.**
>
> *The right-hand figure of a whole number is called a* **figure of the first order**; *the next at the left of it, a* **figure of the second order**; *and so on for other orders.*

2. In the number 3056, name the figure in the first order. Name the one in the second order.

3. What does the *zero* in the third order show?

> *Writing numbers according to any system is called* **notation.**
>
> *Reading numbers that have been written according to any system of notation is called* **numeration.**
>
> *Our system of notation is the* **Hindu notation,** *because it was invented by the Hindus.*

4. In 235, what does the 5 stand for? The 3? The 2? 235 = 2 hundreds + 3 —— + ——.

5. What is the value of each figure in the following:

385; 2963: 3063; 17,842?

6. What is the value of each 5 in 555? Compare the value of each with the next one at the left.

7. How many *ones* make 1 *ten?* How many *tens* make 1 *hundred?*

8. The **units** in the first place at the right, or lowest **order**, are **ones**; the **units** in the second order are **tens**; the **units** in the third order are **hundreds**; those in the fourth order are **thousands.**

Since ten units of any order make one unit of the next higher order, our system of writing numbers is called a **decimal system.**

Decimal comes from a word meaning *ten.*

9. What does the zero show in each of the following:
302; 1053; 260; 3078?

10. Could we write such numbers as three hundred six if we had no zero?

11. Write in figures twenty-five thousand, two hundred.

12. Write one hundred nine thousand, three hundred three.

13. Write eighty-six thousand, forty-three.

14. Write seven hundred seven thousand, three hundred nine.

15. Write one hundred nine thousand, three.

16. Write the population of these cities in figures:
South Bend, fifty-three thousand, six hundred eighty-four;
Davenport, forty-three thousand, twenty-eight.

17. READING AND WRITING MILLIONS

1. Are larger numbers than thousands ever needed? For what?

> *Large numbers are separated into* **groups of three figures each,** *beginning at the right. The first three make* **ones'** *group, the next three* **thousands'** *group, and the next* **millions'** *group.*
>
> *Each group is read as if it stood alone, and then the name of the group is added. Do not use* **and** *in reading whole numbers.*

2. 35,567,370 is read:

35 million, 567 thousand, 370.

NOTE. — The group name is not used in reading *ones.*

3. Read the population of each of the 3 largest cities of the United States in 1910:

New York, 4,766,883; Chicago, 2,185,283; Philadelphia, 1,549,008.

4. The following is the population of each of the six greatest cities of the world. Read them:

London (1908), 7,323,327; New York, 4,766,883; Paris (1906), 2,763,393; Chicago, 2,185,283; Berlin (1908), 2,048,000; Tokio (1908), 2,040,148.

5. The distance from the earth to the sun is 92,900,000 miles. Read this distance.

Write in figures:

6. Five million, three hundred five thousand, five hundred one.

7. Twenty-eight million, fifty-seven thousand, six hundred.

8. Two hundred five million, thirty-six thousand, five hundred seventy.

9. Eighteen million, three hundred fifty-nine.

10. Six million, six thousand, six.

Write in figures the numbers used in the following statements:

11. The mineral production of the United States in 1907 included

Two hundred ninety-five million tons of bituminous and eighty-five million tons of anthracite coal;

One hundred sixty-six million barrels of petroleum;

Forty-five million tons of high-grade and eleven million tons of low-grade iron ore;

Two million, five hundred thousand tons of phosphate rock, and

Eight hundred sixty-nine million tons of copper.

12. The industries of this country that subsist wholly or mainly upon wood pay the wages of more than one million, five hundred thousand men and women.

Our forests now cover five hundred fifty million acres, or about one fourth of the United States.

Since 1870, forest fires have destroyed a yearly average of fifty lives and fifty million dollars' worth of timber.

Not less than fifty million acres of forest is burned over yearly.

13. According to the census of 1910 the population of Massachusetts is three million, three hundred sixty-six thousand, four hundred sixteen.

18. ADDING TWO NUMBERS OF TWO DIGITS EACH

One often wishes to know the sum of two numbers of two digits each without a pencil. Thus, if I buy 48 ¢ worth of steak and 26 ¢ worth of sausage, I should see without a pencil that in all I must pay 74 ¢.

To add 43 and 39 it is easier to add 30 to 43, then add 9 to the 73. To do this well requires much practice in *two special* drills before the general drill.

TABLE I

	A	B	C	D	E	F	G	H	I	J
1.	65 30	27 30	67 40	36 10	85 80	83 40	12 90	87 60	28 80	13 10
2.	37 70	48 50	95 90	63 10	78 40	47 50	36 30	54 60	92 60	56 50
3.	68 30	47 20	87 90	83 70	69 40	18 40	91 70	24 20	52 10	66 80
4.	96 30	97 30	16 70	28 60	87 30	79 20	85 50	96 80	76 70	87 70
5.	28 90	29 50	55 70	49 40	63 80	56 30	14 20	43 80	78 50	68 60

TABLE II

	A	B	C	D	E	F	G	H	I	J
1.	72	87	64	75	58	83	89	37	86	14
	8	9	2	5	3	4	1	1	9	4
2.	61	64	98	39	16	32	76	79	45	47
	4	7	8	9	1	9	6	7	1	7
3.	47	79	84	87	48	48	56	63	36	73
	3	8	5	5	1	7	7	2	8	1
4.	59	36	42	99	64	95	46	78	99	32
	8	3	7	3	6	3	5	5	5	2
5.	92	94	39	63	91	64	22	92	68	26
	6	8	6	3	1	9	1	5	7	8

Problems

1. Mary purchased 2 lb. of meat for her mother for 48 ¢ and some vegetables for 35 ¢. How much did she spend?

2. Frank sold 36 papers one evening and 53 the next. How many did he sell in the two evenings?

3. Alice picked 43 pansies from one bed and 26 from another. How many did she pick in all?

When Tables I and II can be given rapidly and accurately, drill upon the following:

TABLE III

	A	B	C	D	E	F	G	H	I	J
1.	81 81	14 64	35 38	22 72	55 59	18 39	16 26	37 89	92 79	31 55
2.	34 45	47 48	81 52	65 67	14 98	63 85	92 88	73 39	51 44	91 66
3.	24 99	83 23	52 64	92 93	85 76	23 21	63 26	96 57	71 77	53 18
4.	67 47	72 15	93 34	25 35	41 79	96 49	46 87	91 68	58 97	66 78
5.	48 28	34 67	14 46	12 86	62 87	79 59	73 97	78 56	29 54	85 79

Give the sums from dictation as well as from sight.

Problems

1. Henry paid 45 ¢ for a knife and 75 ¢ for a book. How much did the two cost?

2. If there are 34 people on one street car and 48 on another, how many are there on both?

3. 45 cars pass a corner between 6 A.M. and 6 P.M., and 27 cars during the rest of the day. How many cars pass the corner in a whole day?

4. 68 trains arrive at a certain station in the morning and 53 in the afternoon. How many in all?

5. A freight train contains 18 box cars and 26 flat cars. How many cars of both kinds in the train?

6. If a freight train contains 19 empty cars and 24 loaded cars, how many cars in the train?

7. One car contains 34 tons of coal and another 37 tons. How many tons in the two cars?

8. It is 85 miles from Chicago to Ottawa, and 14 miles from Ottawa to La Salle. How far is it from Chicago to La Salle?

9. It takes 38 hours to go from St. Louis to Colorado Springs, and 56 hours to go from Colorado Springs to San Francisco. How many hours does it take to go from St. Louis to San Francisco?

10. In furnishing his house, Mr. Martin bought for the parlor five chairs for $28 and a rug for $24. What was the total cost?

11. For the dining room he bought a table for $29 and 6 chairs for $18. How much did all cost?

12. He bought a couch for $17 and a bookcase for $22. Find the cost of both.

13. For the kitchen he bought a range for $16 and a cabinet for $13. Find the cost of both.

14. He bought a bed for $23 and a dresser for $18. How much did both cost?

15. A curtain 92 inches long and 39 inches wide is to have a ruffle around one side and one end. How many inches of ruffle does it take?

16. A room is 16 ft. wide and 22 ft. long. How much picture molding does it take to go along one side and one end? To go clear around the room?

17. If you buy two pieces of ribbon at the remnant counter, one containing 43 inches and the other 39 inches, how many inches of ribbon do you get in both pieces?

18. John tied two pieces of cord together for his top string. One contained 18 inches and the other 27 inches. He used 2 inches in tying the knot. How long was his top string?

19. January has 31 days and February has 28. How many days in the two months?

20. If 14 inches of snow fell in December, and 19 inches in January, how many inches fell during both months?

21. There are 16 boys and 18 girls in a class. How many children in the class?

22. Fred paid 35 cents for a knife and 29 cents for a screwdriver. Count his change back from $2.

19. RAPID WORK IN SUBTRACTION

1. Count backward by 6's from 100.
2. Count backward from 97 by 8's; by 7's; by 9's.
3. How much is $10.00 less $4.25?
4. From $5.00 take $2.75.
5. From 1000 take 250; take 375; take 500.
6. Count by 12's from 150 to 6.

Give differences as rapidly as possible at sight:

7.	67 30	59 40	75 60	86 50	94 71	47 32	85 53	53 23	78 36	59 48

8.	1246 300	3721 600	4986 880	5478 5406	3966 3900	2791 2500

9.	539 408	827 325	396 190	417 95	824 704	391 270	864 453	249 127

At sight subtract each one of the following numbers from $1.00:

	A	B	C	D	E	F	G	H
10.	11¢	88¢	44¢	74¢	52¢	70¢	36¢	13¢
11.	35¢	61¢	82¢	14¢	81¢	33¢	22¢	65¢
12.	83¢	30¢	23¢	57¢	89¢	43¢	95¢	29¢
13.	20¢	81¢	73¢	63¢	21¢	94¢	56¢	99¢
14.	92¢	19¢	63¢	45¢	96¢	18¢	50¢	46¢
15.	64¢	75¢	24¢	90¢	34¢	39¢	78¢	38¢

16–21. *Give the differences between any two numbers in the same line in adjacent columns.*

Problems

1. Two years ago Robert was 45 inches tall. Now he is 52 inches tall. How much has he grown in the two years?

2. William sold 31 copies of the *News* and 23 copies of the *American* one afternoon. How many more copies of the *News* than of the *American* did he sell?

3. Harry earned 38 cents Friday and 52 cents Saturday. How much more did he earn on Saturday?

4. Tom is 12 years old and his father is 41. How much older than Tom is his father?

5. A class studying soils found that a pan of soil when wet weighed 42 ounces. After it had been heated for a long time over a fire, it weighed only 28 ounces. What weight of water did the soil contain originally?

6. Of 73 ounces of sandy loam, 37 ounces were found to be sand. How many ounces of other soil were there?

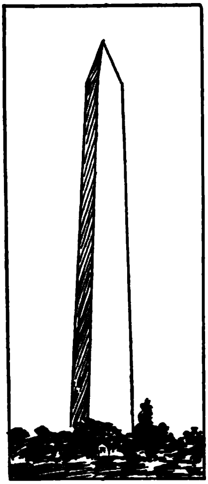
7. If a man earns $ 94 a month and spends $68, how much does he save?

8. If a farmer raised 43 bu. of potatoes, and wished to keep 16 bu. for his own use, how many bushels could he sell?

9. If he raised 31 bu. of onions, and wished to keep 8 bu. for his own use, how many bushels of onions could he sell?

10. If you bought 2 yd. of ribbon and used 64 in., how many inches would you have left?

11. The table on page 47 gives the heights of some of the world's most famous structures. How much higher is the Eiffel Tower than the Metropolitan Life Building? Than Cologne Cathedral? Than the Great Pyramid? Than the Capitol at Washington?


Washington Monument

HEIGHTS OF FAMOUS STRUCTURES	
Eiffel Tower, Paris	984 ft.
Metropolitan Life Building, N.Y.	700 ft.
Singer Building, N.Y.	612 ft.
Washington Monument	556 ft.
Cologne Cathedral	512 ft.
Strassburg Cathedral	465 ft.
Great Pyramid, Egypt	451 ft.
St. Peter's, Rome	433 ft.
Capitol, Washington	288 ft.
Bunker Hill Monument	221 ft.

12. How much higher is the Singer building, New York City, than the Washington Monument? Than Strassburg Cathedral? Than the Great Pyramid?

13. How much higher is the Great Pyramid of Egypt than St. Peter's, Rome? Than the Capitol at Washington?

Drills in Addition and Subtraction

Add and check:

1.	2.	3.	4.	5.
457	432	445	454	579
544	324	389	894	683
326	749	748	976	495
247	477	476	789	683

6.	7.	8.	9.	10.
6731	6942	3792	4679	5763
978	7678	9876	8946	9473
6143	5134	5472	9478	8146
927	7568	8946	8967	7981
843	4520	6478	3654	1897
549	9624	3961	4678	9384

11–46. The following table shows the amount of cotton produced in the United States for the past 36 years.

Find the increase or decrease, year by year, from 1874 to 1909.

YEAR.	BALES.	YEAR.	BALES.	YEAR.	BALES.
1874 . .	4,170,388	1886 . .	6,550,215	1898 . .	11,180,960
1875 . .	3,832,991	1887 . .	6,513,624	1899 . .	11,235,383
1876 . .	4,669,288	1888 . .	7,017,707	1900 . .	9,439,559
1877 . .	4,485,423	1889 . .	6,935,082	1901 . .	10,425,141
1878 . .	4,811,265	1890 . .	7,313,726	1902 . .	10,701,453
1879 . .	5,073,531	1891 . .	8,655,518	1903 . .	10,758,326
1880 . .	5,757,397	1892 . .	9,038,707	1904 . .	10,123,686
1881 . .	6,589,329	1893 . .	6,717,142	1905 . .	13,556,841
1882 . .	5,435,845	1894 . .	7,527,211	1906 . .	11,319,860
1883 . .	6,992,234	1895 . .	9,892,766	1907 . .	13,550,760
1884 . .	5,714,052	1896 . .	7,162,473	1908 . .	11,581,829
1885 . .	5,669,021	1897 . .	8,714,011	1909 . .	13,828,846

20. RAPID WORK IN MULTIPLICATION

Multiply 35 by 7.

WORK

$$\begin{array}{r} 35 \\ \underline{7} \\ 35 \\ \underline{21} \\ 245 \end{array} \quad \text{or} \quad \begin{array}{r} 35 \\ \underline{7} \\ 21 \\ \underline{35} \\ 245 \end{array}$$

By multiplying the tens' digit first, then adding the *tens* from the product of *ones*, then annexing the *ones'* digit from the product of the *ones*, we do not need a pencil when multiplying a number of two orders by a number of one order.

Thus, $7 \times 35 = 3 \times 7 + 3$ with 5 annexed.

1. Give the product of 46 by 8. Glance at 6 and 8, think 48. Then think $8 \times 4 + 4 = 36$, then annex 8. The product is 368.

2. Where did you get the 4 you added to 8×4?

3. Where did you get the 8 which you annexed?

21. DRILL FOR RAPID WORK IN MULTIPLICATION

Give the products quickly:

	A	B	C	D	E	F	G	H	I	J
4.	37 2	41 5	23 2	36 3	85 6	39 2	94 9	56 9	45 2	96 2
5.	83 4	49 3	57 7	53 2	15 9	82 5	38 8	74 8	97 3	43 7
6.	17 3	66 4	35 8	95 9	42 8	49 8	14 7	64 7	48 5	34 8
7.	52 8	94 2	21 9	73 5	24 7	63 9	98 6	54 4	91 6	37 9
8.	19 6	81 6	65 6	57 8	99 5	27 7	44 3	13 3	26 9	75 4
9.	71 7	12 9	93 4	33 7	16 3	72 6	25 5	62 2	88 4	46 3
10.	86 9	55 8	18 5	87 6	29 4	73 4	92 4	32 5	61 7	59 5

22. MULTIPLYING BY A NUMBER OF THREE DIGITS

Find the product of 376×485.

WORK

$$\begin{array}{r} 485 \\ \underline{376} \\ 2910 \\ 3395 \\ \underline{1455} \\ 182360 \end{array}$$

Read the first *partial product.* Where was the first figure placed?

Read the second partial product. Where was the first figure placed?

Read the third partial product. Where was the first figure placed?

What is the sum of the partial products?

Find the products of:

1. 375×893.	4. 643×785.	7. 734×867.
2. 463×786.	5. 836×974.	8. 843×796.
3. 527×389.	6. 349×738.	9. 384×963.

23. MULTIPLIERS CONTAINING ZEROS

1. Find the product of 306×847.

A

$$\begin{array}{r} 847 \\ \underline{306} \\ 5082 \\ 000 \\ \underline{2541} \\ 259182 \end{array}$$

B

$$\begin{array}{r} 847 \\ \underline{306} \\ 5082 \\ \underline{2541} \\ 259182 \end{array}$$

Since the zeros in the second partial product of the work marked *A* do not affect the complete product, they are omitted in practice. The work is written as in *B*. Where is the first figure of the second partial product written?

Find the product of:

2. 305×946.	6. 403×763.	10. 207×964.
3. 708×264.	7. 906×542.	11. 309×728.
4. 890×387.	8. 380×675.	12. 730×645.
5. 406×843.	9. 450×783.	13. 901×768.

Find the product of:

NOTE. — Select as multiplier the factor that will give but two partial products when possible.

14. 509 × 648.	21. 507 × 809.	28. 764 × 904.
15. 704 × 945.	22. 494 × 608.	29. 8701 × 906.
16. 907 × 893.	23. 904 × 889.	30. 9076 × 807.
17. 395 × 607.	24. 879 × 707.	31. 7809 × 703.
18. 809 × 694.	25. 609 × 984.	32. 3009 × 847.
19. 907 × 648.	26. 685 × 982.	33. 9070 × 369.
20. 876 × 394.	27. 809 × 698.	34. 8095 × 908.

Problems involving Multiplication

1. A dealer bought 218 tons of coal at $5.27 per ton, and sold the lot for $1667.70. How much did he gain?

2. A dealer bought 209 barrels of flour at $4.52 per barrel, and sold it for $6.20 per barrel. How much did he gain on the whole purchase?

3. A man after selling his stable sold 2 matched horses at $225 each, 1 black horse at $175, 2 bay horses at $135 and $160, 4 gray team horses at $75 each, and 3 single carriages to a dealer at $68 each. How much did he receive for all?

4. There are 238 teachers in the schools of a city. If the number of pupils to each teacher averages 38, find the total number of pupils.

5. A farmer raised 26 bushels of wheat per acre on 49 acres. How much is it worth at 87¢ per bushel?

6. My garden is 138 feet wide and 172 feet long. How many square feet does it contain?

7. A farmer sold 96 sheep and 57 lambs. He got $6.45 each for the sheep and $3.95 each for the lambs. How much did he get for all?

8. If a boy earns $3.75 per week selling papers and $2.50 more selling butter and eggs, how much can he earn in 26 weeks?

9. When 9 pounds of tea are worth $4.86, how much will 72 pounds cost? Solve in two ways.

24. MEANING OF ABSTRACT AND CONCRETE NUMBERS

1. Of these numbers, which apply to things that can be seen, handled, or measured? 5 books; 3 ft.; $9; 15; 6 gal.; 23; 10 lb.; 14; 7 strokes.

2. Which of these numbers are not connected with things of any kind?

> *Numbers that are applied to things of any kind are called* **concrete** *numbers.*

3. Name five concrete numbers.

> *Numbers used without reference to things of any kind are called* **abstract** *numbers.*

4. Name some abstract numbers.

5. In what respect are 9 ft. and 11 ft. alike? Can you think of any respect in which 5 gal. and 8 hr. are alike?

> **Like numbers** *have units of the same name and kind.*

6. Mention three like numbers; four that are unlike. Why are all abstract numbers like numbers?

7. Using the objects, actually multiply 2 pencils or 2 books by 3. Which of these numbers do you see and handle? Which is abstract? Of what use is the abstract number?

8. $6 \times \$8 = \$48$. How many numbers are used in this example? Which of them are like numbers? Why?

9. Which is more, $2 \times 3 \times 4$ inches or $3 \times 4 \times 2$ inches?

Principles in Multiplication. — 1. *Only one factor can be concrete.*

2. *The product and the concrete factor must be like numbers.*

3. *The order in which the factors are used will not affect the product.*

25. INTERCHANGING MULTIPLIER AND MULTIPLICAND

A farmer sold 238 sheep at \$7 each. How much did he get for them?

Since the amount received for 1 sheep was \$7, the amount received for 238 was $238 \times \$7$.

What is the *multiplier?* Which number is the *multiplicand?*

When there are 3 figures in the multiplier, how many partial products have we?

When there is but one figure in the multiplier, we get the whole product at once.

$$\begin{array}{r} \$7 \\ \underline{238} \\ 56 \\ 21 \\ \underline{14} \\ \$1666 \end{array}$$

Compare $238 \times \$7$ with $7 \times \$238$. Which is shorter, to find $238 \times \$7$ or $7 \times \$238$? Find each.

$$\begin{array}{r} \$238 \\ \underline{7} \\ \$1666 \end{array}$$

In practice we select the number for multiplier that will make the least work. To do this we have to think of it as an abstract number. From the real multiplicand we know what things the product represents.

Find the cost of:

1. 644 chairs at $3.
2. 197 desks at $5.
3. 623 sofas at $7.
4. 865 tables at $9.
5. 346 beds at $8.

Change:

6. 425 weeks to days.
7. 785 yards to feet.
8. 649 gallons to quarts.
9. 427 days to hours.
10. 416 pounds to ounces.

26. FACTORS AND MULTIPLES

Since the multiplier and multiplicand *make* the product, they are called **factors** of the product.

Exercises

1. What are the products of the following factors: 9 and 8; 15 and 6; 20 and 5; 25 and 3; 16 and 6?
2. If 3 is one factor of 21, what is the other?
3. 12 is a factor of 60. Find the other factor.
4. Find the factors of 22; of 34; of 26; of 35.

Give the factors smaller than 14 *that produce the following:*

5. 28, 32, 33.
6. 35, 36, 39.
7. 42, 45, 48.
8. 49, 52, 54.
9. 65, 66, 72.
10. 77, 78, 81.
11. 84, 88, 91.
12. 96, 99, 104.
13. 108, 110.
14. 117, 121.
15. 130, 132.
16. 143, 156.

Instead of saying that one number will *contain* another number, we say that it is a **multiple** of the number. Thus, 36 is a multiple of 2, or 3, or 4. Give four other numbers of which 36 is a multiple.

Exercises

1. 35 is a multiple of what two numbers?
2. Which of the following are multiples of 2? Of 3?

   4, 5, 9, 12, 15, 18, 16, 24, 27, 32, 54.
3. Beginning with 4, name all the multiples of 2 to 100.
4. Name all multiples of 3 to 99.
5. Name all multiples of 4 to 100.
6. 38 is a multiple of what two factors?
7. Give the multiples of 5 to 200.
8. $3 \times \$8 = \$$——. Name two factors that make \$24.

27. DIVISION

Our skill in division depends largely upon our skill in multiplication. One does not know that $\frac{1}{11}$ of 99 is 9 until he first knows that $9 \times 11 = 99$.

Exercises

1. The product is 121 ft.; one factor is 11. Find the other.
2. —— $\times$ 9 miles = 108 miles, hence 108 miles $\div$ 9 miles = ——; $7 \times$ —— $= \$77$, hence $\$77 \div 7 = \$$——.

> *In division the* **product** *becomes the* **dividend.**
> *The* **known factor** *becomes the* **divisor.**
> *The other factor, when found, is the* **quotient.**

3. If the *dividend* is 91 days and the *divisor* is 7 days, what is the quotient? If the divisor is 7, what is the quotient?

4. The dividend is 96 ¢. The divisor is 8. What is the quotient?

5. One factor is 12 feet. The product is 84 feet. What is the other factor?

6. Product = \$96; multiplier = 8; multiplicand = what?

7. Product = \$150; multiplicand = \$25; multiplier = what?

8. \$7560 ÷ 15. **9.** 8472 ft. ÷ 12. **10.** 1080 in. ÷ 15.

11. 6432 ft. ÷ 96. **13.** 5600 mi. ÷ 34.

12. 6400 yd. ÷ 84 yd. **14.** 8476 ÷ 68.

15. One factor of \$475,000 is \$250. Find the other.

16. The product of two numbers is 100,000. One factor is 125. Find the other.

17. Product = \$1675. One factor = \$67. The other factor is what?

18. One factor is 197. The dividend is \$90,817. What is the other factor?

Divide and check:

19. 3468 by 75.	**26.** 57,632 ÷ 143.	**33.** 846,792 by 837.
20. 7923 by 64.	**27.** 28,931 ÷ 151.	**34.** 478,329 ÷ 926.
21. 8495 by 85.	**28.** 73,201 ÷ 173.	**35.** 384,731 ÷ 692.
22. 3048 by 54.	**29.** 864,371 by 426.	**36.** 483,009 ÷ 741.
23. 5703 by 72.	**30.** 698,428 by 573.	**37.** 624,642 ÷ 888.
24. 17,863 ÷ 121.	**31.** 386,471 by 648.	**38.** 911,783 ÷ 951.
25. 19,831 ÷ 131.	**32.** 694,328 by 901.	**39.** 684,732 ÷ 895.

Minimum Weights of Produce

The following are minimum weights of certain articles of produce according to the laws of the United States:

	Per Bushel		Per Bushel
Wheat	60 lb.	Sweet Potatoes	55 lb.
Corn, in the ear	70 lb.	Onions	57 lb.
Corn, shelled	56 lb.	Turnips	55 lb.
Rye	56 lb.	Clover Seed	60 lb.
Buckwheat	48 lb.	Flaxseed	56 lb.
Barley	48 lb.	Timothy Seed	45 lb.
Oats	32 lb.	Blue Grass Seed	44 lb.
White Beans	60 lb.	Corn Meal	48 lb.
White Potatoes	60 lb.	Ground Peas	24 lb.

NOTE. — Do not use a fraction of a bushel, but give the answer to the nearest bushel.

1. The capacity of a certain car is 65,000 pounds. With how many bushels of shell corn may it be loaded?

2. How many bushels of sweet potatoes can be loaded in a car whose capacity is 38,000 pounds?

3. A farmer sold 38,500 pounds of corn in the ear. How much did he get for it at 54¢ a bushel?

4. A farmer had 40 acres of oats and the same of wheat. He had 51,200 pounds of oats and sold them at 48¢ a bushel. He had 48,000 pounds of wheat which he sold at 87¢ a bushel. From which field did he receive more? How much?

5. One year a field produced 56,800 pounds of wheat. The next year with better preparation of the soil and greater care in selecting seed the same field produced 62,500 pounds. How much was the increase worth at 85¢ a bushel?

6. From an acre a gardener dug 15,000 pounds of white potatoes. At this rate, how many bushels will 40 acres produce?

7. Make a table showing how many bushels a car will carry of each of the articles named in the above table. Call the capacity of the car 48,000 pounds.

28. MEASURES OF TIME

60 seconds (sec.)	**= 1 minute (min.)**
60 minutes	**= 1 hour (hr.)**
24 hours	**= 1 day (da.)**
7 days	**= 1 week (wk.)**
12 months (mo.)	**= 1 year (yr.)**
100 years	**= 1 century**

It takes the earth nearly $365\frac{1}{4}$ days to revolve around the sun. In the calendar 365 days make a **common year.** Hence, in order to correct the error made in considering 365 days a year, an extra day is added to all years divisible by 4, except the century years, which must be divisible by 400. These years containing an extra day are **leap years.**

The extra day is added to the month of February, which has but 28 days in a common year. Four of the months have 30 days each. They are April, June, September, and November. The remaining seven have 31 days each.

"Thirty days have September,
April, June, and November.
All the rest have thirty-one,
Save February, which alone
Has twenty-eight, and one day more
We add to it one year in four."

Tell which of the following years will be leap years: 1912; 1922; 1940; 1956; 2000; 2100; 2300; 2400.

Exercises in Measures of Time

1. How many seconds in 15 minutes? In 1 hour?

2. How many minutes in 8 hours? In 1 day?

3. How many hours in 1 week? In the month of July?

4. How many days in 6 wk.?

5. Your vacation lasts 12 weeks, or how many days?

6. If you are 11 years old, how many months old are you?

7. George's brother is $2\frac{1}{2}$ years old. How many months old is he?

8. A month is what part of a year?

9. Give as a fraction of a year each of the following: 2 mo.; 3 mo.; 4 mo.; 5 mo.; 6 mo.; 7 mo.; 8 mo.; 9 mo.; 10 mo.; 11 mo.

10. When rent is $600 per year, how much is that for 6 months? For 3 months? For 9 months? For 2 months?

11. If you are in school 9 months of the year, what part of the year is that? What part of the year are you out of school?

12. If you are in school 10 months of the year, what part of the year are you out of school?

29. AVOIRDUPOIS MEASURES OF WEIGHTS

The weights used in weighing all common articles, as groceries, grain, and meat, are called **Avoirdupois weights.** There are two other systems of weights. One is used to weigh gold, silver, and other valuable metals and gems. It is called **Troy weight.** The other system is used by druggists in filling prescriptions. It is called **Apothecaries' weight.**

It is not necessary to learn the last two named tables, for commonly all reference is to *Avoirdupois weight.*

16 ounces (oz.) = 1 pound (lb.)
2000 pounds = 1 ton (T.)

Sometimes the "**hundredweight**" (**cwt.**), meaning 100 lb., is used as a unit. In weighing some ores at the mine, 2240 lb. is called a ton. To distinguish one from the other, 2240 lb. is called a **long ton**, and 2000 lb. a **short ton.**

Problems in Weight

1. How many ounces in 12 pounds? In 20 pounds?

2. How many pounds in 8 tons? In 35 tons?

3. How many tons may be put in a car whose capacity is 48,000 pounds? In one whose capacity is 72,000 pounds?

4. Four men together bought a car load of coal for $5.20 a ton, delivered. The car load weighed 40,000 lb. How many tons did each man get? How much did it cost each? How much did each man save if the dealers would have charged $6.75 a ton to deliver it?

5. From 15 acres a farmer sold 62,500 pounds of timothy hay. How much did he get for it at $16 a ton? How much was this for the hay produced on 1 acre?

6. A man bought 8 tons of coal. The first load contained 4350 pounds, the second 4750 pounds, the third 3900 pounds. How many pounds must the fourth load contain?

7. During the months of January, February, March, and April, a family used 200 pounds of coal a day. How much did it cost at $6.75 a ton?

Making and Receipting Bills

1. Check the following bill. That is, see if it is correct:

NEWARK, N. J., *May 10, 1910.*

HAHNE & CO.

SOLD TO *Miss Louise Bacon*

14 yards English Serge @ $1.25	*$17*	*50*		
$1\frac{1}{4}$ yards Velvet @ 4.00	*5*	*00*		
2 yards Silesia @ .15		*30*		
			$22	*80*

PAID
Hahne & Co.

Make and receipt bills for the following, using any names you wish:

2. Find cost of the following bill of material for basketry:

9 lb. plain raffia	@ $0.20	per lb.
5 lb. colored raffia	@ .50	per lb.
3 lb. ash splints	@ 1.25	per lb.
3 lb. No. 1 reed	@ .95	per lb.
5 lb. No. 2 reed	@ .75	per lb.

3. The following supplies were ordered for a sewing class. What was the amount of the bill?

3 boxes thread	@ $0.48	per box.
20 yd. cambric	@ .15	per yd.
18 yd. toweling	@ $.12\frac{1}{2}$	per yd.
7 yd. damask for scarfs	@ .39	per yd.

4. Bill for bookbinding material. Find the total amount.

7 yd. linen canvas	@ $0.17	per yd.
25 yd. cord for lacings	@ .02	per yd.
18 sheets cover paper	@ .03	per sheet.
8 sheets press board	@ .10	per sheet.

5. Write a bill for this basketry material:

The Board of Education of Lake Forest, Ill., bought of Vaughan's Seed Store, 84–86 Randolph Street, Chicago,

100-lb. lot natural raffia	@ 12¢	per lb.
10-lb. lot white raffia	@ 18¢	per lb.
3 lb. Indian red raffia	@ 40¢	per lb.
4 lb. seal brown raffia	@ 40¢	per lb.
2 lb. orange raffia	@ 40¢	per lb.
2 lb. black raffia	@ 40¢	per lb.
4 lb. olive green raffia	@ 40¢	per lb.
5-lb. lot No. 4 rattan	@ 40¢	per lb.
5-lb. lot No. 2 rattan	@ 50¢	per lb.
"Indian Basketry" — James	$2.50	

Miscellaneous Problems

1. The receipts at a fair were as follows: 260 nickels, 145 dimes, 84 quarter-dollars, 32 half-dollars, and 12 one-dollar bills. How much money was received in all?

2. A lot of land cost $1500. The house on it cost $4652 more than the lot, and the garage $600 less than the lot. Find the entire cost.

3. The sidewalk in front of my house is 52 ft. long and 4 ft. wide. How many square feet does it contain? How much did it cost at 27¢ per square foot?

4. If a horse eats six quarts of oats in a day, how much will a span of horses eat in six weeks?

5. A man who lives eight miles from his work rides on his bicycle to his work and back every day. How many miles does he ride in a year? Allow fifty-two Sundays, eight holidays, and ten days for illness.

6. I hire eight men at $1.75 a day, four men at $3.25, and five boys at $4 a week. How much money shall I need to pay them all for a week's work?

7. During the winter I burn ten tons of coal, at a cost of $62.50. I pay $0.25 a ton for housing. Find the cost of the coal per ton.

8. A young man owes his tailor $85. If he begins Monday morning and pays $6.25 every morning during the week, how much will he owe him on Saturday evening?

9. A collector has received $84.75. How much more must he collect to have $125 and his day's pay, $2.25?

10. I have ordered ten tons of coal. I have received three loads, weighing 3400, 5760, and 6300 pounds. How many pounds should there be in the last load?

11. A druggist owes a physician $257.83, and the physician owes the druggist $175. If the druggist pays $25, how much does he still owe the doctor?

12. A corn crib contains 72,000 pounds. How many bushels remain if 12,000 pounds are removed? Allow 56 pounds to the bushel.

13. A five-acre field of corn in 1908 yielded only ten bushels to the acre. Under scientific cultivation in 1909 the same field yielded 80 bushels to the acre. How many bushels more were raised in 1909 than in 1908?

14. According to the census returns of 1910, Detroit has a population of 465,766; Indianapolis, 233,650; and St. Louis, 687,029. What is the total population of these three cities?

15. I lose one-eighth of $2476.48. How much have I left?

16. If 144 blackboard crayons are put into a box, how many boxes will be needed for 10,000 crayons?

17. In a building containing 18 schoolrooms, there are 630 pupils. How many pupils may there be in each room?

18. Find the sum of the product and the difference of 834 and 629.

19. What shall I pay in all for the following articles?

12 yd. of silk	@ $1.75	per yard.
4 yd. of lace	@ .96	per yard.
34 yd. of carpet	@ 1.25	per yard.

20. A stenographer receives $45 a month. If her expenses are $385 a year, how much can she save?

21. What must you add to the sum of the smaller of these two columns to make it equal the larger?

$478.32	$641.28
946.21	147.63
87.50	205.75

22. How many five-inch badges will twenty yards of ribbon make?

23. A farmer sells his apples at $1.75 a barrel. If the picking costs him 15¢ a barrel, and the barrel itself 20¢, how much does he make on a crop of 75 barrels?

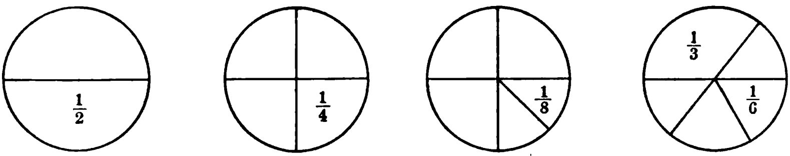
24. A gentleman pays his chauffeur $75 a month; the repairs to his machine cost $375; other expenses are $130. Required, the expense of running the machine for a year.

III. FRACTIONS

30. ORAL REVIEW IN FRACTIONS

1. To get *one half* of a circle we divide it into —— parts.

2. To get *one fourth* of a circle we divide it into —— parts.



3. Compare $\frac{1}{2}$ of a circle and $\frac{1}{4}$ of it. $\frac{1}{2}$ of $\frac{1}{2}$ of a circle = ——; $\frac{1}{4}+\frac{1}{4}=$ ——.

4. If each fourth be divided into 2 equal parts, the whole circle will be divided into how many equal parts?

5. What is each part called? Then how many 8ths of a circle in $\frac{1}{4}$ of it? $\frac{1}{4} \div 2 = \frac{?}{8}$.

SUGGESTION. — "Divided by 2" means divided into 2 equal parts.

6. 4 times $\frac{1}{8}$ of a circle $= \frac{?}{8}$, or $\frac{?}{4}$ of it; $6 \times \frac{1}{8}$ of a circle $= \frac{?}{8}$, or $\frac{?}{4}$ of it.

7. $\frac{1}{4} \div \frac{1}{8} =$ ——; $\frac{1}{2} \div \frac{1}{8} =$ ——.

8. 1 circle − $\frac{3}{8}$ of it = $\frac{?}{8}$ of it.

9. How many $\$\frac{1}{4}$ in a dollar? In $\$\frac{1}{2}$?

10. $\$\frac{1}{4} + \$\frac{1}{2} = \$\frac{?}{4}$. $\$1 - \$\frac{1}{4} = \$\frac{?}{4}$.

11. If you divide a circle into three equal parts, what is each part called? If you divide each of the 3 equal parts into 2 equal parts, what is each part called? Write two thirds.

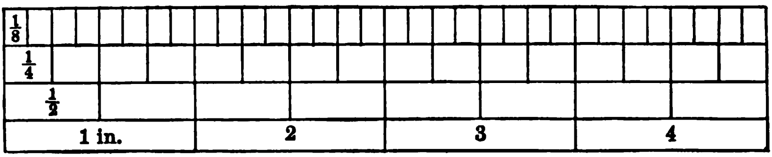
12. $\frac{1}{3}=\frac{?}{6}$; $\frac{1}{3}+\frac{1}{6}=\frac{?}{6}$, or $\frac{1}{?}$.

13. $\frac{1}{3}$ yd. = —— in.; $\frac{2}{3}$ yd. = —— in.

14. $\frac{1}{8}$ bu. = —— qt.; $\frac{3}{8}$ bu. = —— qt.; $\frac{3}{4}$ bu. = —— qt.

15. $\frac{1}{2}$ hr. = —— min.; $\frac{1}{4}$ hr. = —— min.; $\frac{3}{4}$ hr. = —— min.

Oral Review of Fractional Lengths



1. How many inches in a foot?
2. What part of a foot is here represented?
3. How many $\frac{1}{2}$ in. spaces in 1 in.? In 2 in.? In 3 in.?
4. How many $\frac{1}{4}$ in. spaces in $\frac{1}{2}$ in.? In 1 in.? In 3 in.?
5. How many $\frac{1}{8}$ in. spaces in $\frac{1}{4}$ in.? In $\frac{1}{2}$ in.? In 2 in.?
6. How many times is $\frac{1}{8}$ in. contained in $\frac{3}{4}$ in.? In $2\frac{1}{2}$ in.?
7. 6 times $\frac{1}{8}$ in. = $\frac{?}{4}$ in.; 8 times $\frac{1}{4}$ in. = —— in.
8. $1\frac{1}{2}$ in. + $1\frac{1}{2}$ in. = —— in.; $2\frac{1}{2}$ in. + $1\frac{1}{4}$ in. = —— in.
9. $\frac{1}{4}$ in. + $\frac{3}{8}$ in. = $\frac{?}{8}$ in.; $\frac{1}{4}$ in. + $\frac{1}{8}$ in. + $\frac{1}{2}$ in. = —— in.
10. $1\frac{1}{4}$ in. + $\frac{3}{4}$ in. = —— in.; $\frac{3}{4}$ in. + $\frac{1}{8}$ in. + $\frac{1}{8}$ in. = $\frac{?}{8}$ in.

Add:

11.	12.	13.	14.	15.
$1\frac{1}{2}$ in.	$1\frac{1}{4}$ in.	$\frac{3}{4}$ in.	$1\frac{1}{4}$ in.	$1\frac{5}{8}$ in.
$2\frac{1}{4}$ in.	$1\frac{3}{8}$ in.	$1\frac{1}{4}$ in.	$2\frac{1}{8}$ in.	$2\frac{3}{8}$ in.
$3\frac{1}{2}$ in.	$2\frac{1}{4}$ in.	$2\frac{7}{8}$ in.	$1\frac{7}{8}$ in.	4 in.

16. $1\frac{3}{4}$ in. $-$ $1\frac{1}{2}$ in. = ——.

17. $\frac{5}{8}$ in. $-$ $\frac{1}{4}$ in. = ——.

18. $\frac{1}{4}$ in. $-$ $\frac{1}{8}$ in. = ——.

19. $\frac{3}{4}$ in. $-$ $\frac{5}{8}$ in. = ——.

20. $\frac{6}{8}$ in. $-$ $\frac{1}{2}$ in. = ——.

21. 2 in. $-$ $1\frac{1}{4}$ in. = ——.

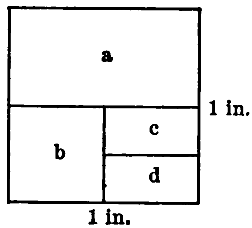
22. 3 in. $-$ $2\frac{1}{8}$ in. = ——.

23. $3\frac{1}{4}$ in. $-$ $2\frac{1}{2}$ in. = ——.

24. Draw an oblong $2\frac{1}{2}$ inches long and $1\frac{1}{2}$ inches wide. What is the distance around it?

The distance around a closed figure, that is, the sum of the lines bounding a surface, is called the **perimeter**.

Oral Review in Comparison of Fractions



1. *a* is what part of a square inch?

2. *b* is what part? *c* is what part?

3. Compare $\frac{1}{2}$ sq. in. and $\frac{1}{4}$ sq. in.; $\frac{1}{4}$ sq. in. and $\frac{1}{8}$ sq. in.

4. If this square represents a piece of valuable metal worth $24, what is *a* worth? What is *b* worth? What is *c* worth?

5. $\frac{1}{8}$ sq. in. + $\frac{1}{8}$ sq. in. + $\frac{1}{4}$ sq. in. = $\frac{1}{2}$ sq. in.; $4 \times \frac{1}{8}$ sq. in. = ——.

6. $\frac{1}{2}$ sq. in + $\frac{1}{4}$ sq. in. = ——; $\frac{1}{4}$ sq. in. $\div$ $\frac{1}{8}$ sq. in. = —— times.

7. $\frac{1}{2} \div 2 =$ ——; $\frac{1}{4} \div 2 =$ ——; $\frac{1}{2} \div 4 =$ ——.

8. $\frac{3}{4} - \frac{1}{8} =$ ——; $\frac{1}{2} - \frac{3}{8} =$ ——; $\frac{1}{4} - \frac{1}{8} =$ ——; $1 - \frac{5}{8} =$ ——.

9. When I divide a square into 6 equal parts, what is each part called?

10. What are five of those parts called?

11. If I divide $30 into 6 equal parts, what part of $30 is each part?

12. Five of those parts are what part of \$30?

13. Find $\frac{5}{6}$ of \$42; $\frac{7}{8}$ of \$40; $\frac{2}{3}$ of 12 feet; $\frac{5}{6}$ of 12 inches.

14. 1 oz. is what part of a pound? 4 oz. is what part of a pound? 8 oz. is what part of a pound?

15. If a pound costs 28¢, how much will 4 oz. cost?

16. At 28¢ a pound how much will 1 lb. 8 oz. cost?

17. At 12¢ a pound how much will 1 lb. 8 oz. cost? How much will 4 oz. cost? 1 lb. 12 oz. will cost how much?

18. At 24¢ a yard how much will 1 yd. 18 in. cost? How much will 9 in. cost? 1 yd. 27 in.?

19. At 40¢ a bushel how much will 1 bu. 3 pk. cost?

Oral Review in Finding Parts of Numbers

1. $\frac{1}{3}$ of 36 inches is how many inches?

2. $\frac{2}{3}$ of 36 inches means what? $\frac{2}{3}$ of 36 inches $= 2 \times$ ——, or——.

3. $\frac{1}{4}$ of 24 lb. $=$ ——; $\frac{3}{4}$ of 12 lb. $= 3 \times$ ——lb., or—— pounds.

Find in like manner:

4. $\frac{3}{4}$ of \$40.
5. $\frac{5}{6}$ of 30 ft.
6. $\frac{3}{7}$ of 28 mi.
7. $\frac{2}{5}$ of 45 lb.
8. $\frac{3}{8}$ of 24 oz.
9. $\frac{4}{9}$ of 27 doz.
10. $\frac{5}{7}$ of \$35.
11. $\frac{5}{8}$ of 48 yd.
12. $\frac{7}{9}$ of 63 gal.
13. $\frac{2}{9}$ of 45 tons.
14. $\frac{6}{7}$ of 42 qt.
15. $\frac{9}{10}$ of \$70.
16. $\frac{7}{9}$ of \$72.
17. $\frac{2}{3}$ of \$33.
18. $\frac{5}{7}$ of 63 hr.
19. $\frac{6}{11}$ of \$66.
20. $\frac{7}{12}$ of 84 hr.
21. $\frac{8}{11}$ of \$44.
22. $\frac{6}{7}$ of 56.
23. $\frac{5}{12}$ of 108.
24. $\frac{7}{12}$ of 144.
25. $\frac{4}{11}$ of 121.
26. $\frac{3}{4}$ of 400.
27. $\frac{5}{6}$ of 600.
28. $\frac{5}{9}$ of 270.
29. $\frac{6}{7}$ of 350.
30. $\frac{4}{11}$ of 1100.

31. 5 is $\frac{1}{4}$ of what number?

32. 8 is $\frac{1}{5}$ of what number?

33. 8 is what part of 24? Of 32? Of 40?

34. 20 min. is what part of an hour? 10 min. is what part?

35. 12 oz. is what part of a pound?

36. How many pounds in a ton? 1000 lb. is what part of a ton? 500 lb. is what part? 1500 lb. is what part?

37. If butter is selling at 24 ¢ a pound, how much will $\frac{1}{4}$ of a pound cost? How much will $2\frac{1}{4}$ lb. cost?

38. At 40 ¢ a pound how much will 12 oz. of tea cost?

39. A boy sells $\frac{2}{3}$ of his papers and has 5 left. How many had he at first?

40. If $\frac{1}{2}$ doz. oranges cost 20 ¢, how much will 5 doz. cost?

41. If 4 qt. of nuts cost 40 ¢, how much will $\frac{1}{4}$ bu. cost?

Oral Review of Addition of Fractions

1. Divide a strip of paper 1 foot long into pieces 1 inch long. How many pieces have you? What part of a foot is each?

2. What is meant by $\frac{3}{12}$ of a foot? What are the terms of this fraction?

3. What does the 12 show? What does the 3 show?

The lower term of a fraction shows (1) *into* **how many equal parts** *the whole has been divided, and hence* (2) **names the fractional units.** *It is called the* **Denominator.**

The upper term shows the **number of fractional units** *in the fraction. It is called the* **Numerator.**

4. In the fraction $\frac{5}{8}$ of a gallon, tell what each term shows.

5. In what respect are 3 feet and 9 feet alike?

6. In what respect are $\frac{7}{12}$ of a foot and $\frac{5}{12}$ of a foot alike?

> *Numbers whose units have the same name are* **Like Numbers.**

7. Add 1 inch, 2 inches, and 4 inches.

8. Add $\frac{1}{12}$ ft., $\frac{2}{12}$ ft., and $\frac{4}{12}$ ft.

9. $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{?}{12}$. $\frac{4}{12}$ and $\frac{3}{12}$ are *like* fractions. Why?

10. $\frac{1}{3} + \frac{1}{6} = \frac{1}{?}$; $\frac{1}{12} + \frac{1}{12} + \frac{1}{3} =$ ——; $\frac{2}{3} + \frac{1}{6} =$ ——.

Add the following:

11. $\frac{1}{2} + \frac{1}{3}$.

12. $\frac{1}{12} + \frac{5}{6}$.

13. $\frac{1}{12} + \frac{1}{6} + \frac{1}{3}$.

14. $\frac{1}{12} + \frac{1}{6} + \frac{2}{3}$.

15. $\frac{1}{3} + \frac{5}{12}$.

16. $\frac{1}{6} + \frac{2}{3}$.

17. 6 in. + 9 in. = —— in., or 1 foot and —— inches.

18. $\frac{6}{12} + \frac{9}{12} = \frac{15}{12}$, or $1\frac{3}{12}$; $\frac{7}{12} + \frac{10}{12} = \frac{?}{12}$, or $1\frac{?}{12}$.

19. Before unlike numbers can be added, how must they be changed?

31. REDUCTION OF FRACTIONS

1. Which is most valuable, 1 half-dollar, 2 quarter-dollars, 5 dimes, or 10 nickels?

2. Compare the size of the units in these numbers.

3. Express 3 quarts in smaller units.

4. Represent 2 bushels in smaller units.

5. Change $\frac{3}{4}$ to a larger unit.

6. A gallon is how many quarts? How many pints?

7. Which of the following numbers contains the largest number of units? The largest units? How do the numbers compare in value?

9 in. $\frac{3}{4}$ ft. $\frac{1}{4}$ yd.

8. Change the form of the following numbers without changing their value:

12 pt. 2 bu. 16 oz. $\frac{8}{12}$ $\frac{1}{8}$.

From these exercises we learn that the same value may be expressed in units of different size. As we lessen the size of the units, we increase their number, and as we lessen the number of the units, we increase their size.

> *Changing the form of a number without changing its value is* **Reduction.**

9. Draw rectangles to show that $\frac{4}{8} = \frac{2}{4} = \frac{1}{2}$.

As in Example 9, *show the following:*

10. $\frac{3}{6} = \frac{1}{2}$.
13. $\frac{6}{12} = \frac{3}{6} = \frac{1}{2}$.
16. $\frac{12}{16} = \frac{6}{8} = \frac{3}{4}$.
11. $\frac{3}{9} = \frac{1}{3}$.
14. $\frac{8}{16} = \frac{4}{8} = \frac{1}{2}$.
17. $\frac{1}{4} = \frac{2}{8} = \frac{4}{16}$.
12. $\frac{4}{12} = \frac{1}{3}$.
15. $\frac{4}{12} = \frac{2}{6} = \frac{1}{3}$.
18. $\frac{2}{3} = \frac{4}{6} = \frac{8}{12}$.

In the examples given we have illustrations of the following principles:

> I. *Both terms of a fraction may be divided by any number without changing the value of the fraction.*
>
> II. *Both terms of a fraction may be multiplied by any number without changing the value of the fraction.*

Change to smallest terms:

19. $\frac{2}{12}$; $\frac{10}{12}$; $\frac{3}{12}$; $\frac{9}{12}$; $\frac{4}{12}$; $\frac{8}{12}$.

20. $\frac{3}{18}$; $\frac{15}{18}$; $\frac{6}{18}$; $\frac{12}{18}$; $\frac{5}{18}$; $\frac{15}{18}$.

21. $\frac{6}{24}$; $\frac{18}{24}$; $\frac{4}{24}$; $\frac{20}{24}$; $\frac{8}{24}$; $\frac{16}{24}$.

22. $\frac{4}{20}$; $\frac{8}{20}$; $\frac{12}{20}$; $\frac{16}{20}$; $\frac{5}{20}$; $\frac{25}{30}$.

23. $\frac{6}{30}$; $\frac{12}{30}$; $\frac{18}{30}$; $\frac{24}{30}$.

24. $\frac{6}{36}$; $\frac{30}{36}$; $\frac{4}{36}$; $\frac{8}{36}$.

25. $\frac{28}{36}$; $\frac{32}{36}$; $\frac{4}{48}$; $\frac{20}{48}$.

26. $\frac{6}{60}$; $\frac{18}{60}$; $\frac{42}{60}$; $\frac{54}{60}$.

27. Which is the larger, $\frac{1}{2}$ or $\frac{1}{4}$? $\frac{1}{3}$ or $\frac{1}{6}$? $\frac{1}{4}$ or $\frac{1}{8}$?

> ***When a fraction is changed to its smallest terms, its units are made the largest possible.***

28. Give the different numbers of 12ths that can be changed to larger units.

29. Can 3ds, 5ths, 7ths, 11ths, or 13ths be changed to larger units?

30. What is the largest unit to which $\frac{75}{100}$ can be changed?

Change to largest units:

31. $\frac{8}{12}$; $\frac{9}{12}$; $\frac{10}{12}$; $\frac{12}{16}$; $\frac{14}{16}$; $\frac{6}{16}$; $\frac{15}{18}$; $\frac{30}{50}$; $\frac{50}{75}$.

32. $\frac{14}{32}$; $\frac{20}{32}$; $\frac{30}{36}$; $\frac{18}{42}$; $\frac{36}{42}$; $\frac{33}{44}$; $\frac{20}{45}$; $\frac{24}{60}$; $\frac{24}{64}$.

Exercises in Reduction of Fractions

Change the following fractions as indicated:

1. $\frac{9}{12}$ to fourths.
Dividing both terms by 3, $\frac{9}{12} = \frac{3}{4}$.

2. $\frac{8}{12}$ to thirds.

3. $\frac{12}{16}$ to fourths.

4. $\frac{8}{16}$ to halves.

5. $\frac{12}{16}$ to fourths.

6. $\frac{6}{8}$ to fourths.

7. $\frac{10}{16}$ to eighths.

8. $\frac{10}{12}$ to sixths.

9. $\frac{10}{15}$ to thirds.

10. $\frac{14}{16}$ to eighths.

11. $\frac{8}{10}$ to fifths.

12. $\frac{6}{9}$ to thirds.

13. $\frac{18}{20}$ to tenths.

14. $\frac{15}{20}$ to fourths.

15. $\frac{12}{20}$ to fifths.

16. $\frac{10}{20}$ to halves.

17. $\frac{1}{3}$ to ninths.

Multiplying both terms by 3, $\frac{1}{3} = \frac{3}{9}$.

18. $\frac{2}{5}$ to tenths.

19. $\frac{3}{8}$ to sixteenths.

20. $\frac{2}{3}$ to twelfths.

21. $\frac{3}{4}$ to twelfths.

22. $\frac{7}{8}$ to sixteenths.

23. $\frac{2}{3}$ to ninths.

24. $\frac{2}{3}$ to twelfths.

25. $\frac{3}{4}$ to eighths.

26. $\frac{3}{4}$ to sixteenths.

27. $\frac{5}{6}$ to twelfths.

28. $\frac{5}{6}$ to eighteenths.

29. $\frac{4}{5}$ to fifteenths.

30. $\frac{4}{5}$ to twentieths.

31. $\frac{5}{8}$ to twenty-fourths.

32. $\frac{6}{7}$ to fourteenths.

33. How do you change a fraction with large terms to an equal fraction with smaller terms?

34. How may you make the terms of a fraction larger without changing its value?

35. When is a fraction in its smallest terms?

> *A fraction is in its smallest terms when no factor will divide them both.*

32. ADDING AND SUBTRACTING FRACTIONS

1. Add 3 feet, 2 yards, 24 inches.

2. To what *common unit* did you change the numbers?

3. Add 3 qt., 1 gal., 3 pt. To what *unit* did you change them?

4. From 75 minutes subtract 1 hour. What change in form did you make; that is, to what common unit did you change the two numbers?

5. Add $\frac{1}{2}$ and $\frac{1}{4}$. To what common unit did you change the fractions?

> *Before integers or fractions can be added or subtracted, they must have* **Like Units.**

6. Add $\frac{1}{2}$ and $\frac{1}{8}$. Can $\frac{1}{2}$ be changed to 8ths?

7. Add $\frac{1}{3}$ and $\frac{1}{4}$. Can $\frac{1}{3}$ be changed to 4ths? To what like unit can both be changed? $\frac{1}{3}+\frac{1}{4}=\frac{4}{12}+\frac{3}{12}=\frac{7}{12}$.

Exercises

Change these fractions to like units, and find their sum and their difference:

1. $\frac{1}{5}, \frac{1}{10}$.	4. $\frac{5}{6}, \frac{3}{10}$.	7. $\frac{1}{12}, \frac{4}{9}$.	10. $\frac{2}{9}, \frac{1}{6}$.
2. $\frac{1}{2}, \frac{1}{3}$.	5. $\frac{2}{3}, \frac{1}{2}$.	8. $\frac{2}{9}, \frac{4}{5}$.	11. $\frac{8}{9}, \frac{7}{8}$.
3. $\frac{2}{9}, \frac{1}{6}$.	6. $\frac{5}{6}, \frac{1}{4}$.	9. $\frac{3}{8}, \frac{5}{6}$.	12. $\frac{3}{10}, \frac{3}{5}$.

13. Add $1\frac{1}{2}$, $2\frac{1}{4}$, 3. $1\frac{1}{2}+2\frac{1}{4}+3=1\frac{2}{4}+2\frac{1}{4}+3=6\frac{3}{4}$.

Add the following mixed numbers:

14.	15.	16.	17.	18.
$3\frac{1}{3}$	$3\frac{1}{2}$	$1\frac{1}{4}$	$1\frac{1}{2}$	$2\frac{1}{8}$
$2\frac{1}{6}$	$1\frac{2}{3}$	$7\frac{3}{4}$	$7\frac{3}{8}$	$3\frac{3}{4}$
$4\frac{1}{2}$	$4\frac{1}{6}$	$6\frac{1}{2}$	$6\frac{1}{4}$	$6\frac{7}{8}$

19. How many oranges in a box containing $\frac{2}{3}$ doz., $\frac{5}{6}$ doz., and $\frac{11}{12}$ doz.?

33. IMPROPER FRACTIONS AND MIXED NUMBERS

1. What is your idea of $\frac{5}{4}$ of an apple?

2. Why can we not *properly* speak of $\frac{5}{4}$ of one apple?

A **proper** *fraction is always less than one.*

3. In adding fractions have you ever found a result that contained more fractional units than a whole one contains?

4. Add $\frac{3}{4}$ ft. + $\frac{2}{4}$ ft. Is $\frac{5}{4}$ ft. greater or less than 1 ft.? How many 4ths greater?

5. In $1\frac{1}{4}$ ft. how many different units are represented?

A fraction equal to or greater than a whole one is an **improper** *fraction.* *A number made up of a whole number and a fraction is a* **mixed number.**

A whole number is often called an *integer*.

Exercises

1. Which of these are fractions and which integers?
   8 feet; $\frac{2}{3}$ inch; \$42; $\frac{5}{6}$ rd.; $\frac{22}{3}$ yd.; \$$\frac{6}{4}$; $\frac{9}{8}$ qt.

2. What does $\frac{4}{4}$ of a dollar equal? $\frac{12}{12}$ of a foot?

3. Is \$$\frac{5}{4}$ more or less than \$1? How much?

4. Is $\frac{18}{12}$ ft. more than 1 foot? How much?

5. Tell which of these are improper fractions:
   $\frac{2}{3}$ yd.; $\frac{9}{8}$ gal.; $\frac{23}{12}$; $\frac{6}{7}$; $\frac{5}{4}$; $\frac{7}{8}$.

6. What is the value of \$$\frac{7}{4}$? Of $\frac{15}{12}$ ft.? Of $\frac{10}{4}$ gal.?

7. Change the following to mixed numbers:

\$$\frac{3}{2}$; \$$\frac{5}{4}$; $\frac{7}{2}$ qt.; $\frac{8}{3}$ yd.; $\frac{7}{4}$ bu.; $\frac{19}{16}$ lb.; $\frac{11}{2}$ qt.

8. How many 4ths in $2\frac{1}{4}$? In $1\frac{3}{4}$? In $3\frac{3}{4}$?

9. Change $\frac{138}{5}$ to a mixed number.

WORK

5 fifths)138 fifths
$27\frac{3}{5}$

Therefore $\frac{138}{5} = 27\frac{3}{5}$.

EXPLANATION.—5 fifths = 1; in 138 fifths there are as many 1's as there are 5's in 138, or $27\frac{3}{5}$.

Change to mixed numbers:

10. $\frac{63}{8}$.	15. $\frac{128}{14}$.	20. $\frac{88}{11}$.	25. $\frac{71}{12}$.
11. $\frac{48}{9}$.	16. $\frac{681}{10}$.	21. $\frac{48}{17}$.	26. $\frac{86}{13}$.
12. $\frac{43}{6}$.	17. $\frac{281}{8}$.	22. $\frac{65}{14}$.	27. $\frac{75}{16}$.
13. $\frac{107}{12}$.	18. $\frac{47}{16}$.	23. $\frac{98}{11}$.	28. $\frac{97}{16}$.
14. $\frac{347}{13}$.	19. $\frac{55}{14}$.	24. $\frac{64}{15}$.	29. $\frac{58}{14}$.

30. Which is more, $\frac{234}{16}$ ft. or $14\frac{9}{16}$ ft.?

Problems in Addition and Subtraction of Fractions

1. A certain rug is made of $\frac{1}{4}$ flax, $\frac{1}{3}$ hemp, and the rest jute. What part is jute?

2. A lady bought $18\frac{3}{4}$ yards of silk for a dress and had $2\frac{1}{8}$ yards left. How much did she use?

3. I filled my coal bin $\frac{2}{3}$ full in the fall. In the spring it was yet $\frac{1}{6}$ full. What part of a binful had I used?

4. A girl bought three remnants of ribbon. One piece contained $3\frac{1}{4}$ yards, another $2\frac{1}{2}$ yards, and the third $6\frac{3}{4}$ yards. How much did the ribbon cost her at 10¢ a yard?

5. After spending $\frac{1}{4}$ of his money for a suit and $\frac{1}{6}$ of it for an overcoat, a boy had what part left?

6. Lucile had $2\frac{5}{6}$ yards of braid. She used $1\frac{1}{3}$ yards. How much had she left?

7. A milliner trimmed three hats one day. She used $2\frac{3}{4}$ yd. of silk velvet for one, $3\frac{1}{4}$ yd. for another, and $4\frac{1}{4}$ yd. for the third. How much did the velvet cost her at \$2.40 a yard?

8. From a remnant of lace $1\frac{1}{8}$ yd. long, 18 inches were cut. What part of a yard remained?

9. If you live $1\frac{1}{4}$ miles from school and James lives $\frac{7}{8}$ of a mile nearer, how far does he live from the school?

10. John is $4\frac{3}{4}$ ft. tall and Henry is 3 in. taller. How tall is Henry?

11. In June Harry weighed $87\frac{3}{4}$ pounds. In October he weighed $93\frac{1}{2}$ pounds. How much had he gained?

12. If $3\frac{1}{3}$ in. should be cut from each end of a yard stick, how much would remain?

Drill Exercises

1. $\frac{1}{2}+\frac{3}{4}$.	9. $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}$.	17. $\frac{3}{4}-\frac{1}{2}$.	25. $\frac{7}{8}-\frac{5}{16}$.
2. $\frac{3}{4}+\frac{2}{3}$.	10. $\frac{1}{2}+\frac{2}{3}+\frac{3}{4}$.	18. $\frac{7}{8}-\frac{3}{4}$.	26. $\frac{15}{16}-\frac{3}{8}$.
3. $\frac{2}{3}+\frac{3}{5}$.	11. $\frac{1}{5}+\frac{1}{2}+\frac{7}{10}$.	19. $\frac{5}{6}-\frac{1}{3}$.	27. $\frac{13}{14}-\frac{3}{7}$.
4. $\frac{1}{2}+\frac{2}{3}$.	12. $\frac{5}{6}+\frac{3}{4}+\frac{2}{3}$.	20. $\frac{9}{10}-\frac{3}{5}$.	28. $\frac{6}{7}-\frac{5}{14}$.
5. $\frac{3}{5}+\frac{1}{2}$.	13. $\frac{7}{8}+\frac{1}{4}+\frac{1}{2}$.	21. $\frac{7}{8}-\frac{3}{16}$.	29. $\frac{3}{8}-\frac{1}{4}$.
6. $\frac{1}{2}+\frac{5}{6}$.	14. $\frac{9}{10}+\frac{4}{5}+\frac{1}{2}$.	22. $\frac{3}{4}-\frac{5}{12}$.	30. $\frac{2}{3}-\frac{1}{4}$.
7. $\frac{2}{3}+\frac{4}{5}$.	15. $\frac{5}{6}+\frac{7}{12}+\frac{3}{4}$.	23. $\frac{2}{3}-\frac{5}{9}$.	31. $\frac{5}{6}-\frac{1}{2}$.
8. $\frac{1}{4}+\frac{5}{6}$.	16. $\frac{3}{8}+\frac{3}{4}+\frac{1}{2}$.	24. $\frac{4}{5}-\frac{3}{10}$.	32. $\frac{7}{8}-\frac{3}{4}$.

Drill Table

Using any two adjacent fractions in either columns or rows, give their sums, then their differences.

	A.	*B.*	*C.*	*D.*	*E.*	*F.*	*G.*	*H.*	*I.*	*J.*
1.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{5}{8}$	$\frac{7}{16}$	$\frac{1}{2}$	$\frac{1}{3}$	$\frac{7}{9}$	$\frac{2}{3}$	$\frac{5}{6}$	$\frac{1}{2}$
2.	$\frac{2}{3}$	$\frac{1}{6}$	$\frac{3}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{5}{6}$	$\frac{2}{3}$	$\frac{8}{9}$	$\frac{1}{2}$	$\frac{3}{5}$
3.	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{7}{8}$	$\frac{3}{4}$	$\frac{2}{3}$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{2}{3}$	$\frac{4}{5}$	$\frac{7}{10}$

Exercises with Mixed Numbers

1. $2\frac{1}{2} + 3\frac{1}{4} + 7\frac{3}{8}$.	**8.** $7\frac{2}{3} - 5\frac{1}{6}$.	**15.** $6\frac{1}{2} - 3\frac{1}{4}$.
2. $5\frac{3}{4} + 6\frac{7}{8} + 5\frac{1}{2}$.	**9.** $8\frac{3}{4} - 6\frac{1}{2}$.	**16.** $3\frac{5}{8} - 1\frac{1}{4}$.
3. $7\frac{2}{3} + 6\frac{5}{9} + 8\frac{1}{3}$.	**10.** $9\frac{7}{8} - 8\frac{3}{4}$.	**17.** $6\frac{7}{8} - 3\frac{3}{4}$.
4. $6\frac{1}{2} + 7\frac{3}{8} + 9\frac{5}{16}$.	**11.** $7\frac{5}{6} - 3\frac{5}{12}$.	**18.** $3\frac{2}{3} - 1\frac{1}{6}$.
5. $6\frac{3}{4} + 5\frac{5}{6} + 8\frac{1}{3}$.	**12.** $18\frac{1}{2} - 12\frac{1}{6}$.	**19.** $7\frac{3}{4} - 5\frac{1}{3}$.
6. $7\frac{1}{2} + 6\frac{3}{5} + \frac{9}{10}$.	**13.** $17\frac{5}{6} - 8\frac{1}{4}$.	**20.** $3\frac{5}{6} - 1\frac{3}{4}$.
7. $8\frac{3}{4} + 7\frac{2}{3} + 1\frac{5}{6}$.	**14.** $15\frac{8}{9} - 6\frac{2}{3}$.	**21.** $5\frac{3}{4} - 3\frac{1}{2}$.

22. Subtract $3\frac{7}{8}$ from $7\frac{1}{4}$.

SOLUTION

$$\begin{array}{r} 7\frac{1}{4} = 7\frac{2}{8} = 6\frac{10}{8} \\ 3\frac{7}{8} = \underline{3\frac{7}{8}} \\ 3\frac{3}{8} \end{array}$$

EXPLANATION. — Since $\frac{7}{8}$ is larger than $\frac{2}{8}$, 1, or $\frac{8}{8}$, is taken from 7 and added to $\frac{2}{8}$. Thus we think $6\frac{10}{8} - 3\frac{7}{8} = 3\frac{3}{8}$.

23. $9\frac{1}{4} - 6\frac{3}{8}$.	**26.** $10\frac{1}{2} - 7\frac{3}{4}$.	**29.** $16\frac{1}{6} - 12\frac{3}{4}$.
24. $8\frac{1}{2} - 3\frac{5}{6}$.	**27.** $12\frac{1}{8} - 6\frac{3}{4}$.	**30.** $12\frac{1}{8} - 9\frac{7}{16}$.
25. $9\frac{1}{4} - 7\frac{5}{6}$.	**28.** $15\frac{1}{3} - 6\frac{5}{6}$.	**31.** $8\frac{1}{2} - 3\frac{11}{16}$.

32. From a box of butter weighing $7\frac{1}{4}$ lb., $3\frac{1}{2}$ lb. were sold at one time and $2\frac{3}{4}$ lb. at another. How much remained?

34. MULTIPLICATION OF A FRACTION BY AN INTEGER

1. 5×7 in. = —— in.
2. 5×7 yd. = —— yd.
3. 6×3 ft. = —— ft.
4. 5×7 eighths = —— eighths.
5. 5×7 twelfths = —— twelfths.
6. 6×3 fifths = —— fifths.
7. Write Exercises 4, 5, and 6 in fractional form.

Thus, $5 \times \frac{7}{8} = \frac{35}{8}$, etc.

> *A fraction is multiplied by an integer by multiplying the numerator, which shows* **how many things**, *and placing the product over the denominator, which shows* **what the things are.**

Exercises and Problems

Find the products of the following:

1. $5 \times \frac{2}{3}$.	5. $5 \times \frac{4}{7}$.	9. $4 \times \frac{3}{7}$.	13. $9 \times \frac{7}{10}$.
2. $6 \times \frac{3}{5}$.	6. $8 \times \frac{3}{5}$.	10. $7 \times \frac{7}{8}$.	14. $6 \times \frac{4}{5}$.
3. $4 \times \frac{2}{5}$.	7. $2 \times \frac{4}{9}$.	11. $3 \times \frac{3}{11}$.	15. $8 \times \frac{7}{9}$.
4. $4 \times \frac{5}{6}$.	8. $6 \times \frac{5}{11}$.	12. $5 \times \frac{5}{8}$.	16. $10 \times \frac{6}{7}$.

17. How much will 6 lb. of crackers cost at $6\frac{1}{2}$ cents a pound?

SOLUTION.

$$\begin{aligned} 6 \times 6¢ &= 36¢ \\ 6 \times \tfrac{1}{2}¢ &= \underline{\ \ 3¢} \\ 6 \times 6\tfrac{1}{2} &= 39¢ \end{aligned}$$

Find the cost of the following:

18. 8 lb. of meat at $16\frac{1}{4}$ cents a pound.
19. 12 yd. of ribbon at $8\frac{1}{2}$ cents a yard.
20. 6 qt. of berries at $8\frac{1}{2}$ cents a quart.

21. 13 qt. of milk at $6\frac{1}{2}$ cents a quart.

22. 8 lb. of lard at $16\frac{1}{2}$ cents a pound.

23. 8 lb. butter at $30\frac{1}{2}$ cents a pound.

24. Find the cost of 9 cans of tomatoes at $16\frac{2}{3}$¢.

25. At $\$1\frac{3}{4}$ each, how much will 12 books cost?

26. Find the cost of 36 yd. of carpet at $\$1\frac{1}{4}$.

27. If 5 dozen oranges cost $\$1\frac{3}{4}$, how much will 20 dozen cost? Suggestion.—20 is how many times 5?

28. If 6 bushels of apples cost $\$2\frac{1}{2}$, how much will 18 bushels cost at the same rate? Suggestion. —— $\times\ \$2\frac{1}{2}$ = ——.

35. MULTIPLYING AN INTEGER BY A FRACTION

1. How do you find $\frac{3}{8}$ of an apple?

2. Find $\frac{3}{8}$ of 24.

3. Multiplying by a fraction has not the same meaning as multiplying by an integer. To multiply a number by $\frac{3}{8}$ means to find $\frac{3}{8}$ of the number. Which term of the fraction is used as a divisor? Which term of the fraction is used as a multiplier?

4. Just as $3 \times 4 = 4 \times 3$, so $24 \times \frac{3}{8} = \frac{3}{8} \times 24$. Find $24 \times \frac{3}{8}$ by the method of § 34 and $\frac{3}{8} \times 24$ by the method of this section, and thus see that both results are the same.

5. Look at the statement in § 34 and make one statement for both sections. Begin thus, "The product of a fraction and an integer is found by ——."

Note.—Unless the integer is a multiple of the denominator, it is best to perform the multiplication first.

Find the cost of the following articles:

6. $\frac{3}{4}$ lb. candy @ 10¢.
7. $\frac{2}{3}$ yd. ribbon @ 16¢.
8. $\frac{5}{6}$ doz. oranges @ 25¢.
9. $\frac{2}{9}$ yd. lace @ 30¢.
10. $\frac{7}{8}$ lb. butter @ 20¢.
11. $1\frac{1}{2}$ yd. ribbon @ 30¢.
12. $2\frac{1}{2}$ lb. crackers @ 10¢.
13. $3\frac{1}{3}$ yd. cotton @ 10¢.
14. $4\frac{1}{8}$ lb. meat @ 15¢.
15. $5\frac{1}{4}$ lb. coffee @ 25¢.

36. THE MEANING OF MULTIPLICATION BY A FRACTION

1. If you did not know the product of $4 \times \$7$, in what other way could $4 \times \$7$ be found? What then does it mean to multiply by an integer?

2. Is it proper to say $\frac{2}{3}$ times \$6? That is, can you take \$6 $\frac{2}{3}$ times and add as in Exercise 1?

> *When the multiplier is a fraction, the multiplication sign* (×) *should be read "of."*

For convenience in writing, multiplication by a fraction is usually written with the sign of multiplication instead of the word "of," but the sign must be *read* "of." Thus, $\frac{2}{3} \times \$6$ is read $\frac{2}{3}$ of \$6.

Read and find the product of the following:

3. $\frac{5}{8} \times 6$.
4. $\frac{3}{7} \times 9$.
5. $\frac{3}{11} \times 8$.
6. $\frac{2}{9} \times 12$.
7. $\frac{3}{4} \times 16$.
8. $\frac{2}{9} \times 21$.
9. $\frac{7}{8} \times 24$.
10. $\frac{5}{9} \times 16$.

11. When a journey of 64 miles is $\frac{3}{4}$ completed, how many miles remain to be traveled?

12. What time is it when $\frac{2}{3}$ of the day of 24 hours has passed?

37. MULTIPLYING BY A MIXED NUMBER

1. How much are $2\frac{3}{4}$ lb. of butter worth at 32¢ a pound?

Evidently, 2 lb. are worth	2×32¢, or 64¢.
And $\frac{3}{4}$ lb. is worth	$\frac{3}{4}$ of 32¢, or 24¢.
Hence $2\frac{3}{4}$ lb. are worth	64¢ + 24¢, or 88¢.

2. Find the product of $25\frac{3}{4} \times 28$.

WORK

$$\begin{array}{r} 28 \\ \underline{25\frac{3}{4}} \\ 4)\underline{84} \\ 21 = \frac{3}{4} \times 28 \\ 140 = 5 \times 28 \\ \underline{560 = 20 \times 28} \\ 721 = 25\frac{3}{4} \times 28 \end{array}$$

Find the product of the following:

3. $16\frac{2}{3} \times 24$.

4. $8\frac{1}{3} \times 375$.

5. $12\frac{1}{2} \times 360$.

6. $22\frac{5}{6} \times 42$.

7. $14\frac{7}{8} \times 96$.

8. $17\frac{5}{6} \times 48$.

9. $42\frac{2}{3} \times 84$.

10. $17\frac{4}{9} \times 36$.

11. $18\frac{5}{6} \times 96$.

12. $48\frac{2}{3} \times 51$.

13. Find the cost of 8 yards of gingham at $16\frac{2}{3}$¢ a yard.

$8 \times 16\frac{2}{3}$¢ $= 8 \times 16$¢ $+ 8 \times \frac{2}{3}$¢ $= 128$¢ $+ 5\frac{1}{3}$¢ $= \$1.33\frac{1}{3}$.

NOTE. — The usual practice of traders is to charge to the nearest cent. Hence in this transaction the purchaser would pay \$1.33 for the gingham. Had the fraction been $\frac{1}{2}$¢ or more, the purchaser would have paid \$1.34 instead of \$1.33.

Make and receipt bills for the following:

14.

$16\frac{3}{4}$ yd. of silk at 85¢.
12 yd. of lining at $16\frac{2}{3}$¢.
1[illegible] of braid at 12¢.

15.

14 yd. of braid at $8\frac{1}{3}$¢.
$8\frac{3}{4}$ yd. of ribbon at 10¢.
$16\frac{1}{2}$ yd. muslin at 8¢.

[illegible]ggs at 32¢.
[illegible]ast at 24¢.
[illegible]rd at 18¢.

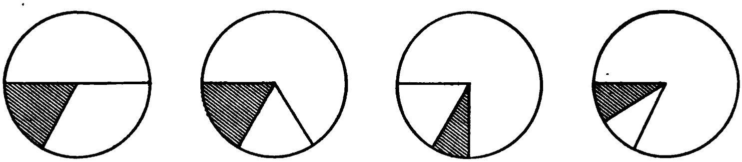
17.

$5\frac{1}{2}$ bu. apples at \$1.20.
$10\frac{1}{4}$ bu. potatoes at 80¢.
$2\frac{1}{4}$ bu. turnips at 60¢.

38. MULTIPLYING A FRACTION BY A FRACTION

We have seen that to multiply an integer by a fraction is to find a part of the integer. Likewise, to multiply a fraction by a fraction is to find a part of the fraction.

Exercises



$\frac{1}{3}$ of $\frac{1}{2} = \frac{1}{6}$ $\frac{1}{2}$ of $\frac{1}{3} = \frac{1}{6}$ $\frac{1}{3}$ of $\frac{1}{4} = \frac{1}{12}$ $\frac{1}{2}$ of $\frac{1}{6} = \frac{1}{12}$

1. Divide $\frac{1}{2}$ a circle by 3; that is, find $\frac{1}{3}$ of $\frac{1}{2}$ a circle.

Remember that to divide by 3 may mean to separate whatever is divided — a number, a whole thing, or a part of a thing — into 3 equal parts.

2. Observe in the circles what it is to divide $\frac{1}{3}$ by 2; $\frac{1}{4}$ by 3; $\frac{1}{6}$ by 2.

3. Draw oblongs to show $\frac{1}{2}$ of $\frac{1}{4} = \frac{1}{8}$. Show $\frac{1}{4}$ of $\frac{1}{2}$.

As in Exercise 3, *show:*

4. $\frac{1}{3}$ of $\frac{1}{4}$.	7. $\frac{1}{5}$ of $\frac{1}{2}$.	10. $\frac{1}{4}$ of $\frac{1}{4}$.
5. $\frac{1}{4}$ of $\frac{1}{3}$.	8. $\frac{1}{2}$ of $\frac{1}{6}$.	11. $\frac{1}{2}$ of $\frac{1}{8}$.
6. $\frac{1}{2}$ of $\frac{1}{5}$.	9. $\frac{1}{6}$ of $\frac{1}{2}$.	12. $\frac{1}{8}$ of $\frac{1}{2}$.

13. What does it mean to find $\frac{3}{4}$ of 8? $\frac{2}{5}$ of 10?

14. What is it to find $\frac{2}{3}$ of $\frac{4}{5}$ of a thing?

15. Find $\frac{2}{3}$ of $\frac{4}{5}$.

SUGGESTION. — $\frac{4}{5}$ divided into 3 equal parts; that is, $\frac{1}{3}$ of $\frac{4}{5}$ is $\frac{4}{15}$, for 5ths divided into 3 equal parts are 15ths. Now since $\frac{2}{3}$ of anything is 2 times $\frac{1}{3}$ of it, we are to take $2 \times \frac{4}{15}$, which are $\frac{8}{15}$.

Find:

16. $\frac{2}{3}$ of $\frac{1}{4}$.

17. $\frac{2}{3}$ of $\frac{3}{4}$.

18. $\frac{2}{3}$ of $\frac{5}{6}$.

19. $\frac{3}{4}$ of $\frac{2}{3}$.

20. $\frac{3}{4}$ of $\frac{1}{2}$.

21. $\frac{3}{4}$ of $\frac{1}{4}$.

22. $\frac{2}{3}$ of $\frac{3}{5}$.

23. $\frac{3}{5}$ of $\frac{2}{3}$.

24. $\frac{2}{3}$ of $\frac{4}{5}$.

Observe that:

> *The product of two or more fractions is a fraction whose numerator is the product of the numerators, and whose denominator is the product of the denominators of the fractions.*

39. MULTIPLYING A MIXED NUMBER BY A FRACTION OR BY ANOTHER MIXED NUMBER

We have seen that an improper fraction may be changed to a whole or mixed number. Likewise, by reversing the work, a whole or mixed number may be changed to an improper fraction.

$$\tfrac{13}{4} = 3\tfrac{1}{4}, \text{ so } 3\tfrac{1}{4} = \tfrac{13}{4}$$

SOLUTION. — $1 = \frac{4}{4}$, and hence $3 = 3 \times \frac{4}{4} = \frac{12}{4}$. So $3\frac{1}{4} = \frac{12}{4} + \frac{1}{4} = \frac{13}{4}$.

In practice we multiply 3 by 4, add 1, and place the result over 4.

Exercises

1. Change $2\frac{3}{4}$ to an improper fraction.

Change to improper fractions and multiply:

2. $2\frac{2}{3} \times 1\frac{1}{4}$.

3. $2\frac{1}{4} \times 5\frac{1}{3}$.

4. $3\frac{7}{8} \times 5\frac{1}{2}$.

5. $1\frac{5}{6} \times 3\frac{1}{3}$.

6. $3\frac{1}{4} \times 2\frac{1}{2}$.

7. $7\frac{1}{8} \times 6\frac{1}{4}$.

8. $2\frac{1}{5} \times 3\frac{2}{3}$.

9. $1\frac{5}{6} \times 5\frac{1}{3}$.

10. $2\frac{1}{6} \times 5\frac{1}{2}$.

11. $1\frac{2}{3} \times 5\frac{3}{4}$.

12. $2\frac{1}{4} \times 5\frac{1}{3}$.

13. $6\frac{1}{2} \times 1\frac{7}{8}$.

Practical Problems in Multiplication

1. A dressmaker found that she had used $21\frac{3}{4}$ yards of cloth in making up 4 coats. If they were all cut from the same pattern, how many yards in each coat?

2. Find the cost at 0.17\frac{1}{2}$ a foot of putting molding around a room 13 ft. by $16\frac{1}{2}$ ft.

3. It takes 2 ounces of butter for a plain cake. How much will it cost at $0.32 a pound?

4. At $0.36 a pound find the cost of 3 ounces of butter.

5. Find cost of making a cheese soufflé from the recipe below. Cheese costs $0.20 per pound; milk $0.08 per quart; eggs $0.24 per dozen; bread $0.05 a loaf, one loaf making a quart of crumbs.

Recipe: $\frac{1}{4}$ lb. cheese, 1 pt. milk,
1 qt. bread crumbs, 4 eggs and seasoning.

6. Allowing 0.37\frac{1}{2}$ per person per day, how much will meals cost for a family of four for the month of July?

7. Find the cost of 3 pecks of potatoes at $1.20 per bushel.

8. If 1 quart of milk costs $0.08, find the cost of 3 gallons.

9. Find the cost of 13 ounces of butter at $0.32 per pound.

10. At $4\frac{1}{8}$ cents per pound find the cost of one barrel of flour (196 lb.).

11. At 45 cents per yard for 50-inch burlap, what would be the cost of a pencil case $6\frac{1}{2}$ inches wide by 10 inches long for each of 72 pupils?

12. The pattern of an envelope is 11 inches by 14 inches. What would be the cost of supplying a class of 40 pupils from paper measuring 22 inches by 28 inches at $5\frac{1}{2}$ cents a sheet?

Problems of a Grocery

Apples	pk.	$0.40	Oranges	doz.	$0.40
Butter	lb.	0.36	Peaches	½ pk.	0.30
Cabbage	lb.	0.04	Pickles	qt.	0.20
Cauliflower	hd.	0.20	Potatoes	pk.	0.36
Celery (3 bunches)		0.10	Rice	lb.	0.08
Coffee	lb.	0.38	Sugar	lb.	0.06
Eggs	doz.	0.32	Tomatoes	lb.	0.06
Flour	24½ lb.	0.70	Corn	doz.	0.15

Write out sales checks and receipts for the following:

1. ½ pk. apples; 2½ lb. butter; 2 hd. cauliflower; 1 pk. peaches.

2. 3½ lb. head cabbage; 6 bunches celery; 1½ doz. oranges; 1 pt. pickles; 1½ doz. corn.

3. 1 sack flour; 1½ doz. eggs; 1½ lb. coffee; 5½ lb. tomatoes; 2½ doz. corn.

4. ½ bu. peaches; ½ pk. potatoes; 7½ lb. sugar; 3½ lb. rice.

5. 1¾ lb. butter; ½ pk. apples; 2 hd. cauliflower; 1½ doz. eggs.

6. 10 lb. tomatoes; 1½ pk. potatoes; 1½ pk. peaches; 9 oranges; 3¼ doz. corn.

7. 2½ lb. coffee; 18 eggs; 3 pt. pickles; 1 lb. 8 oz. coffee.

8. 6¼ lb. head cabbage; 2 lb. 4 oz. rice; ¾ pk. potatoes.

9. 3½ lb. butter; 2½ doz. eggs; 5 lb. tomatoes; 5½ lb. sugar.

10. 1 pk. peaches; ½ pk. potatoes; 3 pt. pickles.

Problems on Market Reports

FRUITS	FARM AND GARDEN
PEARS. — Best, $1.25 per bu.	POTATOES. — Michigan, 55¢ per bu.
APPLES. — Best, $2.25; fair grades, $1.50 per bbl.	SWEET POTATOES. — Jersey, $1.40; Virginia, 85¢ per bu.
PEACHES. — Good, $2 per bu.; fancy, $2.50 per bu.	CHEESE. — Full cream, Michigan, $10\frac{1}{4}$¢ per lb.
GRAPES. — Concords, 25¢ per 10-lb. basket; Niagara, 25¢ per 10-lb. basket.	EGGS. — Regular receipts, 19¢; candled, $22\frac{1}{2}$¢ per doz.
CRANBERRIES. — $6.75 per bbl.	BUTTER. — State creamery, 22¢; extra dairy, 17¢ per lb.
LEMONS. — California, $4.50 per box.	VEGETABLES. — New carrots, 45¢ per bu.; celery, 25¢ per doz.

From the market report shown above, find the cost of the following:

1. 365 bu. potatoes.
2. 12 cases, 30 doz. each, eggs, candled.
3. 378 10-lb. baskets Concord grapes.
4. 216 boxes California lemons.
5. 17 barrels cranberries.
6. 34 bu. peaches, fancy.
7. 48 bbl. best apples.
8. 53 bbl. apples, fair grade.
9. 17 barrels, $2\frac{1}{2}$ bu. each, sweet potatoes, Virginia.
10. How much more would the same quantity of Jersey sweet potatoes cost?
11. 97 lb. extra dairy butter.
12. 54 bushels best pears.
13. 312 baskets of Niagara grapes.
14. 38 cases, 30 doz. each, eggs, regular receipts.
15. 19 full cream cheese, 14 lb. each.
16. 17 bu. new carrots and 36 doz. celery.

Problems of the Meat Market

The following printed notice was distributed by a grocer to all of the houses in his neighborhood:

JENSEN'S MEAT MARKET

864 South St. — Telephone 651
Special Sale — September 17

Spring and Hen Chickens per lb.	18¢
Native Hind Quarter Lamb per lb.	16¢
Short Leg of Lamb per lb.	18¢
Loin or Rib Lamb Chops per lb.	22¢
Pork Loin Roast per lb.	18¢
Beef Tenderloin per lb.	22¢
Native Rib Roast, Beef, per lb.	17¢
Native Beef Pot Roast per lb.	$12\frac{1}{2}$¢
Salted Native Beef Tongue per lb.	18¢
Native Sirloin Steak, choice cuts, per lb.	19¢

Find the cost to customers of the following:

1. A chicken weighing $3\frac{1}{2}$ lb.
2. $1\frac{1}{2}$ lb. beef tenderloin.
3. 4 lb. 4 oz. native hind quarter lamb.
4. 6 lb. native beef pot roast.
5. $4\frac{1}{2}$ lb. short leg of lamb.
6. $1\frac{1}{8}$ lb. native sirloin steak.
7. $4\frac{1}{8}$ lb. pork loin roast.
8. $5\frac{1}{4}$ lb. native rib roast, beef.
9. $2\frac{1}{2}$ lb. lamb chops and $3\frac{1}{4}$ lb. salted beef tongue.
10. Which is the cheaper, and how much, to buy a 4-pound chicken or to buy $3\frac{1}{2}$ lb. of beef tenderloin?

11. In which case do you get more for your money, when you buy 3 lb. chicken of which $\frac{1}{3}$ is waste, or when you buy 2 lb. beef tenderloin with no waste?

12. In which do you get more for your money, when you buy 3 lb. pork loin roast of which $\frac{1}{6}$ is waste, or when you buy $2\frac{1}{2}$ lb. beef tenderloin with no waste?

Problems of the Delicatessen Shop

1. What are some of the things bought at a delicatessen shop?

2. How much do you pay for $\frac{3}{8}$ lb. of boiled ham, at 40¢ a pound?

3. How much do you pay for 1 lb. 2 oz. of veal loaf, at 30¢ a pound?

4. Boiled tongue sells at 48¢ a pound. What is the cost of 7 oz.?

5. Roast beef sells at 40¢ a pound. What is the cost of $2\frac{3}{8}$ lb.?

6. Dried beef costs 32¢ a pound. How much must you pay for 10 oz.?

7. Frankfurts cost 14¢ a pound. Find cost of $3\frac{1}{2}$ lb.

8. Star Sausage costs 40¢ a pound. If you buy 1 lb. 2 oz., how much does it cost you?

9. Corn beef hash sells for 13¢ a pound at the delicatessen. Find the cost of $1\frac{1}{4}$ lb.

10. Spaghetti sells for 12¢ a pound. How much will 12 oz. cost?

11. Potato salad sells for 10¢ a pound. Find the cost of $1\frac{3}{4}$ lb.

12. Roast spring chicken costs 33¢ a pound. How much will you pay for a roast chicken weighing $3\frac{1}{4}$ lb.?

13. At the delicatessen shop, baked beans sell at 15¢ a pint. What is the cost of $1\frac{1}{2}$ pints?

14. Find the cost of $1\frac{3}{4}$ lb. butter at 32¢ a pound.

15. If you pay 10¢ for $\frac{1}{2}$ pt. of cream, how much is that per gallon?

16. Swiss cheese sells at 40¢ a pound, Limburger at 22¢, and cottage cheese at 10¢. Make problems about these.

17. If jumbles are sold at 40¢ a pound, how much must I pay for 10 oz.?

Problems of the Fish Market

Boneless herring per lb. .	20¢	Smoked sturgeon per lb. .	64¢
Salt whitefish per lb. . .	16¢	Smoked salmon per lb. .	40¢
Royal fat mackerel per lb.	25¢	Smoked whitefish per lb. .	20¢
Salt salmon per lb. . . .	15¢	Smoked halibut per lb. .	32¢
Matjes herring, 2 for . .	15¢	Whole codfish per lb. . .	$12\frac{1}{2}$¢

Above are the prices at which fish sold in a market one day. *Find the cost of:*

1. 2 lb. whole codfish.

2. 3 lb. 2 oz. salt whitefish.

3. 1 lb. 4 oz. smoked salmon.

4. 2 lb. 8 oz. boneless herring.

5. 12 oz. smoked sturgeon.

6. $3\frac{1}{4}$ lb. salt salmon.

7. 1 lb. 4 oz. whitefish.

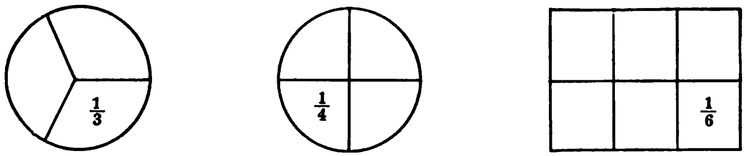
8. 2 lb. 10 oz. mackerel.

9. $\frac{1}{2}$ doz. Matjes herring.

10. 2 lb. 12 oz. smoked halibut.

DIVISION

40. DIVISION OF ONE BY A FRACTION



1. How many *thirds* of a circle in one circle? $1 \div \frac{1}{3} =$ ——.

2. How many fourths of a circle in one circle? $1 \div \frac{1}{4} =$ ——.

3. $1 \div \frac{1}{6} =$ ——. $1 \div \frac{1}{5} =$ ——. $1 \div \frac{1}{8} =$ ——.

4. How many $\$\frac{1}{4}$ in $\$1$? 1 ft. $\div \frac{1}{2}$ ft. = ?

5. How many times is $\frac{3}{4}$ contained in 1, or $\frac{4}{4}$?

SUGGESTION. — From the circle notice that $\frac{3}{4}$ is contained in 1, or $\frac{4}{4}$, 1 time, with $\frac{1}{4}$ of the circle remaining; but $\frac{1}{4}$ is $\frac{1}{3}$ of $\frac{3}{4}$, hence $1 \div \frac{3}{4} = 1\frac{1}{3}$ or $\frac{4}{3}$.

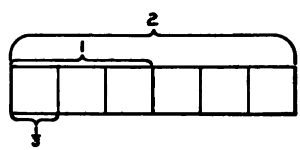
> *When* 1 *is divided by a fraction, the result is the fraction with its terms interchanged.*

6. Give quickly: $1 \div \frac{3}{8}$; $1 \div \frac{5}{6}$; $1 \div \frac{2}{7}$; $1 \div \frac{3}{7}$; $1 \div \frac{5}{16}$.

41. DIVISION OF AN INTEGER BY A FRACTION

1. Which is greater, the quotient of $4 \div 2$ or $8 \div 2$? The quotient of $1 \div \frac{1}{4}$ or $2 \div \frac{1}{4}$?

2. Here we have a rectangle divided by $\frac{1}{3}$ of the rectangle. The $\frac{1}{3}$ is contained 3 times. In the two rectangles the $\frac{1}{3}$ is contained 6 times.



3. As in Exercise 2, show that $3 \div \frac{1}{2}$ is 3 times as great as $1 \div \frac{1}{2}$.

In the same way show that

4. $3 \div \frac{1}{4}$ is 3 times as great as $1 \div \frac{1}{4}$.

5. $4 \div \frac{1}{3}$ is 4 times as great as $1 \div \frac{1}{3}$.

6. $5 \div \frac{1}{5}$ is 5 times as great as $1 \div \frac{1}{5}$.

Give the following:

7. $1 \div \frac{1}{4}$.
8. $2 \div \frac{1}{4}$.
9. $3 \div \frac{1}{4}$.
10. $4 \div \frac{1}{4}$.
11. $1 \div \frac{2}{3}$.
12. $2 \div \frac{2}{3}$.
13. $3 \div \frac{2}{3}$.
14. $4 \div \frac{2}{3}$.
15. $1 \div \frac{3}{4}$.
16. $2 \div \frac{3}{4}$.
17. $1 \div \frac{2}{5}$.
18. $2 \div \frac{2}{5}$.
19. $3 \div \frac{2}{5}$.
20. $1 \div \frac{3}{8}$.
21. $2 \div \frac{3}{8}$.

When any integer is divided by a fraction the result is the product of the dividend and the fraction with its terms interchanged.

Practical Problems in Division

1. Into how many pieces $\frac{3}{4}$ of a yard long can 24 yards of ribbon be cut?

2. If carpet is $\frac{3}{4}$ of a yard wide, how many strips are needed for a room 6 yards wide?

3. Some girls made fudge for a fair. They put $\frac{3}{8}$ of a pound in each 15¢ box. How many boxes did they need for 9 pounds of fudge?

4. How many cards $3\frac{1}{2}'' \times 5\frac{1}{3}''$ can be cut from a card $14'' \times 16''$?

5. From $10\frac{1}{2}$ yd. of ribbon, badges $\frac{1}{4}$ yd. long were cut. How many did it make?

6. A seamstress found that she could average making $2\frac{1}{4}$ aprons a day. How long will it take her to make 90 aprons?

7. If it takes $\frac{3}{8}$ yd. of lace to trim a neck band, how many such neck bands can be trimmed from 12 yards?

8. If your mother puts up 4 quarts of jelly in $\frac{1}{2}$ pint glasses, how many glasses will she need?

42. THE DIVISION OF A FRACTION BY A FRACTION

1. 6 ft. will contain 2 ft. how many times?
2. $6 will contain $2 how many times?
3. 6 ninths will contain 2 ninths how many times?
4. $\frac{6}{8} \div \frac{2}{8} = ?$ $\frac{6}{9} \div \frac{2}{9} = ?$ $\frac{6}{12} \div \frac{2}{12} = ?$ $\frac{6}{15} \div \frac{2}{15} = ?$

> ***Fractions with like units are divided one by the other, by dividing the numerator of the dividend by the numerator of the divisor.***

5. 6 qt. will contain 3 pt. how many times?
6. 3 bu. will contain 2 pk. how many times?
7. In Exercises 5 and 6, what change was made in the dividends before dividing?
8. 3 fourths will contain 3 eighths how many times?

> ***Evidently fractions with unlike units must be changed to fractions with like units before dividing.***

In case the numerator of the dividend is smaller than that of the divisor, indicate the quotient by writing the numerator of the dividend over that of the divisor. Thus, $\frac{7}{12} \div \frac{9}{12} = \frac{7}{9}$.

9. $\frac{2}{3} \div \frac{5}{6}$.

10. $\frac{3}{4} \div \frac{5}{8}$.

11. $\frac{2}{3} \div \frac{2}{9}$.

12. $\frac{3}{4} \div \frac{3}{16}$.

13. $\frac{7}{8} \div \frac{3}{16}$.

14. $\frac{3}{4} \div \frac{1}{2}$.

15. $\frac{7}{8} \div \frac{3}{4}$.

16. $\frac{5}{6} \div \frac{1}{3}$.

17. $\frac{7}{8} \div \frac{1}{4}$.

18. $\frac{11}{12} \div \frac{2}{3}$.

19. $\frac{7}{12} \div \frac{3}{4}$.

20. $\frac{15}{16} \div \frac{3}{8}$.

21. $\frac{11}{14} \div \frac{3}{7}$.

22. $\frac{2}{3} \div \frac{3}{4}$.

23. $\frac{2}{3} \div \frac{4}{5}$.

24. $\frac{3}{4} \div \frac{2}{3}$.

43. DIVIDING ANY NUMBER BY ANY OTHER NUMBER

Either the dividend or the divisor or both may be an integer, a fraction, or a mixed number. The method of dividing given in Section 42 may be followed in all cases provided that *both dividend and divisor are first changed to* **like units**.

1. $2\frac{2}{3} \div 5 = \frac{8}{3} \div \frac{15}{3} = 8 \div 15 = \frac{8}{15}$.

2. $5 \div 2\frac{1}{4} = \frac{20}{4} \div \frac{9}{4} = 20 \div 9 = \frac{20}{9} = 2\frac{2}{9}$.

3. $1\frac{3}{4} \div 1\frac{1}{8} = \frac{7}{4} \div \frac{9}{8} = \frac{14}{8} \div \frac{9}{8} = 14 \div 9 = 1\frac{5}{9}$.

Drill Exercises

1. $2\frac{2}{3} \div 1\frac{1}{2}$.

2. $5\frac{1}{4} \div 2\frac{1}{2}$.

3. $6\frac{2}{3} \div 1\frac{1}{2}$.

4. $7\frac{3}{4} \div 5\frac{2}{3}$.

5. $1\frac{2}{5} \div 2$.

6. $2\frac{3}{4} \div 7\frac{1}{2}$.

7. $6\frac{3}{8} \div 2\frac{1}{4}$.

8. $3\frac{2}{3} \div 1\frac{1}{4}$.

9. $12\frac{1}{2} \div 2\frac{2}{3}$.

10. $24\frac{1}{2} \div 3\frac{1}{4}$.

11. $17\frac{1}{2} \div 6\frac{1}{4}$.

12. $16\frac{2}{3} \div 5$.

13. $26 \div 3\frac{1}{2}$.

14. $42 \div 6\frac{2}{3}$.

15. $16 \div 3\frac{3}{4}$.

16. $26 \div 8\frac{1}{3}$.

17. $42 \div 3\frac{1}{2}$.

18. $3\frac{1}{2} \div 6\frac{2}{3}$.

19. $2\frac{1}{4} \div 8\frac{1}{3}$.

20. $1\frac{2}{3} \div 2\frac{1}{4}$.

21. $\frac{3}{4} \div 1\frac{7}{8}$.

22. $2\frac{1}{12} \div 3\frac{5}{6}$.

23. $\frac{7}{12} \div \frac{2}{3}$.

24. $8 \div 9\frac{1}{2}$.

25. $9\frac{1}{2} \div 8$.

26. $6\frac{3}{4} \div 2\frac{5}{8}$.

27. $5\frac{3}{4} \div 5\frac{9}{12}$.

Practical Problems in Division. Paper Cutting

1. From a sheet of cardboard 22 inches by 28 inches, how many cards $3\frac{2}{3}$ inches by 4 inches can be cut so that there is no waste?

2. Can cards $5\frac{1}{2}$ inches by 7 inches be cut from a sheet 22 inches by 28 inches without waste? How many?

3. From a sheet of blotting paper 18 inches by 24 inches, how many small blotters $4\frac{1}{2}$ inches by 8 inches can be cut?

4. From the same sheet how many blotters $2\frac{1}{4}$ inches by 4 inches can be cut?

5. How much is wasted by cutting from the same sheet blotters $4\frac{1}{2}$ inches by $5\frac{3}{4}$ inches?

6. Demy paper measures $15\frac{1}{2}$ inches by $18\frac{1}{2}$ inches. Cutting crosswise and then lengthwise we make 4 equal sheets. What are the dimensions of each sheet?

7. A sheet of medium paper 18 inches by 22 inches is cut into 12 equal sheets by cutting twice lengthwise and three times crosswise. What is the size of each small sheet?

8. How many sheets 6 inches by 9 inches of manila drawing paper can be cut from 500 sheets of paper 24 inches by 36 inches?

44. RATIO

The quotient of two like numbers is called their **ratio.** Thus the ratio of 6 to 2 is 3, for $6 \div 2 = 3$. The ratio of 2 to $6 = \frac{1}{3}$, for 2 is $\frac{1}{3}$ of 6.

With fractions we can express the ratio of any two numbers of like units. Thus the ratio of 7 to 3 is $2\frac{1}{3}$, for $7 \div 3 = 2\frac{1}{3}$. The ratio of 3 to 7 is $\frac{3}{7}$, for $3 \div 7 = \frac{3}{7}$.

Give the ratios:

1. 8 to 2; 2 to 8.
2. 12 to 4; 4 to 12.
3. 10 to 5; 5 to 10.
4. 16 to 4; 4 to 16.
5. 18 to 6; 6 to 18.
6. 20 to 4; 4 to 20.
7. 14 to 7; 7 to 14.
8. 24 to 6; 6 to 24.
9. 32 to 8; 8 to 32.
10. 28 to 7; 7 to 28.
11. 3 to 5; 5 to 3.
12. 4 to 9; 9 to 4.
13. 5 to 11; 11 to 5.
14. 6 to 15; 15 to 6.
15. 5 to 12; 12 to 5.
16. 8 to 20; 20 to 8.
17. 6 to 25; 25 to 6.
18. 7 to 12; 12 to 7.
19. 8 to 13; 13 to 8.
20. 9 to 15; 15 to 9.

21. What is the ratio of $\frac{1}{2}$ lb. to $\frac{1}{4}$ lb.?

SOLUTION. — Since $\frac{1}{2} \div \frac{1}{4} = 2$, the ratio is 2.

22. What is the ratio of $\frac{1}{4}$ lb. to $\frac{1}{2}$ lb.?

SOLUTION. — Since $\frac{1}{4}$ is but $\frac{1}{2}$ of $\frac{1}{2}$, the ratio is $\frac{1}{2}$.

Give the ratios:

23. $\frac{1}{3}$ to $\frac{1}{6}$; $\frac{1}{6}$ to $\frac{1}{3}$.
24. $\frac{1}{2}$ to $\frac{1}{8}$; $\frac{1}{8}$ to $\frac{1}{2}$.
25. $\frac{1}{4}$ to $\frac{1}{12}$; $\frac{1}{12}$ to $\frac{1}{4}$.
26. $\frac{1}{3}$ to $\frac{1}{9}$; $\frac{1}{9}$ to $\frac{1}{3}$.
27. $\frac{1}{4}$ to $\frac{1}{16}$; $\frac{1}{16}$ to $\frac{1}{4}$.
28. $\frac{1}{6}$ to $\frac{1}{12}$; $\frac{1}{12}$ to $\frac{1}{6}$.
29. $\frac{3}{8}$ to $\frac{5}{4}$; $\frac{5}{4}$ to $\frac{3}{8}$.
30. $\frac{3}{8}$ to $\frac{7}{16}$; $\frac{7}{16}$ to $\frac{3}{8}$.
31. $\frac{4}{5}$ to $\frac{3}{10}$; $\frac{3}{10}$ to $\frac{4}{5}$.
32. $\frac{3}{4}$ to $\frac{11}{16}$; $\frac{11}{16}$ to $\frac{3}{4}$.
33. $\frac{5}{9}$ to $\frac{2}{3}$; $\frac{2}{3}$ to $\frac{5}{9}$.
34. $\frac{5}{8}$ to $\frac{3}{16}$; $\frac{3}{16}$ to $\frac{5}{8}$.

Practical Use of Simple Ratios

1. If a $5\frac{1}{2}$-lb. roast costs \$1.21, how much will an 11-lb. roast cost at the same rate?

SUGGESTION. — Since 11 lb. is just twice as much as $5\frac{1}{2}$ lb., the cost will be twice as much.

2. At 40 cents a pound, how much will 4 ounces of spices cost? Compare 16 and 4.

3. When $\frac{1}{2}$ yd. of ribbon costs 18 cents, how much should $\frac{1}{4}$ yd. cost?

4. When $\frac{1}{2}$ pound of tea is worth 40¢, how much shall I pay for $\frac{1}{4}$ pound?

5. When $\frac{1}{4}$ bushel of potatoes is worth 20¢, how much shall I pay for $\frac{1}{2}$ bushel?

6. When $\frac{1}{2}$ acre yields 10 bushels of wheat, what should $\frac{1}{4}$ acre yield?

7. When $\frac{1}{4}$ barrel of flour weighs 49 pounds, what will $\frac{1}{2}$ barrel weigh?

8. When 5 things cost $8, how much will 10 of the same kind cost? How much will 15 cost?

9. When I can buy 6 large marbles for 25¢, how much shall I have to pay for 12? For 24?

10. A farmer had 18 calves. He sold 6 for $90. At this rate, what sum should he receive for the other 12?

11. 3 collars for 25¢ makes 6 cost how much? 9 will cost how much? What is the price per dozen?

12. 32¢ per peck is how much per bushel? How much per quart?

13. I have 5 acres of beans in one field and 15 in another. If I gather 98 bushels from the small field, how many bushels shall I expect from the larger?

14. If I get 285 bushels from the 15-acre field, how many shall I expect to get from the 5-acre field?

15. When 4 cords of wood cost $23.40, what sum shall I have to pay for 24 cords?

16. When $\frac{1}{4}$ ton of coal is worth \$1.75, how much must I pay for $\frac{1}{2}$ ton? How much for 1 ton?

17. When $\frac{1}{8}$ of a farm is worth \$3200, how much is $\frac{1}{2}$ of it worth? ($\frac{1}{2}$ is how many times as large as $\frac{1}{8}$?)

18. A farmer sold 20 bushels of potatoes. This was $\frac{1}{4}$ of all he raised. How many did he keep?

19. When onions are 35¢ per peck, what will 6 bushels cost?

20. When a 20-lb. cheese is worth \$2.90, how much will a 10-lb. cheese cost? Find from your answer the cost of a 30-lb. cheese. The cost of a 90-lb. cheese.

IV. MENSURATION

45. LINEAR OR LENGTH MEASURE

1. How wide is your book? How long is it? What *unit of measure* did you use in answering these questions?

2. How long is your schoolroom? How wide is it? What unit of measure did you use in answering these questions?

3. What unit of measure do you use in asking for cloth, braid, or ribbon at a dry goods store?

4. How many inches in a foot? Feet in a yard?

5. Take a strong cord and measure off $16\frac{1}{2}$ ft. on it. This makes another kind of unit used in measuring larger distances, as the length of a field. It is called a **rod**.

6. Estimate a rod on the floor, then measure it. Was your estimate correct?

7. How many steps do you take in walking one rod? (Try it and count them.)

8. Step off 2 rods; 4 rods; 10 rods.

9. If you are asked the distance between two cities or the length of a state, you use a longer unit called **miles**. Do you know some place 1 mile away? 320 rods make one mile. How many of your steps make one mile? Step off one mile when you take a long walk.

Remember the following table of length measure:

12 inches (in. or ″)	**= 1 foot (ft. or ′)**
3 feet	**= 1 yard (yd.)**
$16\frac{1}{2}$ feet or $5\frac{1}{2}$ yards	**= 1 rod (rd.)**
320 rods	**= 1 mile (mi.)**

Exercises in Length Measure

1. How many inches in a yard? In 12 yards?

2. How many yards in a mile?

3. How many feet in a mile?

4. Estimate and then measure the length of your schoolroom in rods.

5. Estimate and then measure the length of your school grounds in rods.

6. How many yards in a mile?

7. Estimate the length of your blackboard in feet. Then measure it and see how nearly accurate your judgment is.

8. Estimate, then measure the length of your schoolroom.

9. Estimate, then measure the length of the block upon which your school building stands, first in yards, then in rods.

10. Estimate 10 rods out of doors, then measure it. How nearly did you estimate?

NOTE. — Have pupils estimate both short and long distances as suggested above, to develop accuracy in judgment of distance.

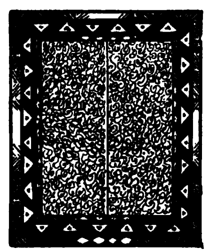
11. The doctor directs a convalescent to walk $\frac{1}{4}$ mile on the porch every day. The porch is 40 feet in length. How many times must the man walk across the porch to fulfill the doctor's directions?

12. If the promenade deck of an ocean liner allows passengers to walk 160′, how much more than $\frac{1}{2}$ a mile did Mr. Jones walk on Wednesday, if he crossed the deck 18 times?

13. The best walker on board crossed the 160-foot deck 40 times. Is that farther than from your house to school? Is it more or less than a mile? How much?

14. An aviator reached an altitude of 6000 feet in his flying machine. Was that more or less than a mile, and how much?

15. Mrs. Johnson bought 4 yards of brussels carpet (27″ wide) and had it cut in two lengths and stitched. To this was added a border 12″ wide. How long and how wide was the completed rug?



16. How many yards of border were used, if corners were mitered?

17. Carpet is not sold by the inch or foot, but by the yard. Find the cost of the rug, if the body is $1.50 per yard and the border 65¢ per yard. Allow $2 for making.

18. A park 600 ft. long and 450 ft. wide. How far does one walk in going twice around the park?

46. THE MEASURE OF SURFACES

1. Draw a square inch upon cardboard. How many such squares will cover a surface 3 inches wide and 4 inches long?

2. Upon the blackboard draw a square foot.

3. Cut a square foot from a piece of paper. How many such squares will cover a surface on the floor 3 feet wide and 5 feet long?

4. Draw a square yard upon the blackboard.

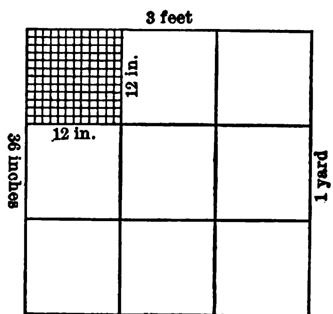
5. How many square yards of paper will cover a surface of the wall 2 yards wide and 4 yards long?

> *In measuring surfaces, as walls, floors, fields, etc., the unit is a square of some size and we call the measure square measure.*

6. Draw a square foot upon the blackboard. Divide it into square inches. How many?

7. Draw a square yard upon the blackboard. Divide it into square feet. How many?

8. On the floor of your schoolroom or upon the playground draw a square rod. 160 such squares make 1 acre, a unit used in measuring land.



Scale $\frac{1}{24}$. One square yard

9. How long is a square rod in feet?

10. How long is the perimeter of a square rod?

Remember the following table of square measure:

144 square inches (sq. in.)	**= 1 square foot**	**(sq. ft.)**
9 square feet (sq. ft.)	**= 1 square yard**	**(sq. yd.)**
$30\frac{1}{4}$ square yards (sq. yd.)	**= 1 square rod**	**(sq. rd.)**
160 square rods (sq. rd.)	**= 1 acre (A.)**	

Problems in Square and Linear Measure

1. Draw an oblong or rectangle 3 inches wide and 4 inches long.

2. Divide it into oblongs 1 inch wide and 4 inches long.

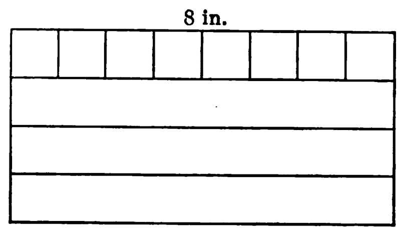
3. How many square inches in one of these oblongs?

4. How many square inches in the whole oblong?

Draw on a "scale of $\frac{1}{4}$," that is, represent an inch by $\frac{1}{4}$ of an inch, and find the area of rectangles with these dimensions:

5. 8 inches long and 4 inches wide.

SUGGESTION. — This may be divided into 4 strips 8 inches long and 1 inch wide, each containing —— sq. in. The area then is equal to 4 × 8 sq. in., or 32 sq. in. In what other way could you have divided it?



Scale $\frac{1}{4}$

6. 12 in. long and 5 in. wide.

7. 9 in. long and 7 in. wide.

8. 16 in. long and 12 in. wide.

9. 10 in. long and 7 in. wide.

10. 12 in. long and 8 in. wide.

11. 18 in. long and 4 in. wide.

12. How many square feet are there in the floor of a room 16 feet long and 12 feet wide?

13. How many square inches are there in a yard of ribbon 3 inches wide?

14. How many square yards of carpet will be needed for a room 18 feet long and 15 feet wide?

15. How many square feet are there in the top of a table $7\frac{1}{2}$ feet long and 4 feet wide?

16. A man concreted the sidewalk in front of his house at a cost of $0.75 per square yard. The walk is 9 feet wide and 42 feet long. What did it cost him?

17. Mr. A's farm is 175 rods long and 84 rods wide. How many acres does it contain?

18. What is the whole area of the sides and ends of a strawboard box 12 inches by 8 inches by 4 inches?

19. How much will it cost to bind the top edge of a strawboard box 8 inches long and 7 inches wide, with paper tape at 4 cents a foot?

20. How many feet of gummed tape are required to bind the edges of a bristol board tray 11 inches long, 4 inches wide, and 1 inch high?

21. How many square inches in a table runner 2 yards long and $\frac{1}{2}$ yard wide?

22. Six threads of a table runner 15 inches wide and 54 inches long are to be drawn for hemstitching. What is the combined length of the drawn threads?

23. How many square inches of paper would be required for a 52-page post-card album, each page measuring 8 inches by 6 inches?

47. MEASURING RECTANGULAR SURFACES: SCALE DRAWING

1. Describe a rectangle. What name do you give to a rectangle whose sides are equal? What do you call a rectangle which is longer than it is wide?

2. Draw a rectangle 4″ long by 3″ wide. What is the length of the perimeter; that is, what is the distance around the rectangle?

3. Divide this rectangle into inch squares. How many rows are there? How many squares in each row? What is the area or the number of square inches in this rectangle? You have drawn this rectangle on the scale of 1; that is, 1 inch in your drawing represents 1 inch of the rectangle. Draw the rectangle on the scale of 2; that is, let 2 inches in the drawing represent 1 inch of the rectangle.

4. Draw the rectangle again on the scale of $\frac{1}{2}$; that is, let $\frac{1}{2}$ inch in your drawing represent 1 inch in the rectangle.

5. Suppose you were asked to draw a rectangle 4′ long and 3′ wide. Can such a rectangle very well be drawn on paper? If you draw this large rectangle on the scale of $\frac{1}{12}$, an inch in the drawing will represent a foot in the rectangle, and your drawing will measure 3″ by 4″. Draw the rectangle on this scale.

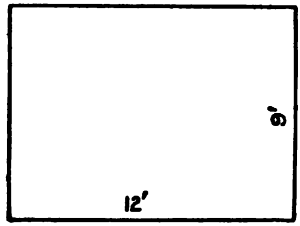
6. What is the area of a rectangle measuring 3′ by 4′?

7. Suppose a rectangle drawn 3″ wide and 4″ long represents a rectangle 3 *yards* wide and 4 *yards* long. What is the scale of such a drawing; that is, what does 1 inch in your drawing represent in the rectangle? What is the area of such a rectangle?

8. Suppose that an inch in the drawing represents a *rod* in the rectangle; what is the area of the rectangle?

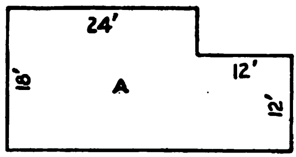
9. Suppose that an inch in a 3″ by 4″ drawing represents a *mile* in a rectangle; what is the area of the rectangle?

10. Make a drawing of a rug, 9′ by 12′. Draw on a scale of $\frac{1}{12}$, — that is, one foot to the inch. What is the perimeter of this rug? What is its area?



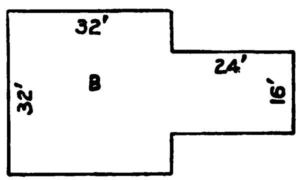
Scale $\frac{1}{96}$.

11. Find the perimeter of the rectangle shown in the drawing marked *A*. On what scale is the drawing made?



12. Divide the rectangular figure *A* into two rectangles and find the area of each. What is the area of the entire rectangular surface?

13. Find the area of the rectangular surface represented by the drawing *B*. You will need to separate it into two rectangles.

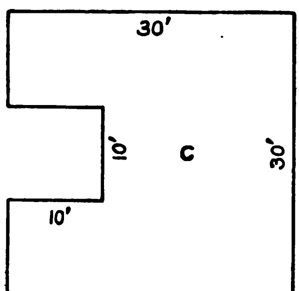


14. What is the distance around the entire lot of land which the drawing may represent?

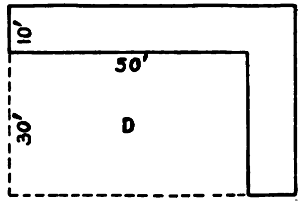
15. *C* represents a lot of land. What is the scale of the drawing?

16. What is the distance around this lot of land?

17. Divide the lot into three rectangles and find the area of each. How many square feet in the area of the lot?



18. Drawing *D* represents a walk on two sides of a corner lot measuring 30′ by 50′. The sidewalk is 10′ wide. What is the length of the entire walk? Be careful not to count the length of the walk at the corner of the lot twice.



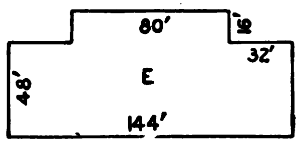
Scale 40′ to 1″.

19. What is the area of the sidewalk?

20. What is the area of the lot?

21. Reproduce this drawing on the scale of 20′ to the inch. Draw it again on the scale of 10′ to the inch.

22. Find the perimeter of the lot of land represented by the drawing *E*.

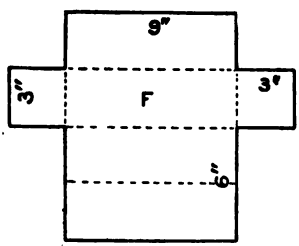


Scale 96′ to 1″.

23. Find the area of this lot of land.

24. Reproduce the drawing on the scale of 32′ to an inch.

25. From tagstock or cardboard cut out the rectangular surface represented in the drawing *F*, making your drawing on a scale of 1. Fold along the dotted lines to make a square prism.



26. How many square inches of cardboard were used in making the prism in Exercise 25? How many were wasted?

27. Draw a rectangular plan of a box 8″ × 4″ × 2″ as in Exercise 25, and make the box.

Exercises in the Cutting of Cards and Paper

1. A sheet of cardboard is 22″ by 28″. How many cards measuring 4″ by $3\frac{1}{2}$″ may be cut from it with the least possible waste? How many square inches of the sheet will be wasted?

2. How many cards 5″ by 7″ may be cut from a sheet of the same size? How many square inches of the surface of the sheet will be wasted?

3. How many cards 3″ by 9″ may be cut from a sheet of the same size? What part of the whole sheet is wasted?

4. If a sheet of blotting paper measures 19″ by 24″, how many individual blotters measuring $3\frac{1}{2}$″ by 8″ may be cut from it? How many square inches of the surface of the blotter are wasted?

5. From a sheet of the same size, how many blotters 3″ by 8″ may be cut without any waste?

6. Demy paper measures $15\frac{1}{2}$″ by $18\frac{1}{2}$″. How many sheets of letter paper measuring $7\frac{3}{4}$″ by $9\frac{1}{4}$″ can be cut from a single demy sheet?

7. A ream of paper generally contains 500 sheets. How many reams of demy size will be needed to cut 12 reams of letter paper of the size indicated in the preceding exercise?

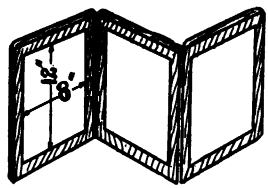
8. A sheet of medium paper, 18″ by 22″, is cut into 12 equal sheets. What may the size of each of the smaller sheets be?

9. How many sheets of folding paper, 7″ by 7″, may be made from a ream of 500 sheets of cover paper, 22″ by 28″?

10. How many pieces of folding paper 5″ square may be cut from a ream of cover paper?

Miscellaneous Problems in Measurement

1. In planning a triple shaving mirror, each glass measures 12″ by 8″. How many feet of oak molding are needed, allowing 1 foot for waste?



2. Find the cost of the glass at $1.50 a square foot.

3. Would it be cheaper to buy the glass at $1\frac{1}{4}$¢ a square inch?

4. A kimono pattern for a doll measures 14″ by 12″. What will be the cost of 6 kimonos cut from pink lawn 36″ wide at 15¢ a yard? Would you be able to buy 28″ of material, or must you buy a yard and have some waste?

5. A piece of vellum 12 inches by 18 inches is required to cover one side of a portfolio. How many students can be supplied from a 36-yard roll of vellum one yard wide?

6. For the foundation covers of an art book it takes 2 pieces of strawboard 9″ × 12″. How many covers can be cut from 1 doz. sheets of board 26″ × 28″?

7. To cover this book, making it with stubs and Japanese lacings, it takes 2 pieces of cloth 11″ × 13″. How many covers can be made from $4\frac{3}{4}$ yd. of material 36″ wide?

8. There are 8 sections in a book 7″ × 10″. Each section contains two double leaves (14″ × 10″). How many leaves can be cut from 10 sheets of construction paper 22″ × 28″? How much is wasted?

9. How many books will that make, and what fraction of a book remains?

10. A pupil requires 115 in. of lacing cord to finish three models. If there are 75 yd. in one spool of cord, how many pupils will one such spool provide for?

11. For a book cover 12″ long it requires a strip of bookbinder's linen 25″ × 5″. How many strips can you cut from $2\frac{1}{2}$ yd. of linen 38″ wide?

12. How many yards will it take to cover 50 box kites with silk 20 inches wide? Each kite requires a piece of silk 10″ × 48″.

13. How much will the silk cost at 70¢ a yard for the 50 kites? How much is that for each kite?

14. How much can be saved by covering the 50 kites with cambric 27″ wide costing 8¢ a yard? How much is wasted?

15. At the store a towel roller costs 35¢. George made one for his mother. He used $1\frac{1}{2}$ ft. of lumber at 8¢ a foot, 2¢ worth of hardware, and 3¢ worth of shellac. Find how much George saved his mother.

16. How much will it cost at 30¢ a square foot for hard leather to cover a table top 40″ × 64″, allowing 2″ extra all around for turning down and tacking?

17. If brass head tacks are used to decorate the edge of this table, how much will they cost at 22¢ a dozen? They are placed 2 inches apart.

18. Miss Hanson expects 150 girls to come to cooking school during a week. Each girl must have a cap of lawn, requiring a piece of lawn 20″ × 10″. Lawn is 36″ wide, and costs 25¢ a yard. Find cost of caps for all the class and the cost per cap.

Kitchen Furnishings and Supplies

1. Find cost of calcimining the walls and ceiling of a kitchen 12 feet wide, 15 feet long, and 10 feet high, at $4\frac{1}{2}$ cents a square foot.

2. Find cost of putting a molding around a kitchen 10 feet wide and 15 feet long, at \$0.17$\frac{1}{2}$ a foot.

3. How many square yards of linoleum will be necessary to cover a kitchen floor 15 feet square? How much will it cost at 65¢ a square yard?

4. Floor oilcloth costs \$0.60 a square yard. How much will it cost to cover a floor 12 feet wide and 15 feet long?

5. It requires one pint of oil to a square yard. Find the cost of oiling a kitchen floor 12 feet long by 10 feet wide. The oil costs \$1.20 a gallon.

6. How much oilcloth will be required to cover a working table 42 inches long and 28 inches wide? The oilcloth is 36 inches wide. How much is wasted?

7. Make and receipt a bill for kitchen furnishings bought of F. G. Smith, 227 Canal St., on Sept. 2, 1910: one gas range, \$26.00; one ice-box, \$25.00; one small top oven, \$1.50; and two kitchen tables, at \$1.65 each.

8. Make and receipt a bill for sundry articles bought of K. T. Jones, October 14, 1910: two dish pans, granite, at \$0.50 each; one sink strainer at \$0.18; one bread box at \$1.00; one cake box at \$0.85; two flour cans at \$1.25 each; four crocks, 2-gallon, at \$0.25 each; three dozen glass jars at \$0.75 a dozen; and one set of spice jars at \$3.50.

9. At \$0.30 a dozen find the cost of 40 pint jars.

10. It takes 5 pints of jelly to fill three dozen glasses. How many will it take to fill twelve dozen such glasses?

11. Bought of L. M. Miller, wholesale grocer, 31 S. Water St., Dec. 7, 1910: 50 lb. of flour (whole wheat) at $2.15; one bbl. wheat flour at $8.50 a bbl.; one 50-lb. sack of sugar at $0.06 a lb.; one 25-lb. keg of butter at $0.25 per lb.; and one crate eggs — 12 dozen — at $0.27 per dozen.

12. How many pounds of butter will a family use in three months at the rate of two pounds a week?

13. What will the milk bill be for one month for a family that uses one quart of milk and one pint of cream a day, if milk costs $0.08 per quart and cream $0.10 per one-half pint?

Miscellaneous Problems

1. Make and receipt a bill for goods bought of "The Success," November 25, 1910: one rug, 9′ × 12′, at $24.00; one dining table at $27.00; six chairs at $3.25 each; four tablecloths at $2.50 each; and five dozen napkins at $1.95 a dozen.

2. When dining-room chairs sell for $48.00 a dozen, find the cost of nine chairs.

3. How many feet of plate rail will be required for a dining room $18\frac{1}{2}$ feet long and $15\frac{1}{2}$ feet wide, deducting 20 feet for window and sideboard space?

4. Find the cost of varnishing a floor 24 feet long and 15 feet wide, when the varnish costs $2.40 a gallon and for every square yard of flooring it takes $\frac{1}{4}$ pint of varnish.

5. Bread and butter knives cost $7.00 a half-dozen. Find cost of one and one half dozen.

6. For a family of six how many table napkins will be used in one month, allowing three per week apiece?

7. There are 5280 feet in a mile. How many feet in $\frac{5}{8}$ of a mile?

8. Into how many pieces $\frac{3}{4}$ of a yard long can 24 yards of ribbon be cut?

9. How many breadths of carpet $\frac{2}{3}$ of a yard wide will be needed for a room 12 feet wide?

10. How much will be paid for $18\frac{3}{4}$ yards of carpeting at $1.60 a yard?

11. $\frac{5}{6}$ of my money is in silver, and I have $15 besides. How much money have I in all?

12. Find the cost of $\frac{3}{4}$ of a ton of ice at 25¢ a hundredweight (100 lb.).

13. When $\frac{5}{8}$ of a pound costs 40¢, what is the price per pound?

14. How many days are there in $\frac{4}{5}$ of a common year?

15. Add $\frac{3}{8}$ of a day, $\frac{5}{6}$ of a day, and eight hours. Give the sum in hours.

16. 72 gallons have been drawn from a tank of oil. It is then $\frac{3}{5}$ full. How many gallons will the tank hold?

17. A boy who works for $3 a week pays 15 cents daily for his lunch and 60 cents a week for car fares. How much has he left Saturday night?

18. How many square yards in a tablecloth measuring 72 inches one way and $4\frac{1}{2}$ yards the other?

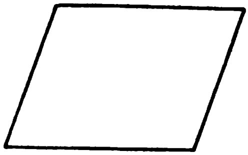
19. How many pounds of sugar at $6\frac{1}{4}$¢ can I buy for $1?

Find the cost of:

20.	$\frac{3}{4}$ yd. calico @ 10¢ $1\frac{2}{3}$ yd. ribbon @ 18¢	21.	$8\frac{1}{2}$ lb. chicken @ 22¢ $6\frac{1}{2}$ lb. steak @ 24¢

48. MEASURING PARALLELOGRAMS

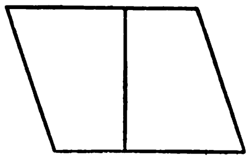
The opposite sides of a rectangle are **parallel.** The angles made by any two sides that meet at a corner are right **angles.** When the opposite sides are parallel, as in the drawing shown here, the drawing is a **parallelogram,** whether the angles are right angles or not. The angles shown in this drawing are **oblique angles.**



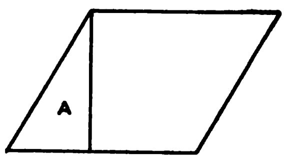
NOTE. — The pupil should be shown how to draw parallel lines, also how to fold and crease paper in parallel lines, and how to draw and to fold right angles.

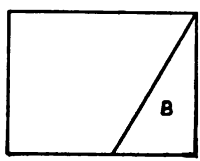
1. Draw a parallelogram with one side 3 inches long, and another side 2 inches long.

2. If you draw a line from one side to another in a parallelogram, so that the line makes right angles with the sides, as in the drawing, the line shows the **height** of the parallelogram. Draw a parallelogram with one side 4 inches and height 3 inches.



3. Take this parallelogram and by a line making right angles with one side, called the **base,** cut off a triangle like the one marked A. Take it away and place it in the position of the one marked B. What kind of figure have you?





4. Fold paper to form a parallelogram. Fold a right triangle as triangle A in the drawing. Tear off A and place it in the position B.

5. Compare the length of the parallelogram with that of the rectangle formed from it. Compare the height of the two. If you have done the work carefully you have found that

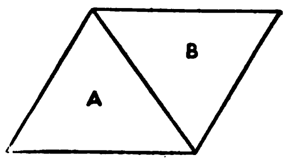
> *The area of a parallelogram is equal to that of a rectangle having the same base and height.*

Give the areas of the following parallelograms:

6. Base 8 in., height 4 in.
7. Base 10 in., height 6 in.
8. Base 12 in., height 5 in.
9. Base 15 in., height 5 in.

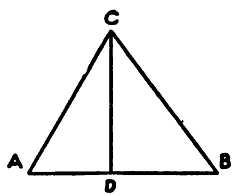
49. MEASURING TRIANGLES

1. Draw a parallelogram. Draw a diagonal dividing the parallelogram into two triangles. Compare them, that is, do they seem to be equal?



2. From paper fold a parallelogram. Fold and tear along a diagonal, thus making two triangles. Place one triangle upon the other and in this way compare them.

3. How many triangles like A, in Exercise 1, will be needed entirely to cover the parallelogram?



4. The corners of a triangle are called its **vertices**. How many vertices has a triangle?

5. A line drawn from one vertex to the opposite side, making right angles with the side, or **base**, is the **height**, or **altitude**, of the triangle. Name the altitude and base of the triangle shown here.

6. The base and height of a triangle or parallelogram are called its *dimensions*. How does the size of a triangle compare with that of a parallelogram with the same dimensions?

7. If you did your work carefully in Exercises 1, 2, and 3, you found that

> *A triangle is just half as large as a parallelogram having the same dimensions.*

Since the area of a parallelogram is equal to that of a rectangle having the same dimensions, it follows that

> *The area of a triangle is equal to one half that of a rectangle having the same dimensions as the base and altitude of the triangle. That is, the area of a triangle is one half of the product of its dimensions.*

8. What is the area of a rectangle whose dimensions are 8 inches and 5 inches?

9. What is the area of a triangle whose base is 8 inches and altitude 5 inches?

10. Find the area of a triangle whose base is 10 inches and altitude 7 inches.

11. Find the area of a triangle whose base is 12 inches and altitude 8 inches.

Give the areas of the following triangles:

12. Base 10 in., height 10 in.

13. Base 12 in., height 5 in.

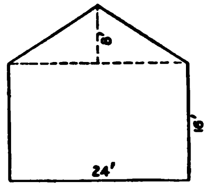
14. Base 20 in., height 6 in.

15. Base 15 in., height 6 in.

16. Base 18 in., height 5 in.

17. Base 14 in., height 7 in.

18. The diagram represents the gable end of a barn.



19. Give the base and altitude of the triangular part, the *gable*. 8 ft. is the height above the *eaves*.

20. What is the area of the triangular part represented in the figure?

21. Find the entire area of the end of the barn.

50. THE MEASURE OF VOLUMES

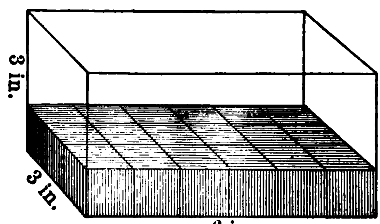
1. How many one-inch cubes will fill a box 2 inches wide, 4 inches long, and 3 inches deep?

2. Make a framework for a cubic foot and cover with paper. How many of these will fill a box 3 feet wide, 3 feet long, and 2 feet deep?

3. Make a framework and cover a cubic yard. How many will fill a room 3 yards wide, 3 yards long, and 4 yards high?

4. How many cubic inches in the bottom layer of this rectangular solid? How many such layers?

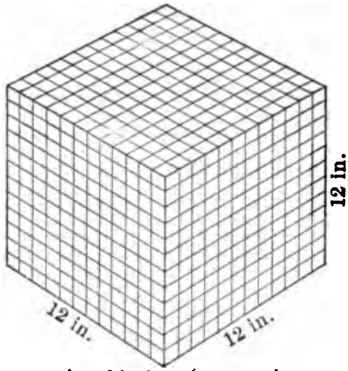
Then there are 3×18 cubic inches in the whole solid, and we say the **volume** is 54 *cubic inches*.



Scale $\frac{1}{4}$

In finding the volume of a rectangular solid, first find the number of cubes in one layer, then the number of layers, and then the total number of cubes.

5. How many cubic inches in one layer of a 12-inch cube? How many layers? Then how many cubic inches in a **cubic foot**?



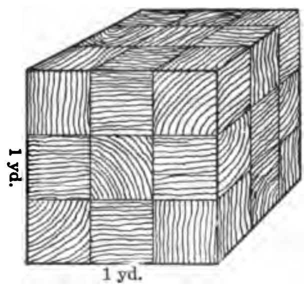
A cubic foot (Scale $\frac{1}{12}$)

6. 2 cu. ft. = —— cu. in.

7. Suppose you should separate a cubic foot into halves, how many cubic inches in each half? In $\frac{1}{4}$ of a cubic foot how many cubic inches?

8. If you should divide a foot cube into 8 equal cubes, how many cubic inches would each one of them contain?

9. How many cubical blocks, each a foot long, would be needed for a pile 8 ft. long, 7 ft. wide, and 3 ft. high?



Scale $\frac{1}{36}$

10. How many cubic feet will fill a box 1 yard long, 1 yard wide, and 1 yard deep? Then how many cubic feet in a cubic yard?

11. How many cubic feet in 2 cubic yards? How many in 8 cubic yards?

In measuring the volumes or capacities of rooms, boxes, bins, etc., the unit is a cube of some size, and we call the measure cubic measure.

12. Give the volume of a coal bin 10 feet long, 10 feet wide, and 6 feet deep.

13. Give the volume of a room 5 yards wide, 6 yards long, and 3 yards high.

Remember the following table of cubic measure:

1728 cubic inches (cu. in.)	**= 1 cubic foot (cu. ft.)**
27 cubic feet	**= 1 cubic yard (cu. yd.)**

Problems in Cubic Measure

1. What is the volume of a rectangular solid $4'' \times 5'' \times 6''$?

2. Does a piece of timber $8'' \times 10'' \times 14''$ contain more or less than a cubic foot? How much?

3. How many cubic inches in a brick $2'' \times 4'' \times 8''$? How many such bricks in a cubic foot?

4. A schoolroom 12 ft. high, 25 ft. wide, and 30 ft. long contains 40 pupils. This is how many cubic feet to each pupil?

5. I am making 3 window boxes, each 1 ft. wide, 1 ft. deep, and 8 ft. long. I asked the gardener to bring me a load (1 cubic yard) of earth to fill them. Is this enough? How much will be left?

6. For 2 small boxes $8'' \times 6'' \times 4'$ I had 3 cubic feet of earth brought. How much was left?

7. How many cakes of soap 2 in. wide and 3 in. long can you lay on the bottom of a box 1 ft. square? If the box is 9 in. deep, and each cake $1\frac{1}{2}$ in. thick, how many cakes will the box hold?

8. How many cubic feet of bricks in a pile 8 ft. long, 4 ft. high, and 3 ft. wide? How many bricks in the pile if 27 bricks contain 1 cu. ft.?

PART TWO: SIXTH YEAR

V. FRACTIONS

51. MEANING AND USES OF FRACTIONS

We are familiar with some of the uses of fractions. To measure a line 5 ft. long with a rule 1 ft. long, we have no need of fractions. We compare the line with the rule and say that the line is 5 times as long; that is, it is 5 ft. long. A *whole number*, or an integer, is sufficient. However, if the line had been more than 5 ft. and less than 6 ft. long, to measure it with a 1 ft. rule would require a *fraction* to express the relation of that part *less* than 1 ft. to the 1 ft. measure.

> *A fraction, then, is a* **relation number.** *And since the relation of one number to another is found by division, we may call a fraction an* **indicated division.**

Exercises

By use of fractions express the relation of:

1. 2 ft. to 3 ft.
2. $3 to $4.
3. $6 to $10.
4. 3 in. to 8 in.
5. 9 oz. to 1 lb.
6. 3 qt. to 1 gal.
7. 7 in. to 1 ft.
8. 3 pk. to 1 bu.
9. 4 da. to 1 wk.
10. 16 da. to 1 mo. (30 da.).
11. 7 mo. to 1 yr.
12. 20 min. to 1 hr.

13. How would you find $\frac{3}{4}$ of an object, as a square?

14. How would you find $\frac{3}{4}$ of a number, as 12, 16, or 20?

15. Which term of a fraction denotes the division? What is it called?

16. Which term of a fraction denotes a multiplication, or shows how many fractional units have been taken? What is it called?

17. When an object has been divided into any number of equal parts, each part is called a *unit fraction*, or a *fractional unit*. Thus, $\frac{1}{4}$ ft.; $\$\frac{1}{5}$; $\frac{1}{8}$ in.; and $\frac{1}{16}$ lb. are unit fractions. Name other unit fractions.

18. Which is larger, $\frac{1}{4}$ ft. or $\frac{1}{6}$ ft.? $\frac{1}{3}$ yd. or $\frac{1}{4}$ yd.? $\frac{1}{8}$ lb. or $\frac{1}{12}$ lb.?

19. Does a large denominator denote a very large or a very small fractional unit?

20. Which is larger, $\frac{5}{8}$ in. or $\frac{7}{8}$ in.? $\frac{11}{16}$ lb. or $\frac{13}{16}$ lb.?

21. Does a large numerator indicate a large or a small fraction?

22. How many 8ths of an inch in one inch? Is $\frac{9}{8}$ in. more or less than 1 in.? How much?

23. Name five fractions that are less than 1; five that are equal to 1; five that are greater than 1.

24. *Name* the unit in each of the following, and also tell *how many* units there are:

$\frac{3}{4}$ in.; $\frac{5}{8}$ gal.; $\frac{7}{8}$ bu.; $\frac{3}{7}$ wk.; $\frac{9}{16}$ lb.

25. Change to mixed numbers:

$\frac{7}{4}$ in.; $\frac{23}{16}$ lb.; $\frac{7}{3}$ yd.; $\frac{17}{12}$ ft.

26. Change to improper fractions:

$3\frac{1}{2}$ ft.; $4\frac{3}{8}$ lb.; $5\frac{7}{12}$ ft.; $3\frac{5}{7}$ wk.

52. REDUCTION OF FRACTIONS: COMMON DENOMINATOR

We have seen that the *number* of units that make up any object may be changed if at the same time their *size* is changed to correspond. Thus we may speak of a gallon as 4 qt., or as 8 pt.; of a yard as 3 ft., or as 36 in. In the same way 15 ft. and 5 yd. represent the same length.

Exercises

1. Give other examples of equal values expressed in units of different sizes.

2. Compare $\frac{1}{4}$ and $\frac{3}{12}$, and tell why the value of $\frac{3}{12}$ is no greater than $\frac{1}{4}$.

3. How does an *increase* in the size of the denominator affect the size of the unit?

4. How does an *increase* in the numerator affect the value of a fraction?

5. How does a *decrease* in the denominator affect the size of the unit?

6. How does a *decrease* in the numerator affect the value of a fraction?

7. If each term of $\frac{8}{18}$ be made $\frac{1}{2}$ as large, how is the value affected? Why is this?

8. Make each term of $\frac{2}{3}$ three times as large. Explain how the change affects the *size* and the *number* of the units.

> *Both terms of a fraction may be multiplied or divided by the same number without changing the value of the fraction.*

9. By the use of this principle, change to 40ths:

$\frac{8}{8}$; $\frac{7}{10}$; $\frac{5}{4}$; $\frac{9}{5}$; $\frac{40}{80}$; $\frac{24}{160}$; $\frac{40}{200}$; $\frac{3}{20}$; $\frac{160}{240}$; $\frac{48}{240}$; $\frac{96}{480}$.

10. Change to 12ths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{5}{6}$.

11. Change to like units: $\frac{3}{2}$; $\frac{3}{4}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{1}{4}$.

12. Change to like units: $\frac{2}{3}$; $\frac{1}{6}$; $\frac{2}{9}$; $\frac{5}{6}$; $\frac{5}{18}$.

13. Change to like units: $\frac{3}{5}$; $\frac{7}{10}$; $\frac{4}{15}$; $\frac{2}{3}$; $\frac{5}{6}$.

> *When fractions are changed to like units they have a* **common denominator.** *The* **smallest** *common denominator to which given fractions can be changed is called their* **least** *common denominator.*

53. LEAST COMMON MULTIPLE

NOTE. — As all fractions used in practical applications may be changed to like units by inspection, this topic need not be taken up at this time.

Since $3 \times 4 = 12$, 12 is called a *multiple* of 3 or of 4. 12 is also a multiple of 2 and 6. If one number will contain another exactly it is a **multiple** of that number.

A number that is not a multiple of any other number is a **prime** number. Thus, 2, 3, 5, 7, 11, 13, 17, 19, and 23 are *prime numbers.*

Exercises

1. Show that 36, 60, 72, and 120 are *multiples* of 12.

2. Show that 50, 60, 80, and 120 are multiples of 10.

3. Show that 60 and 120 are common multiples of 10 and 12.

4. Show that 60 is the least common multiple (L. C. M.) of 10 and 12. That is, show that 60 is the least number that will contain both 10 and 12.

> *The* **least common denominator** *of two or more fractions is the* **least common multiple** *of their denominators.*

5. Find the *least common multiple* of 30 and 42.

WORK

$30 = 2 \times 3 \times 5.$
$42 = 2 \times 3 \times 7.$
$42 \times 5 = 210$, the L. C. M.

EXPLANATION. — A multiple of 30 must contain all the prime factors of 30, and a multiple of 42 all the prime factors of 42. Now, a number whose factors are 2, 3, 5, and 7 will contain $2 \times 3 \times 5$, or 30, and also $2 \times 3 \times 7$, or 42. In practical work we say 30×7, or 42×5, since 7 is not found in 30, or 5 in 42.

6. Find the L. C. M. of 60, 72, and 108.

WORK

$60 = 2 \times 2 \times 3 \times 5.$
$72 = 2 \times 2 \times 2 \times 3 \times 3.$
$108 = 2 \times 2 \times 3 \times 3 \times 3.$
$108 \times 2 \times 5 = 1080$, the L. C. M.

(*a*) What prime factor of 60, needed in the L. C. M., is not found in 108?

(*b*) What prime factor of 72 is not found in 108, that is, found more times in 72 than 108?

(*c*) What is meant by the least common multiple of several numbers?

7. Find the L. C. M. of 60, 84, and 132.

8. What is the least common multiple of 45, 90, 100, 200?

SUGGESTION. — Is a multiple of 90 a multiple of 45?
Is a multiple of 200 a multiple of 100?
Notice that we need simply to find the L. C. M. of 90 and 200.

Find the L. C. M. of:

9. 15, 21, 45.	**12.** 16, 18, 27, 72.	**15.** 16, 25, 80, 100.
10. 16, 32, 48.	**13.** 14, 42, 35.	**16.** 16, 24, 40.
11. 18, 27, 54.	**14.** 12, 18, 48.	**17.** 12, 24, 60.

54. ADDITION AND SUBTRACTION OF FRACTIONS

Add the following:

1. $\frac{3}{4}$, $\frac{5}{6}$, $\frac{7}{12}$.	**7.** $13\frac{1}{3}$, $16\frac{5}{6}$.	**13.** $16\frac{1}{5}$, $3\frac{2}{3}$.
2. $\frac{4}{5}$, $\frac{2}{3}$, $\frac{8}{15}$.	**8.** $7\frac{3}{8}$, $4\frac{9}{16}$.	**14.** $4\frac{3}{4}$, $6\frac{1}{3}$.
3. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{11}{18}$.	**9.** $3\frac{2}{3}$, $4\frac{3}{5}$.	**15.** $3\frac{4}{5}$, $7\frac{1}{2}$.
4. $\frac{3}{4}$, $\frac{7}{8}$, $\frac{9}{16}$.	**10.** $6\frac{7}{8}$, $3\frac{1}{3}$.	**16.** $9\frac{1}{2}$, $3\frac{5}{7}$.
5. $\frac{5}{9}$, $\frac{1}{3}$, $\frac{1}{6}$.	**11.** $7\frac{2}{3}$, $6\frac{3}{4}$.	**17.** $6\frac{3}{4}$, $7\frac{5}{6}$.
6. $\frac{2}{3}$, $\frac{5}{6}$, $\frac{7}{9}$.	**12.** $9\frac{3}{5}$, $10\frac{2}{3}$.	**18.** $7\frac{1}{2}$, $9\frac{2}{3}$.

Fractions of larger terms are added by the following method:

19. Add $\frac{11}{15}$ and $\frac{7}{18}$.

PROCESS

$15 = 3 \times 5.$ $\qquad \frac{11}{15} = \frac{66}{90}$ (why?)

$18 = 3 \times 3 \times 2.$ $\qquad \frac{7}{18} = \frac{35}{90}$ (why?)

$18 \times 5 = 90$, the L. C. M. $\qquad$ Sum $= \frac{101}{90}$ or $1\frac{11}{90}$

EXPLANATION.—90 is the L. C. M. of the denominators. 90ths, then, is the largest common unit to which both 15ths and 18ths can be changed.

Add:

20. $\frac{7}{15} + \frac{4}{21}$.	**23.** $\frac{29}{48} + \frac{29}{32}$.	**26.** $\frac{7}{40} + \frac{17}{30} + \frac{61}{80}$.
21. $\frac{31}{33} + \frac{17}{18}$.	**24.** $\frac{8}{10} + \frac{625}{1000}$.	**27.** $\frac{5}{36} + \frac{19}{32} + \frac{17}{16}$.
22. $\frac{7}{24} + \frac{11}{21}$.	**25.** $\frac{3}{7} + \frac{5}{12} + \frac{23}{42}$.	**28.** $\frac{35}{80} + \frac{49}{112}$.

Subtract:

29. $\frac{2}{3}-\frac{1}{2}$.	32. $\frac{7}{8}-\frac{2}{3}$.	35. $\frac{7}{8}-\frac{2}{7}$.	38. $8\frac{1}{8}-6\frac{3}{4}$.
30. $\frac{5}{6}-\frac{2}{3}$.	33. $\frac{7}{8}-\frac{3}{4}$.	36. $3\frac{1}{2}-1\frac{3}{4}$.	39. $9\frac{1}{2}-3\frac{5}{6}$.
31. $\frac{3}{4}-\frac{2}{3}$.	34. $\frac{5}{6}-\frac{1}{4}$.	37. $7\frac{1}{4}-3\frac{2}{3}$.	40. $10\frac{1}{4}-6\frac{7}{8}$.

Practical Problems in Addition and Subtraction

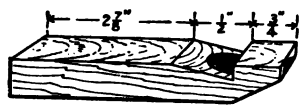
1. I mailed three packages. One weighed $\frac{3}{4}$ lb., one $\frac{7}{8}$ lb., and one $\frac{11}{16}$ lb. What was their total weight?

2. If I buy $\frac{3}{4}$ yd. of lace in one piece, $\frac{5}{12}$ yd. in a second piece, and $\frac{8}{9}$ yd. in a third at a remnant counter, how much do I get in all three pieces?

3. I buy $8\frac{2}{3}$ yd. of goods for one dress and $10\frac{3}{4}$ yd. for another. How many yards do I get for both dresses?

4. If you drive $14\frac{1}{4}$ miles in the forenoon, and $12\frac{7}{8}$ miles in the afternoon, how far do you drive during the whole day?

5. The figure gives the dimensions of a whistle made from a piece of wood. If $\frac{3}{16}''$ must be allowed for waste in making, how long should the piece of wood be from which it is constructed?



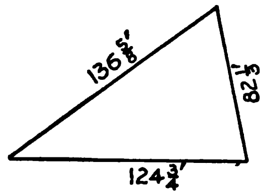
6. A milliner bought two pieces of hat lining. One piece contained $52\frac{1}{3}$ yd., and the other $44\frac{3}{4}$ yd. How many yards in both pieces? Of this she sold $10\frac{3}{4}$ yd., $7\frac{1}{3}$ yd., and $12\frac{5}{12}$ yd. How much did she sell in all?

7. From a bolt of cloth a clerk sold $\frac{2}{5}$ of it at one time and $\frac{1}{4}$ of it at another. How much of it did he sell in all?

8. A farmer built $\frac{3}{4}$ mile of board fence, and $1\frac{1}{8}$ miles of wire fence in one season. How much did he build in all?

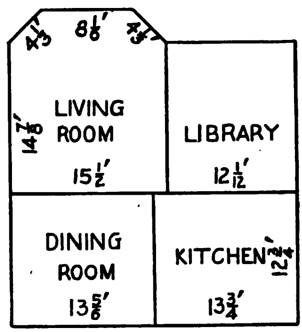
9. A train leaving Chicago at 1.25 P.M. reaches Davenport in $5\frac{3}{8}$ hr. At what time of day does it reach Davenport?

10. A crash towel is to be $27\frac{1}{2}$ in. long when finished. If it takes $\frac{5}{8}$ in. at each end for a hem, how long must the piece of crash be cut in making it?



11. How many feet around this triangle?

12. An iron-holder is to be $5\frac{1}{2}$ in. square when finished. Allowing $\frac{3}{8}$ in. on each side for turning in of material, what must be the length and the width of the pattern?



13. This is the plan of the first floor of a house. Find the number of feet of picture molding required for the living room.

14. Find the amount of picture molding required for the library.

15. Find the number of feet of plate rack for the dining room, allowing $8\frac{1}{3}$ ft. for openings.

16. Find the number of feet of baseboard for the kitchen, allowing $7\frac{1}{2}$ ft. for openings.

The prices of wheat per bushel reported on the Chicago market September 26, 1910, were as follows:

Wheat	Opening	Closing	Highest	Lowest
September . .	$97\frac{1}{8}$¢	$96\frac{3}{4}$¢	$97\frac{3}{8}$¢	$96\frac{3}{4}$¢
December . .	$99\frac{7}{8}$¢	$99\frac{1}{2}$¢	$100\frac{1}{4}$¢	$99\frac{1}{2}$¢
May	$105\frac{1}{2}$¢	105^{1} ¢	$105\frac{7}{8}$¢	$105\frac{1}{4}$¢

17. What was the difference between the opening and the closing prices of September wheat? Between the highest and the lowest prices paid?

18. What was the difference between the opening and the closing prices of December wheat? Between the highest and the lowest prices?

19. What was the difference between the opening and the closing prices of May wheat? Between the highest and the lowest prices?

20. Get a large city daily newspaper and find the prices of wheat, corn, oats, pork, and lard reported on the New York or the Chicago market yesterday. Find the differences between the opening and the closing prices, and between the highest and the lowest prices given for the day.

21. From an iron bar $9\frac{7}{16}$ inches long, a piece $3\frac{3}{4}$ inches long is cut. How much remains?

22. From a piece of cloth containing $14\frac{2}{3}$ yd., one piece, containing $6\frac{3}{8}$ yd., and another, containing $4\frac{3}{4}$ yd., are cut. How much is left?

23. A towel is to be made from a piece of toweling $28\frac{3}{4}$ in. long. If $\frac{5}{16}$ in. is used at each end for a hem, how long will the finished towel be?

24. If $\frac{7}{8}$ yd. is cut from a piece of lace $1\frac{1}{9}$ yd. long, what part of a yard is left?

25. A piece of meat weighing $3\frac{3}{4}$ lb. was roasted. After it was roasted it weighed only $2\frac{5}{8}$ lb. How much weight did it lose in cooking?

26. Apples that weighed $6\frac{7}{8}$ lb. were dried. They made $1\frac{3}{4}$ lb. of dried apples. How much water did the apples contain originally?

55. MULTIPLICATION OF FRACTIONS

1. Compare $\frac{4}{5}$ and $\frac{1}{5}$; $\frac{4}{5}$ and $\frac{2}{5}$; $\frac{12}{21}$ and $\frac{4}{21}$; $\frac{16}{17}$ and $\frac{4}{17}$.

2. Compare $\frac{1}{5}$ and $\frac{1}{20}$; $\frac{1}{6}$ and $\frac{1}{18}$; $\frac{1}{7}$ and $\frac{1}{21}$; $\frac{1}{4}$ and $\frac{1}{12}$.

> *Multiplying the numerator of a fraction increases the* **number** *of equal parts, and hence multiplies the fraction.*
>
> *Dividing the denominator increases the* **size** *of the equal parts, and hence multiplies the fraction.*

Tell which method you use in the following:

3. $4 \times \frac{3}{5}$.	6. $7 \times \frac{8}{14}$.	9. $6 \times \frac{2}{7}$.	12. $15 \times \frac{7}{80}$.
4. $5 \times \frac{3}{10}$.	7. $6 \times \frac{7}{12}$.	10. $8 \times \frac{3}{16}$.	13. $17 \times \frac{19}{34}$.
5. $6 \times \frac{2}{7}$.	8. $5 \times \frac{3}{8}$.	11. $9 \times \frac{15}{18}$.	14. $13 \times \frac{15}{26}$.

15. How do you find $\frac{3}{4}$ of a circle? Of an oblong?

16. Compare $\frac{3}{4}$ and $\frac{3}{16}$. When you divide 4ths into four equal parts, what fractional unit do you get?

17. When you divide 5ths into 4 equal parts, what do you call the parts?

18. Find $\frac{3}{4}$ of $\frac{5}{7}$.

WORK

$$\frac{3}{4} \text{ of } \frac{5}{7} = \frac{3 \times 5}{4 \times 7} = \frac{15}{28}$$

EXPLANATION. — Since to find $\frac{3}{4}$ of a number is to divide it into 4 equal parts and then take 3 of them, we divide $\frac{5}{7}$ into 4 equal parts and have $\frac{5}{28}$; then taking 3 of these we have $\frac{15}{28}$.

19. $\frac{3}{5}$ of $\frac{6}{7}$ = ——.	22. $\frac{2}{5}$ of $\frac{7}{8}$ = ——.	25. $\frac{2}{5}$ of $\frac{4}{7}$ = ——.
20. $\frac{6}{7}$ of $\frac{9}{11}$ = ——.	23. $\frac{2}{9}$ of $\frac{5}{7}$ = ——.	26. $\frac{8}{9}$ of $\frac{10}{11}$ = ——.
21. $\frac{3}{8}$ of $\frac{7}{10}$ = ——.	24. $\frac{3}{7}$ of $\frac{8}{9}$ = ——.	27. $\frac{2}{7}$ of $\frac{11}{12}$ = ——.

Finding a part of a number is called "multiplying by a fraction," and the sign of multiplication ($\times$) is used for "of," and must be read "of"; thus $\frac{3}{8} \times \frac{5}{6}$ is read $\frac{3}{8}$ of $\frac{5}{6}$.

56. CANCELLATION

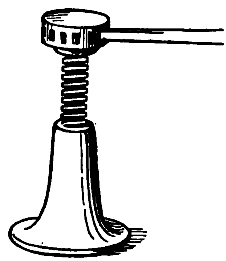
Since the numerator of the product is the product of the numerators, and the denominator of the product the product of the denominators, factors common to both numerator and denominator may be struck out before multiplying. This is called **cancellation.**

Thus $\frac{3}{8} \times \frac{16}{21} = \frac{\cancel{3} \times \overset{2}{\cancel{16}}}{\cancel{8} \times \underset{7}{\cancel{21}}} = \frac{2}{7}$ or simply $\frac{\cancel{3}}{\cancel{8}} \times \frac{\overset{2}{\cancel{16}}}{\underset{7}{\cancel{21}}} = \frac{2}{7}$.

In this way find:

1. $\frac{2}{7} \times \frac{21}{36}$.	**5.** $\frac{4}{11} \times \frac{33}{48}$.	**9.** $\frac{18}{19} \times \frac{76}{90}$.	**13.** $\frac{4}{15} \times \frac{60}{63}$.
2. $\frac{8}{9} \times \frac{33}{40}$.	**6.** $\frac{2}{9} \times \frac{15}{42}$.	**10.** $\frac{14}{15} \times \frac{45}{56}$.	**14.** $\frac{8}{17} \times \frac{85}{96}$.
3. $\frac{10}{12} \times \frac{36}{38}$.	**7.** $\frac{7}{13} \times \frac{39}{49}$.	**11.** $\frac{4}{17} \times \frac{85}{96}$.	**15.** $\frac{2}{5} \times \frac{85}{36}$.
4. $\frac{2}{9} \times \frac{18}{22}$.	**8.** $\frac{2}{15} \times \frac{35}{52}$.	**12.** $\frac{38}{42} \times \frac{63}{99}$.	**16.** $\frac{18}{19} \times \frac{38}{90}$.

Practical Problems in Multiplication



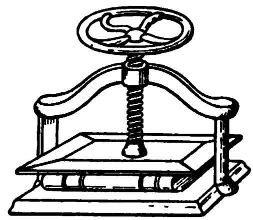
1. This is a picture of a jackscrew, which is used in lifting heavy objects such as the corner of a house or the end of a freight car. There are three threads of the screw to the inch. How high does it lift when turned through one revolution? Through 2 revolutions? Through 3 revolutions? Through 4 revolutions?

2. How far does this jackscrew lift when turned through $\frac{1}{2}$ of a revolution? Through $\frac{2}{3}$ of a revolution? Through $\frac{3}{4}$ of a revolution?

3. If a jackscrew were made so that the screw advanced $\frac{3}{8}$ of an inch at each turn, how far would it lift when turned through 2 revolutions? Through 5 revolutions? Through 12 revolutions?

4. How far would it lift when turned through $\frac{1}{2}$ of a revolution? Through $\frac{3}{4}$ of a revolution? Through $\frac{5}{9}$ of a revolution? Through $\frac{13}{16}$ of a revolution?

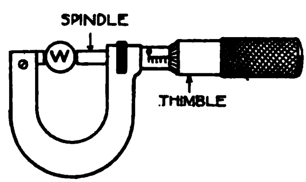
5. This is a picture of a letter press, used for exerting great pressure on books, manuscripts, etc. The distance between the centers of two threads is $\frac{1}{4}$ inch. How far does the screw advance when turned three times? 10 times? 24 times?



6. How far does it advance when making $\frac{1}{2}$ a turn? $\frac{5}{9}$? $\frac{7}{16}$? $\frac{4}{11}$?

7. If the distance between the centers of the threads were $\frac{5}{16}$ of an inch, how far would the screw advance when turned $\frac{4}{5}$ of the way around?

8. This is a picture of an instrument called a micrometer screw, which is used for measuring very small distances, such as the diameter of a wire or the thickness of a sheet of paper. The object to be measured is placed at W, and by turning the thimble with the fingers, the spindle is moved forward against the object. In a micrometer screw of one size, when the thimble makes one turn, the spindle moves forward $\frac{1}{20}$ of an inch. When the thimble is half turned, how far does the spindle move?



9. How far does the spindle move when the thimble is turned $\frac{1}{4}$ of the way around? $\frac{3}{8}$? $\frac{2}{5}$? $\frac{6}{7}$? $\frac{8}{9}$?

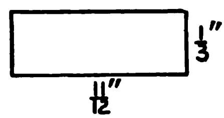
10. If the spindle of a micrometer screw moves forward $\frac{3}{50}$ of an inch when the thimble is turned once, how far will it move at $\frac{1}{2}$ of a turn? $\frac{5}{6}$? $\frac{2}{9}$? $\frac{8}{9}$? $\frac{5}{12}$?

11. When iron castings cool, they shrink. To allow for this, the pattern for a casting must be made $\frac{1}{8}$ of an inch per foot longer, wider, and thicker than the casting is desired to be. If a casting is to be 2 ft. long, how long must the pattern be made?

12. If an iron casting is to be $\frac{2}{3}$ of a foot long, how long must the pattern be?

13. If an iron casting is to be 6 in. wide and 9 in. long, how wide and how long must the pattern be made?

14. What is the area of this rectangle?



15. A box is $\frac{2}{3}$ ft. deep, $\frac{3}{4}$ ft. wide, and $\frac{11}{12}$ ft. long. What part of a cubic foot does it contain?

57. DIVISION OF FRACTIONS

1. How many times is $\frac{2}{3}$ contained in $\frac{8}{3}$? In $\frac{16}{3}$? In $\frac{24}{3}$?

2. At $\$\frac{3}{4}$ a bushel, how many bushels of potatoes can be bought for $\$5\frac{1}{4}$?

3. At $\$\frac{5}{8}$ per yard, how much cloth can be bought for $\$5\frac{5}{8}$?

4. At $\$\frac{3}{8}$ per bushel, how many bushels of apples can be bought for $\$2\frac{5}{8}$?

5. $5\frac{5}{8}$ yd. of cloth are divided into pieces each containing $1\frac{7}{8}$ yd. How many pieces?

6. If you can earn $\$\frac{3}{5}$ a day, how long will it take to earn $6?

7. At $\$\frac{1}{3}$ per pound, how much coffee can be bought for \$2? For \$3? For \$5?

8. If a horse eats $\frac{3}{8}$ bu. of oats a day, how long will 6 bu. last?

Find the value of the following:

9. $\frac{3}{8} \div \frac{3}{4}$.	13. $1\frac{3}{7} \div \frac{2}{7}$.	17. $\frac{2}{3} \div \frac{1}{9}$.
10. $\frac{3}{8} \div \frac{1}{2}$.	14. $1\frac{3}{8} \div \frac{3}{4}$.	18. $\frac{2}{3} \div \frac{5}{6}$.
11. $\frac{9}{10} \div \frac{3}{5}$.	15. $1\frac{5}{6} \div \frac{1}{2}$.	19. $\frac{2}{3} \div \frac{4}{9}$.
12. $\frac{2}{7} \div \frac{3}{14}$.	16. $\frac{9}{10} \div \frac{4}{5}$.	20. $\frac{1}{10} \div \frac{4}{5}$.

21. Describe the method used in dividing one fraction by another.

22. In some cases a second method of division is shorter. We know that $\frac{5}{9} \div \frac{3}{8}$ will give a quotient just $\frac{5}{9}$ as large as 1 or $\frac{9}{9}$ divided by the same divisor. What is $1 \div \frac{3}{8}$? What, then, is $\frac{5}{9} \div \frac{3}{8}$?

In practice we write $\frac{5}{9} \times \frac{8}{3} = \frac{40}{27} = 1\frac{13}{27}$.

23. The quotient of 1 divided by any fraction may be found without a pencil.

Give $1 \div \frac{5}{7}$; $1 \div \frac{3}{16}$; $1 \div \frac{7}{8}$; $1 \div \frac{9}{16}$; $1 \div \frac{7}{12}$; $1 \div \frac{5}{13}$; $1 \div \frac{4}{9}$.

24. Since $\frac{3}{7} \div \frac{5}{6} = \frac{3}{7}$ of $1 \div \frac{5}{6}$, we write $\frac{3}{7} \times \frac{6}{5} = \frac{18}{35}$.

Describe this process of division.

In dividing the following use both methods and tell which one you like better.

25. $\frac{11}{12} \div \frac{3}{4}$.	29. $\frac{7}{8} \div \frac{5}{12}$.	33. $\frac{7}{12} \div \frac{4}{5}$.	37. $\frac{3}{8} \div \frac{2}{3}$.
26. $\frac{4}{5} \div \frac{4}{7}$.	30. $\frac{27}{32} \div \frac{5}{16}$.	34. $\frac{25}{27} \div \frac{5}{18}$.	38. $\frac{11}{34} \div \frac{3}{17}$.
27. $\frac{3}{16} \div \frac{5}{24}$.	31. $\frac{7}{8} \div \frac{3}{5}$.	35. $\frac{5}{44} \div \frac{2}{11}$.	39. $1\frac{2}{5} \div \frac{5}{14}$.
28. $\frac{7}{48} \div \frac{3}{16}$.	32. $\frac{11}{12} \div \frac{6}{7}$.	36. $\frac{3}{8} \div \frac{9}{10}$.	40. $6\frac{1}{4} \div 2\frac{1}{2}$.

41. How many badges $4\frac{1}{6}$ in. long can be made from 5 yards of ribbon?

Practical Problems in Division

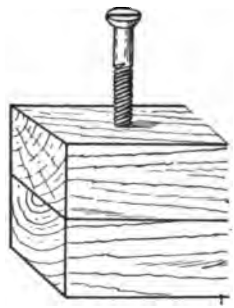
1. For making a garment $\frac{5}{6}$ yd. of cloth is required. How many garments can be made of 15 yd. of the cloth?

2. How many strips of carpet $\frac{3}{4}$ of a yard wide does it take to cover a floor 6 yd. wide?

3. How many pieces of cord, each $\frac{5}{6}$ yd. long, can be cut from a piece of cord 25 yd. long?

4. If a screw $\frac{7}{8}$ in. long has threads $\frac{3}{32}$ in. apart, how many turns with the screwdriver are required to drive it completely into the wood?

5. If a screw is $\frac{9}{16}$ in. long and the threads are $\frac{5}{64}$ in. apart, how many turns does it require to drive it completely into the wood?



6. If a screw is $2\frac{1}{4}$ in. long and the threads $\frac{1}{8}$ in. apart, how many turns does it require to drive it completely into the wood?

7. If, with each turn, a screw moves forward $\frac{5}{48}$ in., how many turns does it take to drive it forward a distance of $\frac{7}{12}$ inch?

8. A rectangle containing $\frac{15}{16}$ sq. ft. is $\frac{3}{4}$ ft. wide. How long is it?

9. A rectangle containing $\frac{11}{12}$ sq. yd. is $\frac{5}{6}$ yd. wide. How long is it?

10. A rectangle containing $5\frac{7}{8}$ sq. ft. is $1\frac{3}{4}$ ft. wide. How long is it?

Problems in Sewing

1. If $\frac{5}{8}$ yd. of lace will make one jabot, how many such jabots can be made from $4\frac{3}{8}$ yd. of lace?

2. If $\frac{3}{8}$ yd. of lace will trim one neckband, how many neckbands can be trimmed from $3\frac{3}{4}$ yd. of lace?

3. There are $12\frac{5}{6}$ yd. of braid used on one sailor suit. How many such suits can be trimmed from $51\frac{1}{3}$ yd. of braid?

4. If a girl can make $\frac{7}{8}$ of a shirt waist in a day, how many can she make in $3\frac{1}{2}$ days?

5. $2\frac{1}{2}$ yd. of thread will stitch one yard of goods. How many yards of goods will a 50-yard spool of thread stitch?

6. How many tucks, each $\frac{1}{4}$ inch wide, can be put in a piece of goods $6\frac{1}{2}$ in. wide?

7. How many tucks, each $\frac{1}{8}$ in. wide, can be put in the goods for a sleeve $15\frac{1}{2}$ in. wide?

8. How many sleeves can be cut out of $4\frac{1}{2}$ yd. of goods, if each sleeve takes $\frac{3}{8}$ yd.?

9. $4\frac{1}{2}$ in. of the length of a skirt is to be taken up with $\frac{3}{8}$ in. tucks. How many tucks must be made?

10. How many stitches, each $\frac{1}{18}$ in. long, does it take to stitch around a neckband, if the neckband measures $13\frac{1}{4}$ in. around?

Making Grape Juice

The following recipe is used in making grape juice:

$1\frac{1}{2}$ cups grapes
1 cup water
$\frac{1}{2}$ cup sugar

1. If we have 3 cups of grapes, that is how many times $1\frac{1}{2}$ cups? Then for 3 cups of grapes we must use how many cups of water?

2. For 3 cups of grapes we must use how many cups of sugar?

3. If we have 2 cups of grapes, how much water must be added? How much sugar?

4. If we have 5 cups of grapes, how much water must be added? How much sugar?

5. If we have $8\frac{1}{2}$ cups of grapes, how much water must be added? How much sugar?

6. If we have $4\frac{2}{3}$ cups of grapes, how much water must be added? How much sugar?

58. RATIO

The **ratio** of one number to another is the quotient of the first divided by the second.

Exercises in Ratio

1. If 15 doz. eggs are worth \$3.15, what are 45 doz. worth?

SUGGESTION. — Compare 15 and 45. Can you solve the problem without a pencil?

2. What is the ratio of 14 to 56? If 14 tons of coal cost \$119, how many times as much will 56 tons cost? How much will 56 tons cost?

3. What is the ratio of 9 to 27? When 9 yards of cloth cost \$11.25, how much will 27 yards cost? Can you solve this without a pencil?

4. What is the ratio of 3 to 12? If 12 barrels of sugar cost \$237, how much will 3 barrels cost?

5. What is the ratio of 17 to 85? If 17 bu. of potatoes are worth \$14.45, how much are 85 bu. worth? Can you solve this without a pencil?

6. What is the ratio of 16 to 64? If a bicyclist rides 16 miles in $1\frac{1}{4}$ hr., how long at this rate will it take him to ride 64 miles?

7. What is the ratio of 12 to 36? If 12 cars will carry 285 tons, how many tons will 36 cars carry?

8. What is the ratio of 13 to 78? If I receive \$45.50 for 13 days' work, how much should I receive for 78 days'?

9. If 3 hats cost a dealer \$6.75, find the rate per dozen.

10. If an article weighing 18 lb. costs \$1.52, how much will one weighing 90 lb. cost at the same rate?

11. What is the ratio of 3 pk. to 1 bu.? How much will 3 pk. of apples cost at \$2 a bushel?

12. What is the ratio of 6 oz. to 1 lb.? How much will 6 oz. of spices cost at 40 ¢ a pound?

13. How much will 8 oranges cost at 45¢ a dozen?

14. What is the ratio of $\frac{1}{8}$ to $\frac{1}{4}$? If $\frac{1}{4}$ A. of land is worth \$32, how much is $\frac{1}{8}$ A. worth?

15. What is the ratio of $\frac{3}{4}$ to $\frac{3}{16}$? If $\frac{3}{16}$ of a farm sells for \$1200, how much will $\frac{3}{4}$ of the farm sell for at the same rate?

16. What is the ratio of $\frac{3}{8}$ to $\frac{2}{3}$? (*Hint.* Change to 24ths.) If $\frac{2}{3}$ of the distance to a certain place is 48 miles, what is $\frac{3}{8}$ the distance to that place?

17. What is the ratio of 8 to 6? If 6 cords of wood are worth \$15.75, how much are 8 cords worth? (Since 8 is $1\frac{1}{3}$ times 6, 8 cords will cost $1\frac{1}{3}$ times \$15.75, or \$15.75 + $\frac{1}{3}$ of \$15.75. Now can you solve without a pencil?)

18. What is the ratio of 12 to 9? If 9 cars can carry 186 tons of coal, how many can 12 cars carry? (Solve quickly by writing $\frac{1}{3}$ of 186 under 186 and adding.)

19. What is the ratio of 20 to 18? If 18 bushels of apples cost \$10.80, how much will 20 bushels cost?

20. If $\frac{3}{16}$ of a farm is worth \$360, find the value of $\frac{5}{8}$ of it.

21. What is the ratio of $2\frac{1}{2}$ to 5? At 5 dozen pencils for \$2.40, how much will $2\frac{1}{2}$ dozen cost?

Give the ratios of the following quantities:

22. 2 bbl. and 18 bbl.

23. 42 gal. and 7 gal.

24. $8 and $64.

25. 1 ft. and 1 yd.

26. 1 bu. and 16 qt.

27. 21 da. and 30 da.

28. 1 lb. and 4 oz.

29. 6 in. and 2 yd.

30. $\frac{3}{4}$ ton and $3\frac{3}{4}$ tons.

31. $2\frac{1}{2}$ yd. and $7\frac{1}{2}$ yd.

32. Make problems including the above quantities and solve them, or propose them to your classmates for solution.

Give the ratios of the following numbers:

33. $\frac{9}{10}$ and $\frac{3}{5}$.

34. $\frac{1}{8}$ and 3.

35. $\frac{2}{5}$ and $\frac{3}{4}$.

36. $\frac{6}{7}$ and $\frac{12}{21}$.

37. $2\frac{1}{2}$ and $1\frac{1}{4}$.

38. $1\frac{1}{8}$ and $4\frac{1}{2}$.

39. $2\frac{3}{8}$ and $\frac{19}{4}$.

40. $\frac{3}{7}$ and $\frac{9}{14}$.

41. 5 and $\frac{1}{3}$.

42. 2 and $\frac{1}{4}$.

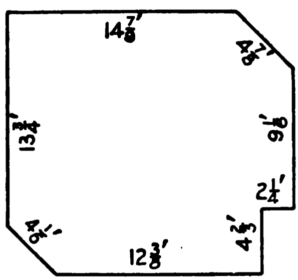
43. 3 and $\frac{1}{5}$.

44. 5 and $\frac{1}{2}$.

45. When $1\frac{1}{8}$ yards of velvet costs $1.75, how much will $4\frac{1}{2}$ yards cost?

Miscellaneous Problems in Fractions

1. This is the plan of a room. How many feet of border are required in papering the room? How many yards?



2. In this room there is a door $3\frac{5}{8}$ ft. wide, one window $3\frac{3}{4}$ ft. wide, one window $4\frac{5}{12}$ ft. wide, and a mantel $5\frac{1}{6}$ ft. wide. What is the sum of all these widths?

3. By subtracting the sum of the widths of the door and the mantel from the total distance around the room, we get the length of the baseboard in the room. How many feet of baseboard are required in finishing the room?

4. A dealer bought 4 carloads of coal at the mine, weighing as follows: $31\frac{5}{8}$ tons, $27\frac{3}{5}$ tons, $29\frac{7}{10}$ tons, and $30\frac{3}{4}$ tons. How many tons in all?

5. On Tuesday night the Mississippi River at Vicksburg stood at $12\frac{3}{8}$ ft. above low-water mark. During Wednesday it rose $1\frac{5}{9}$ ft., on Thursday $2\frac{7}{12}$ ft., on Friday $1\frac{8}{9}$ ft., and on Saturday $\frac{3}{4}$ ft. How high was it above low-water mark on Saturday night?

6. On Sunday the Mississippi fell $\frac{2}{9}$ ft., on Monday $1\frac{3}{4}$ ft., and on Tuesday $2\frac{5}{6}$ ft. How high was it Tuesday night?

7. How much higher was the river the second Tuesday night than it was the first Tuesday night?

8. From Chicago to Geneva, by the Chicago and Northwestern Railroad, is $35\frac{1}{2}$ miles, and from Geneva to De Kalb is $22\frac{4}{5}$ miles. How far is it from Chicago to De Kalb?

9. A five-cent piece weighs $77\frac{4}{25}$ grains. $\frac{1}{4}$ of it is nickel, and $\frac{3}{4}$ of it copper. How many grains of nickel and how many grains of copper does it contain?

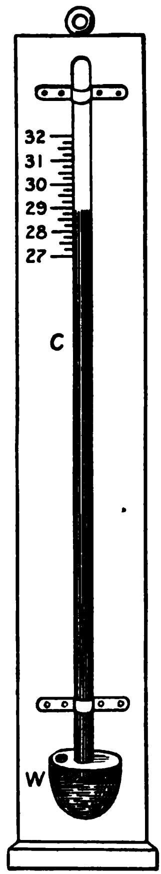
10. The half dime coined from 1792 to 1873, when it was discontinued, contained at first $20\frac{4}{5}$ grains. In 1837 the weight was reduced to $20\frac{5}{8}$ grains. How much was it reduced in 1837? In 1853 the weight was changed again to $19\frac{1}{5}$ grains. How much was it reduced then?

11. The cent weighs 48 grains. Of this, $\frac{19}{20}$ is copper and $\frac{1}{20}$ is tin and zinc. How many grains of copper are there in a cent?

12. In making a suit of clothes, a tailor uses $2\frac{5}{8}$ yd. of cloth for the coat, $\frac{8}{9}$ yd. for the vest, and $2\frac{1}{4}$ yd. for the trousers. How many yards does he use in the whole suit? If he cuts the cloth from a piece containing $9\frac{1}{6}$ yd., how much will then be left?

13. Lead weighs $11\frac{7}{20}$ times as much as water. A cubic foot of water weighs $62\frac{1}{2}$ lb. Find the weight of a cubic foot of lead.

14. This is a picture of a *barometer*, an instrument used for measuring the pressure or weight of the atmosphere. A hollow glass tube, C, sealed air-tight at the top, opens into a well of mercury, W, at the bottom. The downward pressure of the air on the surface of the mercury outside of the tube in the well forces the mercury up into the tube, from which the air has been removed. The scale shows how many inches high the mercury stands in the tube. How high does it stand now?



15. If the cross section of the tube of mercury is 1 sq. in., how many cubic inches of mercury are there in the tube?

16. One cubic inch of water weighs $\frac{29}{50}$ of an ounce. If the liquid in the tube were water, how many ounces would it weigh?

17. Mercury weighs $13\frac{3}{5}$ times as much as water. How many ounces does the mercury in the tube weigh?

18. How many pounds does the mercury in the tube weigh? Then how much is the pressure of the atmosphere on a square inch?

19. Find, by similar computation, the pressure of the atmosphere on a square inch when the barometer reading is 30 inches.

Problems in Printing

These boys are learning to set type. The type is kept sorted in shallow cases. Each boy has a case of type, containing all the letters of the alphabet. He sets up the type in lines in a little instrument held in one hand, called a *stick*. Besides the letters, there are pieces of type, called *quads* and *spaces*, used to space out the words and to fill out incompleted lines.

Type is made in different sizes, called "8-point," "12-point," etc. A *point* is $\frac{1}{72}$ inch. The amount of type in a line, or page, etc., is measured in units called *ems*. An *em* in any size of type is the width of the letter *m* in that type.

1. In 12-point type an em is 12 points. What part of an inch is it? Express in lowest terms.

2. Express the following lengths of lines of 12-point type in inches: 3 ems; 6 ems; 9 ems; 12 ems; 18 ems; 20 ems.

3. In 8-point type an em is 8 points. What part of an inch is it? Express in lowest terms.

4. Express the following lengths of lines of 8-point type in inches: 3 ems; 6 ems; 9 ems; 12 ems; 18 ems; 24 ems.

5. In 6-point type an em is 6 points. What part of an inch is it?

6. Express the following lengths of 6-point type in inches: 3 ems; 6 ems; 9 ems; 12 ems: 18 ems; 24 ems.

7. What part of an inch is an em of 9-point type? 2 ems? 4 ems? 8 ems? 12 ems? 16 ems? 20 ems?

8. What part of an inch is an em of 18-point type? 2 ems? 4 ems? 6 ems? 8 ems? 16 ems? 24 ems?

9. A printer is paid by the thousand ems of any kind of type that he sets. If a printer sets a column of 8-point type 3 inches wide, containing 50 lines, how many ems does he set?

10. A printer sets a column $2\frac{1}{2}$ inches wide and 60 lines long with 12-point type. How many ems does he set?

11. In measuring the widths of newspaper columns, the sizes of cuts for printing pictures, etc., the em of 12-*point type* (pica type) is always used as the unit. An ordinary newspaper column is 13 ems wide. How many inches is that? Verify by measuring the column of some newspaper.

12. How many square inches in a cut 21 ems by 27 ems?

The following is a table of the sizes of the different *quads* and *spaces* used to fill out incomplete lines and to separate words:

1 em quad	= 1 em	= ——	inch.
2 em quad	= 2 ems	= ——	inch.
3 em quad	= 3 ems	= ——	inch.
en quad	= $\frac{1}{2}$ em	= ——	inch.
3 em space	= $\frac{1}{3}$ em	= ——	inch.
4 em space	= $\frac{1}{4}$ em	= ——	inch.
5 em space	= $\frac{1}{5}$ em	= ——	inch.

13. Complete the table by filling the blanks for 12-point type.

14. Complete the above table for 8-point type.

DECIMAL FRACTIONS

59. THE DECIMAL PARTS OF A DOLLAR

1. Compare the values of \$1, 1 dime, 1 cent.

2. $\frac{3}{10}$ of a dime = —— cents; $\frac{4}{10}$ of a dollar = —— dimes.

3. $\frac{8}{100}$ of a dollar = —— cents; $\frac{25}{100}$ of a dollar = —— cents.

4. Compare the value of \$$\frac{1}{2}$, 50 cents, and \$0.50.

5. Compare the value of \$$2\frac{1}{4}$, \$$2\frac{25}{100}$, and \$2.25.

6. What is the common way of writing a number of dollars and cents? Write 1 dollar and 25 cents.

7. Write 2 dollars and 40 cents. 40 cents = —— dimes.

8. In \$2.45, which figure represents dollars? Which one dimes, or tenths of a dollar? Which one cents, or tenths of a dime?

9. 45 cents is what part of a dollar? Read \$2.45 as dollars and hundredths of a dollar.

10. Read \$1.75 as dollars, dimes, and cents, and as dollars and hundredths of a dollar.

> *The point between dollars and cents is called the* **decimal point** *and separates the dollars from the parts of a dollar.*

11. The first figure at the right of the decimal point represents dimes or —— of a dollar. The second figure at the right represents cents or —— of a dollar.

12. Read as dimes and cents, and then as tenths and hundredths of a dollar: \$0.45; \$0.86; \$0.54.

NOTE. — The decimal point is very important. To make it more conspicuous it is well to fill the ones' place with a zero.

60. THE DECIMAL NOTATION

1. What does each 2 represent in 22?

2. How do they compare in value?

3. In $44, what does each 4 represent? Compare their values.

4. In $3.33, tell what each represents as dollars, dimes, and cents. As dollars, tenths of a dollar, hundredths of a dollar.

5. As you move a figure one place to the right, how do you affect the value which it represents?

6. What then should be the value of the number written in the first place at the right of *ones?* The second place at the right? The third place at the right?

> *We use a* **decimal point** (·) *to locate ones. The first place at the right is* **tenths**, *the second*, **hundredths**, *and the third*, **thousandths.**

Thus, 3.46 is read, "three, and forty-six hundredths." 6.375 is read, "six, and three hundred seventy-five thousandths."

> *Numbers written at the* **left** *of the decimal point are* **integers.** *They denote so many* **whole** *things.*
>
> *Numbers written at the* **right** *of the decimal point are* **fractions.** *They denote* **equal parts** *of whole things.*
>
> *Because these equal parts are* **tenths, hundredths, thousandths,** *etc., we call them* **decimal fractions,** *or, more briefly,* **decimals.**

In reading numbers, "and" is used only between the whole number and the fraction, where the decimal point comes.

Read:

7. 6.7.	**12.** 0.63.	**17.** 3.46.	**22.** 9.095.
8. 9.42.	**13.** 1.09.	**18.** 6.04.	**23.** 11.116.
9. 8.53.	**14.** 10.1.	**19.** 7.41.	**24.** 3.435.
10. 9.07.	**15.** 9.09.	**20.** 7.04.	**25.** 0.056.
11. 0.8.	**16.** 9.95.	**21.** 0.95.	**26.** 0.097.

Write:

27. Three, and four tenths. **29.** Eleven hundredths.

28. Forty-five hundredths. **30.** Nine, and six hundredths.

31. Six, and three hundred six thousandths.

32. Seven, and seventy-seven thousandths.

Write the following, using denominators:

33. 0.5; 0.17; 0.08; 0.25; 0.125; 0.008.

34. 0.36; 0.03; 0.003; 0.9; 0.90; 0.09.

Write as decimals:

35. $\frac{3}{10}$.	**38.** $\frac{7}{10}$.	**41.** $6\frac{24}{100}$.	**44.** $4\frac{545}{1000}$.
36. $\frac{17}{100}$.	**39.** $3\frac{4}{10}$.	**42.** $7\frac{13}{100}$.	**45.** $7\frac{603}{1000}$.
37. $\frac{45}{100}$.	**40.** $5\frac{8}{10}$.	**43.** $9\frac{16}{100}$.	**46.** $8\frac{708}{1000}$.

61. ADDITION OF DECIMALS

1. Add 3.8, 7.5, and 6.45.

WORK

```
  3.8
  7.5
  6.45
 -----
 17.75
```

REMARK. — Since only like things can be combined into one sum, we place the decimal points under each other so that the *ones* will come in one column, the *tenths* in another, etc.

Add:

2.	3.	4.	5.	6.
0.75	3.42	5.46	1.83	2.86
1.6	7.886	7.531	2.907	1.945
7.23	0.96	8.205	0.76	0.72
2.5	7.445	0.063	1.093	0.806

7.	8.	9.	10.	11.
23.65	69.04	80.46	8.724	145.84
45.3	40.81	2.73	15.379	902.68
8.74	3.59	16.39	40.903	47.25
49.67	67.2	9.81	28.185	408.91

62. SUBTRACTION OF DECIMALS

Subtract:

1.	2.	3.	4.	5.
3.46	5.73	6.82	9.81	8.03
1.28	2.9	1.93	3.67	1.8

6. Subtract 3.28 from 5.6.

WORK

$$\begin{array}{r} 5.60 \\ \underline{3.28} \\ 2.32 \end{array}$$

REMARK. — Since there are no hundredths in the minuend, we may fill its place with a zero and proceed as in any subtraction.

7. 3.46 from 7.8.
8. 9.13 from 15.7.
9. 6.93 from 9.3.
10. 4.09 from 6.7.
11. 8.16 from 9.
12. 16.453 from 23.84.
13. 14.375 from 29.63.
14. 15.641 from 38.46.
15. 29.365 from 42.6.
16. 41.296 from 50.
17. 42.61 from 63.2
18. 8.296 from 9.05
19. 329.1 from 864.
20. 47.77 from 100.

Practical Problems in Addition and Subtraction

1. At one of its stations the Weather Bureau reported the rainfall for a week as follows: Sunday, 1.4 in.; Monday, 1.1 in.; Tuesday, 0.2 in.; Wednesday, 0.3 in.; Thursday, 0.7 in.; Friday, 0.5 in.; Saturday, 0.6 in. Find the total rainfall for the week.

2. The rainfall in inches for 12 months at a certain place was as follows: 2.26, 2.48, 3.67, 3.19, 2.45, 1.37, 1.65, 2.39, 4.72, 3.31, 2.14, 3.63. What was the total rainfall for the year?

3. The rainfall in inches at Chicago for the 12 months beginning with November, 1908, was as follows: 2.67, 1.18, 1.96, 3.84, 1.63, 7.73, 2.18, 5.09, 1.77, 6.20, 3.60, 1.20. Find the total rainfall for the year.

4. From Chicago to Wheaton is 24.9 mi.; from Wheaton to Geneva is 10.6 mi.; from Geneva to Clinton, Ia., is 102.6 mi.; and from Clinton to Cedar Rapids is 81.3 mi. How far is it from Chicago to Cedar Rapids?

5. A dairyman kept a record of the amount of milk that one of his cows gave in 4 weeks. The amount the first week was 73.6 lb., the second week 78.7 lb., the third week 84.3 lb., and the fourth week 68.8 lb. How much did she give in the 4 weeks?

6. How much coal in 6 carloads weighing as follows: 28.74 tons, 29.38 tons, 31.16 tons, 29.75 tons, 30.27 tons, and 28.13 tons?

7. Wet soil that weighed 4.37 lb. was baked in an oven until perfectly dry, and then weighed again. When dry it weighed only 3.69 lb. How much water did it contain originally?

63. A DECIMAL MULTIPLIED BY AN INTEGER

1. Give the products of the following numbers:

$7	7 ft.	7 miles	7 tenths	0.7	7 hundredths.
3	3	3	3	3	3

2. Multiply 3.56 by 8.

WORK

$$\begin{array}{r} 3.56 \\ 8 \\ \hline 28.48 \end{array}$$

REMARK. — 8 × 6 hundredths are 48 hundredths, or four tenths and 8 hundredths; the 8 is placed in hundredths' place and the 4 carried to the tenths' place.

> *Observe that when a decimal is multiplied by a whole number, the decimal point in the product comes under the decimal point of the multiplicand.*

Multiply:

3.	4.	5.	6.	7.	8.
3.86	5.48	6.93	14.8	16.9	48.9
9	7	8	9	7	6

9.	10.	11.	12.	13.	14.
2.78	63.5	0.375	9.68	86.3	0.963
43	64	47	65	52	85

64. EFFECT OF ANNEXING ZEROS AT THE RIGHT OF THE DECIMAL POINT

1. Compare $0.5 and $0.50.
2. Compare 0.12 ft. and 0.120 ft.
3. Compare $3. and $3.00.
4. Compare 2.5 with 2.500.

5. Does annexing zeros to a number change the value if the decimal point remains unchanged?

The value of a figure depends upon the order which it occupies. If the decimal point is not moved, the order is not changed, and annexing zeros does not change the value.

65. EFFECT OF MOVING THE DECIMAL POINT

1. Compare $0.01 and $1.00. $1.00 is —— times $0.01.

2. Compare $0.05 and $5.00; $2.35 and $235.

3. What is 100 times $4.30? What is $\frac{1}{100}$ of $450?

4. What effect on the value of $4.30 has the changing of the place of the decimal point, so that the number is $430?

5. What is 10 times 2 ft.? How is 0.2 ft. affected by moving the point so that the number becomes 2 ft.?

NOTE.—We do not really write the decimal point when a number is an integer, but we consider it to be at the right of *ones;* thus 2 ft. might be written 2. ft.

6. Compare 0.2 and 2; 0.3 and 3; 0.02 and 2; 0.05 and 5.

7. Compare 0.002 and 2; 0.006 and 6; 0.125 and 125.

Moving the decimal point one place to the right multiplies by 10; *moving it two places to the right multiplies by* 100; *moving it three places multiplies by* 1000.

Multiply the following by 1000:

8. 0.365; 0.045; 0.08; 1.06.

9. 5.345; 7.08; 8.08; 9.2.

Multiply by moving the decimal point:

10. 10 × 3.4.	15. 10 × 2.65.	20. 100 × 2.5.
11. 10 × 0.2.	16. 100 × 3.783.	21. 1000 × 0.065.
12. 100 × 1.25.	17. 100 × 0.1675.	22. 1000 × 0.17.
13. 100 × 3.04.	18. 1000 × 0.243.	23. 100 × 19.3.
14. 100 × 0.16.	19. 100 × 0.5.	24. 1000 × 0.67.

25. Compare $50.00 and $5.00. What change in the place of the decimal point will give $\frac{1}{10}$ of $50?

26. What is $\frac{1}{100}$ of 800? What change is made in the place of the decimal point?

27. Moving the decimal point one place to the left in $700 gives what? What part of $700 is $70?

28. Moving the decimal point two places to the left in 500 gives what? Compare 5 and 500.

> *Moving the decimal point one place to the left divides a number by* 10; *moving it two places to the left divides by* 100; *moving it three places to the left divides by* 1000.

Divide by moving the decimal point:

29. $9750 by 100.	33. $5.50 by 100.
30. 3465 ft. by 1000.	34. 3.54 by 100.
31. 693.5 by 100.	35. 34.6 by 1000.
32. 67.3 by 100.	36. 0.06 by 10.

66. MULTIPLYING A DECIMAL BY A DECIMAL

1. When you divide a number by 10 the quotient is what part of the dividend?

2. 0.1 of 85 is found by dividing 85 by ——.

3. 0.4 of 20 means what? 0.05 of 100 means what?

4. What is 0.4 of 20? 0.05 of 100?

5. When we speak of multiplying a number by $\frac{1}{2}$, do we really multiply the number; that is, really increase it?

6. What do we do to multiply by $\frac{1}{2}$? by $\frac{1}{4}$? by $\frac{1}{5}$?

NOTE. — When the multiplier is a fraction the sign × does not mean "times," but means "of" and must be read "of." For example, 0.1 × 4.5 is read $\frac{1}{10}$ of $4\frac{5}{10}$.

7. 75 ÷ 100, or $\frac{1}{100}$ of 75 = ——. (Written 0.01 × 75.)

Read and then find the value of:

8. 0.1 × 20.	14. 0.1 × 6.5.	20. 0.02 × 5.
9. 0.2 × 20.	15. 0.2 × 6.5.	21. 0.04 × 35.
10. 0.01 × 400.	16. 0.1 × 3.2.	22. 0.5 × 0.25.
11. 0.03 × 400.	17. 0.02 × 3.2.	23. 0.8 × 12.5.
12. 0.01 × 80.	18. 0.01 × 2.5.	24. 0.6 × 0.25.
13. 0.04 × 80.	19. 0.04 × 2.5.	25. 0.05 × 2.4.

Written Exercises

1. Multiply 8.42 by 2.6.

PROCESS

$$\begin{array}{r} 8.42 \\ 2.6 \\ \hline 5.052 \\ 16.84 \\ \hline 21.892 \end{array}$$

EXPLANATION. — 0.6 × 0.02 = 0.012; write 2 in *thousandths'* place and carry the 1 hundredth. 0.6 × 0.4 = 0.24; 0.24 + 0.01 = 0.25; and thus proceed, keeping each figure in its proper order.

Or we may multiply as in whole numbers and give the product as many decimal places as both multiplier and multiplicand contain.

Observe that when a decimal is multiplied by a decimal the product contains as many decimal places as the sum of the places in both multiplier and multiplicand.

Find the products in the following:

2. 0.26×37.5.	8. 40.5×9.32.	14. 1.25×62.5.
3. 4.5×90.8.	9. 7.03×98.2.	15. 0.85×0.24.
4. 6.5×37.2.	10. 0.97×96.3.	16. 0.83×26.4.
5. 0.35×9.6.	11. 0.092×125.	17. 0.19×40.4.
6. 0.82×10.8.	12. 0.34×90.2.	18. 0.37×8.4.
7. 0.37×86.	13. 1.908×62.	19. 0.82×62.5.

Practical Problems with Decimals

1. Water comprises 0.78 of a potato. How much water in 25.4 pounds of potatoes?

2. A bushel of onions weighs 57 lb., of which 0.88 is water. How many pounds of water in a bushel of onions?

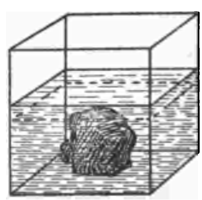
3. Find the area of a rectangle 7.3 in. long and 5.8 in. wide.

4. Find the area of a rectangle 15.43 in. long and 12.25 in. wide.

5. A box is 1.6 ft. deep, 2.5 ft. wide, and 3.2 ft. long. How many cubic feet does it hold?

6. How many cubic inches of soil will a flower box hold that is 5.5 in. deep, 7.5 in. wide, and 36 in. long?

7. The volume of any irregular shaped object may be found by immersing it under water and measuring the amount of water displaced by it. A glass jar is 6 in. wide and 10 in. long. It is filled partly full of water, and the depth of the water measured. A stone is then put into the water so that it is covered, and the depth of the water is again measured. Before the stone was put in, the



water was 5.2 in. deep, and after the stone was put in, it was 7.3 in. deep. How much did the water rise in the jar? How many cubic inches of water did the stone displace? This is the volume of the stone.

8. A piece of broken brick was put into this jar. Before it was put in, the water stood 4.4 in. deep, and after it was put in, the water stood 5.2 in. deep. How much did the water rise? How many cubic inches in the brick?

9. In the same jar a piece of iron was placed. The depths of the water were 6.7 in. and 7.2 in. How many cubic inches in the piece of iron?

10. A cubic foot of water weighs 62.5 lb. Find the weight of 9 cu. ft.

11. Find the weight of 12.25 cu. ft. of water.

12. Ice weighs 0.92 as much as water. Find the weight of a cubic foot of ice.

13. Steel weighs 7.83 times as much as water. Find the weight of a cubic foot of steel.

14. Gold weighs 19.36 times as much as water. Find the weight of a cubic foot of gold.

15. Cork weighs 0.24 times as much as water. Find the weight of a cubic foot of cork.

16. Coal weighs 1.3 times as much as water. What is the weight of 1 cu. ft. of coal?

17. Sheet lead $\frac{1}{16}$ in. thick weighs 3.69 lb. per square foot. How many pounds of this does it require to line the walls and bottom of a tank 2 ft. deep, 3 ft. wide, and 5 ft. long?

18. Sheet copper $\frac{1}{16}$ in. thick weighs 2.888 lb. per sq. ft. How many pounds of this does it require to make a teapot, if 1.3 sq. ft. are used?

67. DIVIDING A DECIMAL BY AN INTEGER

1. What is $\frac{1}{3}$ of 6 tenths? $0.6 \div 3 =$ ——.

2. What is $\frac{1}{4}$ of 84 hundredths? $0.84 \div 4 =$ ——.

Find the value of the following:

3. $0.96 \div 8.$	7. $0.025 \div 5.$	11. $4.08 \div 8.$
4. $0.36 \div 9.$	8. $0.048 \div 8.$	12. $6.44 \div 7.$
5. $4.8 \div 4.$	9. $6.024 \div 6.$	13. $0.84 \div 12.$
6. $12.9 \div 3.$	10. $0.924 \div 12.$	14. $9.63 \div 9.$

> *Evidently, when the divisor is an integer, there are as many decimal places in the quotient as there are in the dividend.*

68. DIVIDING A DECIMAL BY A DECIMAL

1. How many 3's in 9? How many 6's in 18? How many 12's in 36? How many 30's in 90?

2. $4 \div 2 = ?$ $40 \div 20 = ?$ $400 \div 200 = ?$

3. How is the quotient affected when we multiply both dividend and divisor by the same number?

> *Evidently, multiplying both dividend and divisor by the same number does not change the quotient.*

4. How many times is 0.2 contained in 2.4?

SUGGESTION. — 0.2 is contained in 2.4 as many times as 2 is contained in 24, for both dividend and divisor have been multiplied by 10 to get 2 and 24.

Give the quotients:

5. 4.2 ÷ 0.7.	**8.** 6.3 ÷ 0.9.	**11.** 3.4 ÷ 1.7.
6. 6.4 ÷ 0.8.	**9.** 8.4 ÷ 1.2.	**12.** 6.4 ÷ 3.2.
7. 7.2 ÷ 0.9.	**10.** 9.6 ÷ 1.2.	**13.** 4.8 ÷ 2.4.

14. How many times is 0.06 contained in 1.44?

WORK

$$0.06\overline{)1.44} = 6.\overline{)144.}$$
$$24.$$

EXPLANATION. — Here we multiply both dividend and divisor by 100.

Find the value of the following:

15. 3.28 ÷ 0.4.	**19.** 300 ÷ 0.06.	**23.** 25 ÷ 0.005.
16. 0.049 ÷ 0.7.	**20.** 7.2 ÷ 12.	**24.** 0.125 ÷ 0.005.
17. 0.216 ÷ 0.9.	**21.** 14.4 ÷ 1.2.	**25.** 0.096 ÷ 0.08.
18. 6.4 ÷ 0.08.	**22.** 10.8 ÷ 0.009.	**26.** 0.108 ÷ 0.09.

27. How many times is 1.44 contained in 1.728?

WORK

$$\begin{array}{r} 1.2 \\ 1.44)1.728 = 144\overline{)172.8} \\ \underline{144} \\ 28.8 \\ \underline{28.8} \end{array}$$

EXPLANATION. — We multiply both dividend and divisor by 100 by moving the point two places to the right, to get an integral divisor.

The point in the quotient is directly over the point in the dividend.

Should there be a remainder, zeros may be annexed in the dividend and the division continued.

28. 26.6 ÷ 1.6.	**35.** 12.6 ÷ 0.36.
29. 32 ÷ 0.625.	**36.** 2.5 ÷ 0.625.
30. 1.6 ÷ 0.625.	**37.** 0.399 ÷ 1.9.
31. 17.28 ÷ 1.44.	**38.** 6.75 ÷ 0.125.
32. 0.8 ÷ 1.6.	**39.** 8.4 ÷ 0.14.
33. 1.296 ÷ 0.36.	**40.** 17.5 ÷ 0.25.
34. 7.2 ÷ 120.	**41.** 2.56 ÷ 0.016.

42. Divide 3.78 by 15.3.

WORK

```
                0.247+
15.3)3.78=153.)37.8
               30 6
                7 20
                6 12
                1 080
                1 071
                    9
```

Explanation. — We multiply both dividend and divisor by 10 to get an integral divisor.

There being a remainder after each division, we annex zeros.

After carrying the division to thousandths, there is still a remainder. We indicate this by a plus sign (+) in the quotient.

43. 34.6 ÷ 1.75.

44. 89.3 ÷ 2.68.

45. 8.47 ÷ 36.5.

46. 96.7 ÷ 17.3.

47. 36.8 ÷ 9.6.

48. 4.35 ÷ 62.7.

49. 8.63 ÷ 4.27.

50. 8.64 ÷ 1.65.

Note. — Making an integer of the divisor lessens the number of the decimal places in the dividend. Hence, *without making any change before division, we may divide as in integers, and point off as many decimal places in the quotient as the number of places in the dividend exceeds those in the divisor.*

In using this method, remember that every zero annexed to the dividend is counted as one of the places in the dividend.

Practical Problems in Division

1. The average yield of corn in the United States in 1908 was 26.2 bu. per acre. The total yield was 2,668,651,000 bu. On how many acres was corn raised?

2. The average yield of corn in Illinois in 1908 was 31.6 bu. per acre. The total yield of the state was 298,620,000 bu. How many acres of corn were planted in Illinois in 1908?

3. The average yield of corn in Maryland in 1908 was 36.6 bu. per acre. The total yield of the state was 24,705,000 bu. How many acres of corn were planted in Maryland?

4. In 1908–1909 the United States produced 380,000 tons of beet sugar. This was 0.055 of the total amount produced by the world. How many tons were produced by the whole world?

5. In 1908 Louisiana produced 11,550,000 bushels of rice. This was 0.527 of the total amount produced in the United States. How many bushels were produced in the United States?

6. In 1908 Indiana produced 45,169,000 bu. of wheat on 2,721,000 acres. Find to the tenth of a bushel the average amount produced to the acre.

7. Idaho produced 10,897,000 bu. of wheat on 387,000 acres. Find to the tenth of a bushel the average amount produced per acre in Idaho.

8. In 1908 Nevada produced an average of 30 bu. of wheat per acre, which was the greatest yield per acre of all the states. South Carolina produced an average of 9 bu. to the acre, which was the least yield per acre of all the states. Find to hundredths how many times the yield per acre in South Carolina the yield per acre in Nevada was.

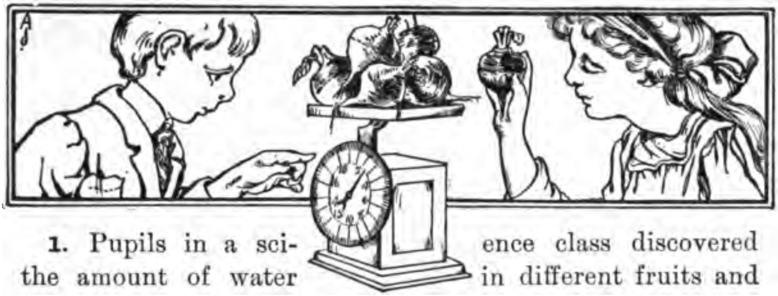
9. At an average speed of 28.4 mi. per hour, how long would it take a train to go from Chicago to New York, a distance of 952 mi.?

10. The United States battleship *Indiana* showed a trial speed of 15.55 knots per hour. At this speed, how long would it take the *Indiana* to go a distance of 260 knots?

11. The trial speed of the battleship *Nebraska* was 19.06 knots per hour. At this speed how long would it take the *Nebraska* to go a distance of 286 knots?

12. Calling a knot 1.15 mi., find the trial speed of the *Indiana* and the *Nebraska* in miles.

Water in Fruit and Vegetables



1. Pupils in a science class discovered the amount of water in different fruits and vegetables.

They found the weight of some beets to be 46.75 oz. When these were cut up and dried, they weighed only 4.5 oz. How many ounces of water evaporated? Find to hundredths what part of the beets was water.

2. Carrots that weighed 23.5 oz. were found when dried to weigh 2.9 oz. Find to hundredths what part of carrots is water.

3. Cucumbers that weighed 14.75 oz. were found when sliced and dried to weigh only 0.7 oz. Find to hundredths the part of cucumbers that is water.

4. Parsnips that weighed 20 oz. were found to weigh only 3.5 oz. when sliced and thoroughly dried. Find to hundredths the part of parsnips that is water.

5. Potatoes weighing 24.25 oz. were sliced and thoroughly dried. When dried they weighed 5.35 oz. Find to hundredths the part of potatoes that is water.

6. A bushel of potatoes weighs 60 lb. How many pounds of water in a bushel of potatoes? (See result of Ex. 5.)

7. One pound of apples when dried weighed only 1.2 oz. Find to hundredths the part of apples that is water.

69. COMMON FRACTIONS CHANGED TO DECIMALS

Since a common fraction may be considered an indicated division, we may change a common fraction to a decimal by proceeding as in division.

1. Change $\frac{3}{4}$ to a decimal.

WORK

$$4\overline{)3.00}$$
$$0.75$$

Since $3 \div 4 = 0.75$, $\frac{3}{4} = 0.75$.

Change the following to decimals:

2. $\frac{1}{2}$.	4. $\frac{1}{5}$.	6. $\frac{3}{5}$.	8. $\frac{17}{20}$.	10. $\frac{6}{25}$.
3. $\frac{1}{4}$.	5. $\frac{2}{5}$.	7. $\frac{4}{5}$.	9. $\frac{11}{20}$.	11. $\frac{17}{40}$.

12. Change $\frac{5}{7}$ to a decimal.

WORK

$$7\overline{)5.00}$$
$$0.71+$$

The quotient is 0.71 with a remainder of .03. Since the next figure of the quotient, were the division carried on, is less than 5, the result is said to be "*true to the nearest hundredth.*" The plus sign indicates a remainder.

13. Change $\frac{2}{3}$ to a decimal.

WORK

$$3\overline{)2.00}$$
$$0.66+$$
$$0.67-$$

Since the third figure of the quotient is 6, we say that "to the nearest hundredth," $\frac{2}{3} = 0.67^-$. The minus sign indicates that 7 is larger than the real quotient.

> *When the next quotient figure, were the division continued, is less than* 5, *a plus sign may be used to indicate the fact; but if it is equal to or greater than* 5, *the preceding figure is increased by* 1 *and a minus sign written.*

Thus, 0.374 is written 0.37+, while 0.375, 0.376, etc., are written 0.38−, when the result is wanted to the nearest hundredth.

Change to decimals, carrying the results to the nearest hundredth:

14. $\frac{2}{3}$.	**17.** $\frac{5}{12}$.	**20.** $\frac{2}{13}$.	**23.** $\frac{11}{12}$.
15. $\frac{1}{6}$.	**18.** $\frac{7}{12}$.	**21.** $\frac{5}{7}$.	**24.** $\frac{3}{17}$.
16. $\frac{2}{7}$.	**19.** $\frac{4}{9}$.	**22.** $\frac{6}{13}$.	**25.** $\frac{5}{11}$.

26. Reduce $5\frac{3}{4}$ to a mixed decimal.

$$\frac{3}{4} = 0.75, \text{ then } 5\frac{3}{4} = 5.75.$$

Reduce to mixed decimals:

27. $5\frac{11}{32}$.	**30.** $4\frac{19}{20}$.	**33.** $6\frac{14}{29}$.	**36.** $12\frac{7}{16}$.
28. $3\frac{13}{16}$.	**31.** $13\frac{7}{8}$.	**34.** $17\frac{2}{3}$.	**37.** $8\frac{9}{32}$.
29. $9\frac{54}{125}$.	**32.** $7\frac{4}{13}$.	**35.** $19\frac{5}{11}$.	**38.** $19\frac{17}{48}$.

70. A RATIO EXPRESSED IN DECIMALS

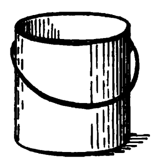
Since a ratio is the quotient of one number divided by another, the ratio may be expressed as a decimal.

Express to hundredths the ratio of:

1. 36 to 15.	**4.** 26 to 65.	**7.** 36 to 131.
2. 84 to 36.	**5.** 84 to 126.	**8.** 87 to 26.
3. 17 to 43.	**6.** 98 to 144.	**9.** 120 to 53.

Practical Problems in Ratio

1. The diameter, or the width across the top of a bucket, is measured and found to be 6 in. By measuring around it with a tape line, the circumference, or the distance around it, is found to be 18.85 in. Find the ratio of the distance around to the distance across, carrying the answer to hundredths.



2. The diameter of a silver half dollar is 1.2 in. The circumference, or distance around it, is found to be 3.77 in. Find the ratio of the circumference to the diameter, carrying the answer to hundredths.

3. A boy measures the wheel of his bicycle, and finds that its diameter is 28 in., and circumference 88 in. Find the ratio of the circumference to the diameter.

4. Measure to the tenth of an inch the diameters and the circumferences of various circular objects of different sizes, and compute the ratios of the circumferences to the diameters.

5. Find the average of these ratios by adding them together and dividing the sum by the number of them. If the work is accurately done, the average of the ratios should be found to be just a little more than 3.14.

6. Calling the ratio of the circumference to the diameter of any circular object 3.14, find the circumference of a wheel whose diameter is 8.5 in.

7. Find the circumference of a grindstone of which the diameter is 38 in.

8. A piece of round steel shafting is 4 in. in diameter. What is the distance around it?

9. The distance around a circular pond is found to be 287 yd. How far is it across the pond?

10. Wishing to know the diameter of a circular water tank, I measured the distance around it, and found it to be 49.5 ft. What was the diameter?

11. The diameter of a wagon wheel is 4 ft. 8 in. How long a tire will be required if we allow 4 in. for welding?

Specific Gravity

The *specific gravity* of any solid or liquid substance is the ratio of its weight to the weight of an equal volume of water. One cubic foot of water weighs 62.5 lb.

1–8. The following table gives the weights in pounds of 1 cu. ft. of each of a number of the most common metals. Compute the specific gravity of each, carrying the answer to the nearest thousandth, and complete the table.

Metal	Weight in Pounds of 1 Cu. Ft.	Specific Gravity
Steel	490	
Cast Iron	450	
Brass	523.8	
Copper	552	
Silver	655.1	
Gold	1200.9	
Lead	709.4	
Platinum	1347	

9–16. The following table gives the weights in pounds of 1 cu. ft. of each of a number of the most common varieties of wood. Find the specific gravity of each, carrying the answer to the nearest thousandth.

Wood	Weight in Pounds of 1 Cu. Ft.	Specific Gravity
Oak	65	
Ash	52.8	
Yellow Pine	34.6	
White Pine	28	
Beech	53.25	
Maple	46.88	
Walnut	41.9	
Cork	15	

VI. WHOLE NUMBERS

71. READING AND WRITING LARGER NUMBERS

1. In 5, 50, 500, and 5000, how does 5 change in value?

2. What value has the zero? What use has it?

3. Which of the following figures has the greatest value, 7, 3, 5, 9, 8? Which of the following, 30, 3, or 300?

4. The value of a figure then depends upon what two things?

5. Write the largest number you can, using just 0, 3, 5, and 8.

6. In Exercise 5, tell why you placed each figure where you did.

7. How many units of any order does it take to make one unit of the next higher order?

8. In the United States system of money, how many cents equal one dime? How many dimes in a dollar?

> *We call our system of writing numbers and our money system a* **decimal system**. *The word* **decimal** *is taken from the Latin* **decem**, *meaning "ten."*
>
> *In a decimal system ten units of any order make one unit of the next higher order.*

9. Read 9,009,000. Of what use are the zeros?

10. How many orders higher is the 9 at the left than the other 9?

11. How many times as great in value is the left-hand 9?

12. How would the 9's be affected in value by removing the zeros in the first period?

13. How is 4000 changed in value when we remove the last zero? The last two? All three?

14. How can you give 387 a value a thousand times as great?

15. Write the largest possible number using these nine figures only: 0, 1, 3, 3, 7, 7, 8, 8, 9.

16. Another zero would enable you to write a number how many times as large?

17. How could you make 365,000 represent a number only one-thousandth part as large?

18. Each zero annexed to a number changes its value how?

19. Each zero removed from the right of a number makes what change?

20. Write 1 thousand 1 hundred 1.

21. Write ten million, two thousand, sixty.

22. Without writing in columns give the sum of 18,000; 200,000; 520; 6.

23. Write the sum of 340; 2,000,000; 700,000; 48,000; 9.

24. Read without using the word *and:* 3005; 2806; 1,003,008.

25. In 281,249,944 why are the figures grouped in threes?

26. Each group is sometimes called a **period.** Give the name of each period beginning with the lowest.

27. What is the value of the highest period? Of the next? Of the lowest? Read the entire number.

Sometimes we need a number of four periods. The next period higher than *millions* is called **billions.**

28. Read:

3,475,635,320	125,370,000,000
15,065,703,490	840,730,045,000

Some Farm Products: The Sugar Industry

NOTE. — This set of problems shows the use of large numbers and gives practice in reading them, without giving difficult computations.

1. In 1909 the world produced 6,775,000 tons of beet sugar and 7,935,000 tons of cane sugar. How much more cane sugar was produced than beet sugar?

2. Germany produces about $\frac{1}{4}$ of the beet sugar of the whole world. If $\frac{1}{4}$ of the product of 1909 was produced by Germany, how many tons did she produce? How much did the rest of the world produce?

3. Cuba produces about $\frac{1}{5}$ of the cane sugar of the world. If she produced $\frac{1}{5}$ of the product of 1909, find the number of tons she produced.

4. Louisiana produces about $\frac{1}{4}$ as much cane sugar as Cuba. What part of the world's product does Louisiana produce?

5. The United States produces about $\frac{1}{7}$ as much beet sugar as Germany. In a recent year the product of the United States was as follows: Colorado, 103,159 tons; Michigan, 79,597 tons; California, 88,347 tons; Utah, 40,820 tons; Idaho, 23,353 tons; and seven other states together produced 48,725 tons. If this was $\frac{1}{7}$ of the product of Germany for that year, how much did she produce? If Germany produced $\frac{1}{4}$ of the world's product that year, find the world's product.

6. The Philippines produce about $\frac{1}{2}$ as much cane sugar as Louisiana. During a recent year the Philippines produced 151,000 tons. If this is $\frac{1}{2}$ the product of Louisiana, and Louisiana produces $\frac{1}{4}$ as much as Cuba, and Cuba produces $\frac{1}{5}$ of the world's product, find the product for that year of Louisiana, Cuba, and of the world.

7. The United States during a recent year produced 384,010 tons of beet sugar and 390,880 tons of cane sugar. We imported that year $4\frac{1}{2}$ times this amount; find the amount imported and the total consumption of sugar in that year.

8. The numbers of tons in the above exercises were numbers of *long tons*. A long ton is 2240 lb. Estimating the population of the United States to be 88,000,000, from the facts of Example 7 find the average number of pounds of sugar consumed by each person.

The Cotton Industry

One of the most important farm products of the United States is cotton. It is our greatest export crop. The value of cotton exported during a recent year was $417,390,000.

1. A farmer in a Southern state gave the following estimate of the cost and income of 3 acres. Cost of production: plowing ground and seeding $4.50; 3 sacks guano at $1; seed $5; four plowings at $1; 3 hoeings at $1.50; picking, hauling, and ginning $10. The income was 600 pounds lint at 9¢ and 36 bushels of seed at 25¢. Find the total cost, income, and profit.

2. At the rate of profit found in Example 1, find the profit from a plantation of 870 acres.

3. After the lint cotton is separated from the seed it is packed into bales of an average weight of about 500 pounds. How many bales on the plantation described in Example 1?

4. The United States produces most of the world's supply of cotton. In 1909 the total cotton product of the world was 17,105,000 bales. Of this the United States produced 13,828,000 bales and the East Indies about one third of the remainder. How many bales did the East Indies produce?

5. Texas produces a little more than one fourth of the cotton produced in the United States. Considering that she produces $\frac{1}{4}$ of the whole crop, how much does she produce when all the rest of the country produces 10,053,000 bales?

6. Mississippi ranks next to Texas in cotton production and produces about half as much. If Mississippi produces $\frac{1}{2}$ as much as Texas, and Texas produces $\frac{1}{4}$ as much as the whole country, find the total product if Mississippi produces 1,735,000 bales.

7. The price of cotton during a single year may vary from $8\frac{3}{4}$ cents to $14\frac{1}{8}$ cents per pound. How will the price of a bale vary at this rate?

8. When the crop of this country is 13,828,000 bales what is the difference between its values at the prices given in Example 7?

The Corn Industry

Our greatest grain crop is corn. In 1909 the corn product of this country was estimated at 2,767,000,000 bushels.

1. The five states having the largest crops of corn in 1908 were as follows: Illinois, 298,620,000 bu.; Iowa, 287,456,000 bu.; Nebraska, 205,767,000 bu.; Missouri, 203,634,000 bu.; and Texas, 201,848,000 bu. If these five states produced as much as all the rest of the country together, what was the total product for that year?

2. When this country produced 2,668,651,000 bushels of corn the estimated price was 60.6¢ per bushel. Find the total value of the crop.

3. If more scientific farming would have increased the value of the crop by $\frac{1}{5}$, how much would that have added to the total wealth of this country?

4. It was estimated that the average yield per acre in 1908 was 26.2 bu. Using the facts of Problem 2, find the number of acres devoted to the cultivation of corn.

5. If in a certain year 108,645,000 acres produced 2,661,-502,500 bushels of corn, find the yield per acre.

The Wheat Industry

The wheat crop is our second greatest grain crop, the corn crop being first. In 1909 the United States produced 724,768,000 bushels of wheat. "Enough wheat, when made into bread, to provide $1\frac{1}{8}$ loaves a day for each inhabitant of the United States for a year; enough, when made into loaves a foot long, to girdle the earth."

1. The United States produces about $\frac{1}{5}$ of the world's product of wheat. Russia ranks next and produces almost $\frac{4}{5}$ as much as the United States. British India ranks third and produces about $\frac{1}{3}$ as much as the United States.

Find the product of Russia, British India, and the rest of the world together when our production was 664,602,000 bushels (the yield of 1907).

2. The three states producing the greatest number of bushels of wheat in 1908 were: Kansas 79,282,000 bushels; Minnesota 68,557,000 bushels; and North Dakota 68,428,000 bushels. This was nearly $\frac{1}{3}$ of the whole product of the country during that year. At this rate, find the total product.

3. At an average price of $87\frac{1}{2}$ cents a bushel, find the value of the crop in each of the three states named in Example 2.

4. The average yield in Minnesota is 14 bushels per acre. How many acres were required to produce the crop named in Example 2?

5. It has been shown by experiment that care in selecting seed will give an average increase of $\frac{1}{4}$ of the former production. What effect would this have upon the value of the crop in each of the states named in Example 2?

6. How would such an increase in production affect the value of the total crop in this country? Call the average yearly yield 664,540,000 bushels, and the average price $88\frac{3}{4}$ cents.

72. Rapid Work in Addition

Rapid and accurate work in arithmetic comes only through much practice. We learn to recognize groups of figures as standing for a sum, just as we recognize a group of letters as standing for a word.

NOTE. — The following tables are to be used for daily drill until a sum can be recognized instantly.

All Possible Combinations of Two Figures

Announce the 45 *sums in half a minute:*

1	1	2	2	4	1	3	3	4	3	1	4	2	4	7
1	3	1	2	1	5	2	3	2	6	7	3	5	6	7
8	9	8	5	6	4	5	5	7	1	5	6	6	8	9
9	9	8	5	1	4	3	4	2	8	6	6	9	6	1
8	7	7	4	9	7	6	7	5	3	2	4	5	7	6
2	3	5	8	3	8	7	9	9	8	9	9	8	4	2

Combinations of Two Place Numbers with One Place Numbers

To add columns quickly requires ability to recognize the sum of such groups as the following:

Announce the 70 *results in one minute and a half or less:*

21 7	41 3	62 9	72 2	44 6	51 5	63 2	33 3	41 7	64 3
18 9	49 9	68 8	45 5	66 5	74 4	15 3	65 4	27 2	41 8
94 2	83 6	36 8	48 6	29 4	88 2	87 3	67 4	14 8	49 3
57 8	46 7	17 9	95 9	23 8	42 9	54 9	55 8	47 4	66 2
35 7	86 5	38 6	79 3	26 9	99 8	76 5	53 9	89 4	56 7
82 9	58 4	92 8	39 7	93 9	85 6	78 5	98 6	43 9	37 6
34 8	93 9	77 6	84 7	96 5	52 9	97 4	73 8	28 9	75 6

Combinations of Two Numbers of Two Figures Each

So much of our work is with two numbers of but two figures each, that we ought to be able to announce at sight the sum of any two such numbers.

It is better to add the tens first and then the units.

Thus in adding 35 and 27, we say 35 and 20 are 55 and 7 are 62.

Announce the 60 *results in two minutes or less:*

12	11	18	17	14	15	17	12	13	16
15	11	13	14	16	16	17	12	14	13
13	14	15	17	12	16	15	12	16	18
17	16	14	15	18	19	15	17	17	18
16	14	19	11	17	19	18	19	12	15
16	13	19	15	18	13	14	18	19	18
14	14	27	19	17	12	18	13	15	21
14	16	18	11	19	14	12	13	19	19
21	76	52	43	32	87	64	87	53	48
18	13	19	17	17	14	12	15	14	19
43	56	43	57	64	47	39	48	78	67
19	14	18	18	19	19	19	18	16	17

When a figure is repeated several times the sum is more quickly found by multiplication than by addition.

Thus in

7	think 4 × 7 + 6,
6	or 5 × 7 − 1.
7	Why?
7	
7	
34	

At sight give sums:

1.	2.	3.	4.	5.	6.	7.	8.
8	7	5	6	8	7	8	8
6	7	9	6	8	9	4	8
6	7	5	8	5	9	4	3
6	9	5	6	8	9	4	8
6	7	5	6	8	9	4	8

In adding columns try to look out for the combinations that make 10, such as 6 + 4, 3 + 7, 3 + 2 + 5, and the rest.

It is useful sometimes to remember that the sum of a sequence of three or five numbers is 3 or 5 times the middle number in the sequence. Thus the sum of 5, 6, 7 is 3 × 6, or 18; the sum of 5, 6, 7, 8, 9 is 5 × 7, or 35.

Copy and find the sum of:

1.	2.	3.	4.	5.	6.
197	757	1345	4787	46,768	48,748
928	836	2769	6432	14,693	98,396
873	793	3876	8978	24,768	47,874
417	581	4245	6479	71,293	56,832
652	629	8347	9463	60,037	91,478
765	924	9038	8779	73,407	16,923
739	786	6705	3678	19,386	94,873
984	937	9477	6482	47,682	73,689
859	724	6841	5962	86,573	69,798
487	692	7238	7389	94,775	54,991
529	496	5467	8917	89,238	98,346
843	739	3094	8998	64,839	79,819
468	496	6298	9843	29,466	54,433

73. RAPID WORK IN SUBTRACTION

Subtraction depends upon addition. To find the difference, think what number added to the smaller makes the larger.

Subtraction Table

Give the 60 *differences in one minute:*

4	5	5	10	5	8	6	6	9	8
2	3	4	4	2	3	4	2	3	5
4	7	9	6	7	12	8	6	7	11
3	2	4	5	5	3	2	3	4	2
7	11	11	14	12	8	12	15	14	13
6	4	5	6	5	6	4	6	5	4
9	8	11	10	10	14	11	11	15	9
6	7	7	8	6	7	8	6	8	7
9	13	10	14	14	12	13	11	16	13
8	6	9	8	9	7	8	9	9	7
21	32	54	76	38	25	42	81	65	43
2	3	5	7	9	6	4	9	7	6

Problems in Addition and Subtraction

The following table gives an estimate of the number of live stock in the different continents in 1909:

Continent	Cattle	Horses	Mules	Sheep	Hogs
North America	91,334,279	27,396,746	4,655,999	62,946,091	60,784,137
South America	75,592,773	9,511,594	865,793	90,075,858	5,742,989
Europe . . .	127,592,645	43,563,225	1,617,608	181,266,488	71,630,599
Asia	112,268,956	11,630,302	56,256	90,590,694	4,824,187
Africa . . .	9,474,115	885,113	296,294	43,901,330	1,276,917
Oceania . . .	12,068,681	2,232,408	1,899	108,646,123	999,976

1. Find the total number of each kind of animal.

2. How many more cattle are in Europe than in North America?

3. How many more horses are in Europe than in North America?

4. How many more sheep are in Europe than in North America?

5. How many more hogs are in Europe than in North America?

The following table is a record of the cheese industry of the United States in a recent year:

State	Factories	Pounds Produced	Value
New York	1,198	132,836,482	$10,812,747
Wisconsin	1,454	109,423,856	10,488,853
Iowa	48	2,829,745	282,078
Illinois	41	5,301,211	426,026
Minnesota	59	3,090,055	307,117
Pennsylvania	120	11,453,424	1,007,815
Other states	690	52,210,099	5,287,124

6. How many cheese factories in the United States? How many pounds of cheese were produced in the United States during that year? What was the total value of the cheese produced?

7. If the United States exported 10,341,335 lb. of American cheese during the same year, how many pounds of this cheese were consumed in the United States?

8. In the same year the United States imported 34,238,459 lb. of cheese from other countries. What was the total amount of cheese consumed in the United States in the year?

9. The total number of male immigrants entering the United States in 1909 was 519,969. Of these 453,297 came from Europe. How many were from other continents?

10. The total number of female immigrants in the same year was 231,817, of which 201,578 came from the countries of Europe. How many came from other countries?

11. How many immigrants in all entered the United States in 1909? How many in all came from Europe?

The following table gives the number of immigrants in 1909 from the different parts of Asia:

Nationality	Male	Female	Total
China	1,773	170	
Japan	1,291	1,820	
India	164	39	
Turkey in Asia	5,792	1,714	
Other Asiatic Countries	112	29	
Total			

12. Find the total number of each nationality, the total number of males, the total number of females, and the grand total from all Asia.

13. In the Chicago public schools the attendance in certain years was as follows: in 1880, 59,562; in 1890, 135,541; in 1900, 255,861; in 1909, 296,427. Find the increase from one period to the next.

74. THE KEEPING OF ACCOUNTS

When a person does not pay cash for goods bought at a store, but has a record kept of the purchases, and pays for all of them at the end of each month or some other period of time, the record of the purchases and payment is called an **account.** Men engaged in business keep accounts of their receipts (money received) and expenditures (money spent).

In every account the record is in two columns, separated by a line running down the middle of the page, or it is kept on opposite pages of the account book.

The debit side of an account. All records of *debt* are called *debits*, and are placed in the *left-hand* column or page of the account.

The credit side of an account. All records of money, or its equivalent, received from a debtor on his account by the one who keeps the account, are called *credits*, and are placed in the *right-hand* column or page.

Balancing an account. The person who keeps an account *balances* it at the end of each month or at the end of some other period of time. That is, he finds the sum of all debits and the sum of all credits, and then finds the difference between these two sums. This difference is called the *balance*. This is entered on the smaller side of the account, thus making the total footings or sums of the two columns equal.

The following illustration shows the form of a ledger account kept by E. L. Holmes & Co., merchants, with Robt. L. Ray, debtor:

Dr.			ROBT. L. RAY					Cr.	
1910					1910				
Jan.	2	Mdse.	28	00	Jan.	6	Cash	20	00
"	7	"	36	00	"	15	"	35	00
"	10	"	10	00	Feb.	15	"	20	00
"	15	"	32	50	Mar.	1	Balance	92	50
Feb.	12	"	18	.75					
"	14	"	42	25					
			167	50				167	50

1. Who has bought goods as shown by the above account? Of whom has he bought them?
2. What do the "Mdse." items of the debit side show?
3. What do the "Cash" items of the credit side show?
4. What does the "Balance" on the credit side show?
5. What would a "Balance" on the debit side have meant?

Exercises

Rule paper, make out the following accounts in the above form, and balance them:

1. John V. Farwell & Co., in account with B. L. Gray, debtor: June 1, 1910, Mdse. $25.48; June 7, Mdse. $76.25; June 25, Mdse. $43.75. Credits, June 15, cash $45; June 20, cash $50.50; June 28, goods returned $24.67. Balance the account July 1, 1910.

2. R. C. Randall & Co., in account with W. R. Brooks, debtor: Mdse. $38.40, Mdse. $96.23, Mdse. $84.68, Mdse. $34.50, Mdse. $26.84. Credits by cash and goods returned, $120, $26.84, $90, $38.50, $84.20. Assign dates.

3. Miller Bros., in account with Wm. R. West, debtor: Mdse. $84.60, Mdse. $96.80, Mdse. $64.98, Mdse. $134.18. Credits by cash and goods returned, $100, $34.60, $75, $26.80. Assign dates.

4. Prepare an account between *The Fair* and yourself as debtor, and balance it.

5. Prepare an account between Mandel Bros. and your teacher as debtor, and balance it.

Find the balance of each of the following accounts:

6. Dr.	6. Cr.	7. Dr.	7. Cr.	8. Dr.	8. Cr.
$987.65	$629.55	$4768.82	$468.34	$649.81	$82.46
1839.76	83.74	947.61	984.59	8439.87	981.32
6482.91	968.71	847.77	1483.22	648.38	641.25
478.85	28.46	3998.64		91.76	239.86
698.47	318.93	8372.91			728.41

9. Dr.	9. Cr.	10. Dr.	10. Cr.	11. Dr.	11. Cr.
$246.94	$839.75	$698.32	$649.83	$356.78	$135.72
839.76	646.81	376.59	478.88	938.12	873.54
842.94	794.32	843.26	694.31	45.23	137.92
327.68	546.78	695.98	883.24	938.83	7639.85
946.32	937.89	831.96	695.64	876.23	736.29

12. The following is the open account of E. R. Walker with the First National Bank:

Dr. E. R. WALKER Cr.

Jan.	11	Check Paid	$500	00	Jan.	4	Balance	$486	87
"	12	" "	57	30	"	10	Deposit	290	00
"	20	" "	235	75	"	11	"	198	75
"	22	" "	11	80	"	21	"	773	40
"	28	" "	97	30	"	25	"	110	00

Find the balance. What would a balance on the credit side show?

13. Balance the following and tell what it shows:

On May 1, E. F. Mason had a balance of $4370.28 to his account in the bank. He deposited on May 1, $269; May 6, $165; May 10, $175; May 15, $180.50; May 20, $290. He withdrew by check the following amounts: May 1, $156; May 10, $450; May 13, $125; May 27, $675.50. What was his balance June 1?

75. AN ACCOUNT WITH CASH

When a man keeps an account of the cash that he receives or spends, he is said to keep a *cash account*, or an *account with Cash*. It is as if he were keeping an account between himself and his pocketbook or his cash drawer. "Cash" becomes *debtor* for all money that a man receives, and "cash" is *credited* with all that he spends.

	Cash	Dr.			Cash	Cr.	
1910				1910			
May 1	Cash on Hand	100	00	May 13	Team Bought	350	00
" 5	Rent Recd.	50	00	" 14	Piano Bought	450	00
" 7	Carriage Sold	75	00	" 18	Clothing Bought	25	00
" 10	Land Sold	655	00	" 22	Balance	55	00
		880	00			880	00
May 22	Cash on Hand	55	00				

1. Cash is charged with having received four amounts, which it owes me; that is, for which it is my debtor. How much on hand at the beginning?

2. What is the total amount that Cash has received?

3. When I take out $450 with which to purchase a piano, Cash has paid back to me how much that it owes me?

4. What other amounts has Cash paid me; that is, for what other amounts should Cash be credited? How much has Cash paid me in all?

5. How much more has Cash received than paid out?

6. For how much is Cash debtor at the beginning of the new account?

Exercises

1. At the beginning of a month, Charles Watson has on hand $4.21. During the month he receives at various times $6.24, $7.36, $8.49, $7.34, $6.75; and he pays out $8.75, $9.81, $8.39.

Rule paper, make out his cash account in the form on page 178, and find his balance at the end of the month.

2. On Monday morning a merchant begins business with $247.84 on hand. He receives $24.75, $86.91, $84.28, $97.25, $164.29. He pays out $18.99, $37.49, $64.91, $83.15. Prepare his cash account for the day in the above form, and balance it.

3. Make out the following account for a day and balance it: Cash on hand at the beginning, $165.28; receipts, $24.35, $6.85, $35.40, $15.87, $42.23, $10.60; expenses, $6.45, $8.75, $25.65, $9.78.

4. Make out and balance the following merchant's account for a week: Cash on hand, $145.35; Monday, Oct. 1, sales, $223.87; Tuesday sales, $194.36; Wednesday sales, $142.95; Thursday sales, $210.12; Friday sales, $187.25; Saturday sales, $364.57. Oct. 2, paid bill of $93.40 for goods. Oct. 3, paid electric light bill of $31.72. Oct. 5, bought new counter, $45.40. Oct. 6, clerks' salaries paid, $80.

5. Make out and balance a cash account of your own for a week, including items of car fare, books, earnings for selling papers, gifts from your father, etc.

76. ALIQUOT PARTS; SHORT METHODS IN MULTIPLICATION

Give the following:

$\frac{1}{2}$ of 10.	$\frac{1}{3}$ of 100.	$\frac{1}{8}$ of 100.	$\frac{1}{7}$ of 100.
$\frac{1}{3}$ of 10.	$\frac{1}{4}$ of 100.	$\frac{1}{12}$ of 100.	$\frac{1}{9}$ of 100.
$\frac{1}{4}$ of 10.	$\frac{1}{6}$ of 100.	$\frac{1}{16}$ of 100.	$\frac{1}{8}$ of 1000.

An **aliquot part** of a number is the quotient resulting from the division of the number by an integer. Thus $16\frac{2}{3}$ is an aliquot part of 100 because $100 \div 6 = 16\frac{2}{3}$.

Name some other aliquot parts of 100:

Memorize this table of aliquot parts:

$25 = \frac{1}{4}$ of 100.	$16\frac{2}{3} = \frac{1}{6}$ of 100.	$6\frac{1}{4} = \frac{1}{16}$ of 100.	$2\frac{1}{2} = \frac{1}{4}$ of 10.
$33\frac{1}{3} = \frac{1}{3}$ of 100.	$12\frac{1}{2} = \frac{1}{8}$ of 100.	$3\frac{1}{3} = \frac{1}{3}$ of 10.	$125 = \frac{1}{8}$ of 1000.

$$25 \times 72 = \tfrac{1}{4} \text{ of } 100 \times 72, \text{ or } \tfrac{1}{4} \text{ of } 7200 = 1800$$

or $$25 \times 72 = \tfrac{1}{4} \text{ of } 72 \times 100, \text{ or } 18 \times 100 = 1800.$$

1. Tell how to multiply by 25; by $33\frac{1}{3}$; by $16\frac{2}{3}$; by $12\frac{1}{2}$; by 125; by $6\frac{1}{4}$; by $3\frac{1}{3}$; by $2\frac{1}{2}$.

Give products at sight:

2. $33\frac{1}{3} \times 18$.	7. $16\frac{2}{3} \times 36$.	12. 25×84.
3. 25×24.	8. $33\frac{1}{3} \times 42$.	13. $12\frac{1}{2} \times 24$.
4. $16\frac{2}{3} \times 54$.	9. $12\frac{1}{2} \times 72$.	14. $16\frac{2}{3} \times 24$.
5. 25×96.	10. 25×32.	15. $12\frac{1}{2} \times 96$.
6. $6\frac{1}{4} \times 48$.	11. $33\frac{1}{3} \times 63$.	16. $6\frac{1}{4} \times 64$.

At sight, give the cost of:

1. 21 lb. coffee at $33\frac{1}{3}$ ct.
2. 48 lb. tea at 25 ct.
3. 24 lb. prunes at $16\frac{2}{3}$ ct.
4. 32 lb. rice at $6\frac{1}{4}$ ct.
5. 48 lb. raisins at $16\frac{2}{3}$ ct.
6. 48 lb. prunes at $12\frac{1}{2}$ ct.

7. 54 lb. steak at $16\frac{2}{3}$ ct.
8. 72 lb. lard at $16\frac{2}{3}$ ct.
9. 96 lb. butter at 25 ct.
10. 48 qt. berries at $12\frac{1}{2}$ ct.
11. 9 yd. cloth at 50 ct.
12. 16 yd. cloth at 25 ct.
13. 12 yd. lace at $8\frac{1}{3}$ ct.
14. 36 yd. braid at $8\frac{1}{3}$ ct.
15. 24 yd. cloth at $16\frac{2}{3}$ ct.
16. 96 yd. cloth at $12\frac{1}{2}$ ct.

Find the products:

1. $3\frac{1}{3} \times 63.72$.
2. $2\frac{1}{2} \times 86.48$.
3. $16\frac{2}{3} \times 193.52$.
4. $16\frac{2}{3} \times 384.54$.
5. 25×36.978.
6. 25×32.847.
7. $33\frac{1}{3} \times 96.126$.
8. $33\frac{1}{3} \times 73.245$.
9. 25×9.368.
10. 25×73.46.
11. $12\frac{1}{2} \times 6.344$.
12. $12\frac{1}{2} \times 73.824$.
13. $6\frac{1}{4} \times 86.375$.
14. $16\frac{2}{3} \times 42.963$.
15. $33\frac{1}{3} \times 19.634$.
16. 25×263.45.
17. $33\frac{1}{3} \times 46.751$.
18. $16\frac{2}{3} \times 32.968$.
19. $12\frac{1}{2} \times 175.32$.
20. $12\frac{1}{2} \times 134.56$.
21. 25×643.96.
22. 125×34.69.
23. 125×28.34.
24. 125×36.82.

77. SPECIAL FRACTIONS

Memorize the following table:

$0.50 = \frac{1}{2}$	$0.33\frac{1}{3} = \frac{1}{3}$	$0.62\frac{1}{2} = \frac{5}{8}$
$0.25 = \frac{1}{4}$	$0.66\frac{2}{3} = \frac{2}{3}$	$0.87\frac{1}{2} = \frac{7}{8}$
$0.12\frac{1}{2} = \frac{1}{8}$	$0.16\frac{2}{3} = \frac{1}{6}$	$0.14\frac{2}{7} = \frac{1}{7}$
$0.06\frac{1}{4} = \frac{1}{16}$	$0.08\frac{1}{3} = \frac{1}{12}$	$0.11\frac{1}{9} = \frac{1}{9}$
$1.25 = 1\frac{1}{4}$	$0.37\frac{1}{2} = \frac{3}{8}$	$0.83\frac{1}{3} = \frac{5}{6}$

NOTE. — There are other relations that are readily seen from these.

Thus, $0.28\frac{4}{7} = \frac{2}{7}$; $0.56\frac{1}{4} = \frac{9}{16}$; $0.58\frac{1}{3} = \frac{7}{12}$; etc.

Give products at sight:

1. 28×0.25.
2. 0.75×84.
3. 32×1.25.
4. 96×0.75.
5. 12×2.25.
6. 16×1.75.
7. 24×2.50.
8. 36×1.25.
9. 14×2.50.

10. 3.5×42. **12.** $24 \times 0.33\frac{1}{3}$. **14.** $18 \times 0.66\frac{2}{3}$.

11. 1.25×64. **13.** $36 \times 1.33\frac{1}{3}$. **15.** $48 \times 1.66\frac{2}{3}$.

Problems in Bills

(Use short methods of multiplication.)

1. Check the following bill:

CHICAGO, ILL., *April 1*, 19*11*

RANDOLPH MARKET AND GROCERY

52–54 STATE ST.

SOLD TO *Harold Johnson,*
56 W. 43d St.

March 14	*4 pkg. rolled oats*	@ *$12\frac{1}{2}$¢*		*50*		
"	*3 lb. cheese*	@ *$16\frac{2}{3}$¢*		*50*		
"	*1 sack G. Med. flour*	@ *95¢*		*95*	*1*	*95*
March 21	*6 cans peas*	@ *$12\frac{1}{2}$¢*		*75*		
"	*$2\frac{1}{2}$ lb. crackers*	@ *10¢*		*25*		
"	*8 lb. sugar*	@ *$6\frac{1}{4}$¢*		*50*		
"	*4 lb. sirloin steak*	@ *$16\frac{2}{3}$¢*		*67*	*2*	*17*
					4	*12*

Make out bills for the following:

2. 3 sacks flour at \$1.25; 4 lb. sirloin roast at $12\frac{1}{2}$¢; 3 lb. sugar-cured corned beef at $8\frac{1}{3}$¢; $3\frac{1}{2}$ lb. loin lamb chops at 18¢; 4 cans Maine lobster at $33\frac{1}{3}$¢.

3. 10 lb. sugar at $6\frac{1}{4}$¢; 6 cans asparagus at $33\frac{1}{3}$¢; 4 pkgs. macaroni at $16\frac{2}{3}$¢; $2\frac{1}{2}$ lb. fancy Japan rice at 8¢; 3 pkgs. prepared flour at $8\frac{1}{3}$¢.

4. 12 lb. prunes at $16\frac{2}{3}$¢; 6 lb. coffee at $33\frac{1}{3}$¢; 48 lb. steak at $12\frac{1}{2}$¢; 32 lb. rice at $6\frac{1}{4}$¢.

5. 48 lb. butter at 25¢; 24 qt. berries at $12\frac{1}{2}$¢; 36 lb. lard at $16\frac{2}{3}$¢; 3 lb. tea at $66\frac{2}{3}$¢.

6. 12 yd. cloth at 50¢; 15 yd. lace at $8\frac{1}{3}$¢; 21 yd. ribbon at $14\frac{1}{7}$¢; 8 yd. silk at $1.25.

7. Make at least four other bills involving the use of the special fractions in the table of Section 77.

78. SHORT METHODS IN DIVISION

The divisors 5, 10, 25 (2.5 or $2\frac{1}{2}$), $33\frac{1}{3}$, 100, and 125 (12.5 or $12\frac{1}{2}$) occur so often in business that a short method of dividing by them should be learned.

1. Divide the following by 10 in the shortest way:
   $160; $2.40; $36.50; 125 lb.; 5280 ft.; 324 tons.

2. How is a number divided by 10?

3. Divide the following by 100 by moving the point:
   $1520; $27; $52; 1250 rd.; 2.6; 36 hr.; $14,000.

4. How is a number divided by 100?

5. $\frac{1}{5}$ = how many 10ths? To find $\frac{1}{5}$ of a number, we find $\frac{2}{10}$ of it. That is, to divide by 5, multiply by 2 and divide by 10, by moving the decimal point.

6. Divide the following by 5:
   1260; $740; $12.50; $445; $1.35; 5280 ft.

7. 25 is what part of 100? Hence, to divide by 25, multiply by —— and move the decimal point ——.

8. Divide the following by 25:
   720; $19.50; $1242; $48.25; 940 lb.; 550 yd.

9. $2\frac{1}{2}$ or 2.5 is what part of 25? Hence, to divide by $2\frac{1}{2}$ or 2.5, multiply by 4 and move the decimal point ——.

10. Divide each number in Ex. 8 by $2\frac{1}{2}$.

11. $33\frac{1}{3}$ is what part of 100? To divide by $33\frac{1}{3}$, multiply by —— and move the decimal point ——.

12. Divide the following by $33\frac{1}{3}$:
400; 1300; \$64; 48 ft.; 1600 lb.; \$270.

13. 125 is what part of 1000? To divide by 125, multiply by —— and move the decimal point ——.

14. Give the short method of dividing by $12\frac{1}{2}$ or 12.5.

15. Divide the following by 125:
12,000; \$25,000; \$420; 7200 lb.; \$19.25; \$126.75.

16. A merchant has a stock of goods worth \$5262. The store fixtures are worth $\frac{1}{10}$ as much. What is the value of the fixtures?

17. For \$6.50, how many yards of cloth can be bought at 25¢ a yard?

18. At $33\frac{1}{3}$¢ a yard, how many yards of carpet can be bought for \$15?

19. How many yards of ribbon at $12\frac{1}{2}$¢ a yard can be bought for 75¢?

20. At 25 mi. an hour, how long will it take an automobile to go 160 miles?

21. At 40 mi. an hour, how far will a train run in 12 min.? In 36 min.?

22. At $33\frac{1}{3}$¢ a hundred, how many picture post cards can be bought for \$5?

23. How many pieces each $12\frac{1}{2}$ in. long can be cut from a piece of ribbon containing 10 yd.?

24. See how many seconds it takes you to divide 98,500 by 125 by long division. See how much less time it takes to perform the division by the short method worked out above.

79. TIME BETWEEN EVENTS

It is often desirable to find the time in years, months, and days, between two dates. Bankers usually find the exact number of days by use of tables. In working without tables, 360 days are considered a year, and 30 days a month, and it is customary to subtract as in the following problem:

1. Find the time between July 4, 1897, and Feb. 22, 1910.

WORK

1910 yr.	2 mo.	22 da.
1897	7	4
12 yr.	7 mo.	18 da.

EXPLANATION. — February is the 2d month and July the 7th month of the year. Since 7 mo. cannot be taken from 2 mo., 1 yr. of the 1910 yr. is changed to 12 mo., which added to 2 mo. gives 14 mo. 14 mo. – 7 mo. = 7 mo. 1909 yr. – 1897 yr. = 12 yr.

Find the time between the following dates:

2. March 4, 1904 and June 20, 1910.

3. August 17, 1902 and April 9, 1905.

4. May 12, 1872 and January 1, 1897.

5. Oct. 16, 1899 and June 2, 1904.

6. December 25, 1908 and April 14, 1909.

7. September 30, 1856 and March 9, 1857.

8. Columbus discovered America Oct. 12, 1492. How long ago was that?

9. When were you born? Find your age to-day in years, months, and days.

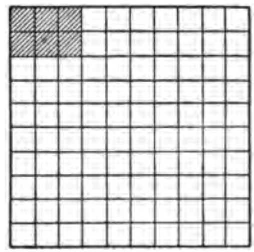
10. Lincoln was born Feb. 12, 1809, and became President of the United States March 4, 1861. At what age did he become President?

11. War with Spain was declared by the United States April 25, 1898, and the treaty of peace ending the war was signed Dec. 12, 1898. How long did the war last?

VII. PERCENTAGE

80. THE MEANING OF PER CENT

Hundredths are also written in another way. Instead of speaking of so many hundredths of anything, we more commonly speak of so many *per cent* of it, which means the same thing.



Thus, if this square is divided into 100 equal parts, each part is 1 *hundredth* of the square, or 1 *per cent* of it.

We write 1 *per cent thus*, 1%.

The shaded part of this square is $\frac{6}{100}$, 0.06, or 6% of it. Which is more easily pronounced, *six hundredths*, or *six per cent?* Which is more easily written?

Exercises

Write as per cent:

1. $\frac{15}{100}$.	4. $\frac{7}{100}$.	7. 0.03.	10. 0.15.
2. $\frac{75}{100}$.	5. $\frac{8}{100}$.	8. 0.08.	11. 0.60.
3. $\frac{6}{100}$.	6. $\frac{50}{100}$.	9. 0.07.	12. 0.40.

13. Draw a square and divide it into 100 equal parts. Shade 5% of them. Shade 15% of them. Shade 25%.

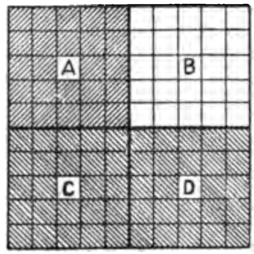
14. Draw a square and divide it into 10 equal parts. Shade 10% of it. 20% of it. 30% of it. 50% of it.

15. If you have 100 words in a spelling "review," and miss 8 of them, how many hundredths do you miss? How many per cent of them do you miss? What per cent is your mark?

16. If you miss 5, what per cent do you have right? If you miss 15? If you miss 20?

17. Out of 100 school days, if you are absent 3 days, what per cent of the 100 days is this? What is your mark of attendance?

18. This square is divided into 100 small squares. How many in the square marked A? Then A = 25% of the whole square. Describe B, C, and D in the same way.



19. The two squares, C and D are what part of the whole square? What per cent of it?

20. Write as per cent:

A = —— of the whole square.
A + B = —— of the whole square.
A + B + C = —— of the whole square.

21. Draw a circle and shade 25% of it. Shade 50% of it. Shade 75% of it.

22. Draw a rectangle whose width is 50% of its length.

23. The width of a certain rectangle is 25% of its length. What is the width if the length is 20 inches? If its length is 24 inches? If its length is 16 feet?

24. If you have 100 marbles, describe in per cent the part you lose when you lose any of the following:

5, 10, 20, 25, 40, 50.

25. If you have a dollar, or 100 cents, and spend 15¢, what per cent of it do you spend?

26. George had 100 pigeons and sold 10% of them. How many did he sell? How many if he sold 15% of them? 25% of them?

81. FRACTIONAL EQUIVALENTS OF CERTAIN PER CENTS

1. Look at the square in the preceding exercises and tell how many per cent of it in $\frac{1}{2}$ of it. In $\frac{1}{4}$ of it.

50 % of anything is $\frac{1}{2}$ of it.
25 % of anything is $\frac{1}{4}$ of it.

2. Henry's kite string was 400 ft. long, but he lost 50 % of it. How much remained?

3. Mr. Sloan bought 20 tons of coal for the winter. In the spring he had 25 % of it left. How much did he use?

4. A pint is what per cent of a quart? Of a gallon?

Give the following:

5. 25 % of 12. **7.** 25 % of 50. **9.** 25 % of 9.

6. 50 % of 8. **8.** 50 % of 84. **10.** 50 % of 11.

Find:

11. $\frac{1}{3}$ of 100. **13.** $\frac{1}{8}$ of 100. **15.** $\frac{1}{12}$ of 100.

12. $\frac{1}{5}$ of 100. **14.** $\frac{1}{6}$ of 100. **16.** $\frac{3}{4}$ of 100.

17. Since $3 \times 33\frac{1}{3} = 100$, $33\frac{1}{3}$ % of anything is $\frac{1}{3}$ of it.

18. Since $5 \times 20 = 100$, 20 % of anything is $\frac{1}{5}$ of it.

19. Since $8 \times 12\frac{1}{2} = 100$, $12\frac{1}{2}$ % of anything is $\frac{1}{8}$ of it.

20. Make similar statements about $16\frac{2}{3}$ %, $8\frac{1}{3}$ %, and 75 %.

A Table of Equivalents

50 % of anything = $\frac{1}{2}$ of it.	$33\frac{1}{3}$ % of anything = $\frac{1}{3}$ of it.
25 % of anything = $\frac{1}{4}$ of it.	$16\frac{2}{3}$ % of anything = $\frac{1}{6}$ of it.
$12\frac{1}{2}$ % of anything = $\frac{1}{8}$ of it.	$66\frac{2}{3}$ % of anything = $\frac{2}{3}$ of it.
20 % of anything = $\frac{1}{5}$ of it.	75 % of anything = $\frac{3}{4}$ of it.

Give the following:

21. 50 % of 17 yd.	27. $66\frac{2}{3}$ % of 30 bu.	33. $66\frac{2}{3}$ % of 36.
22. 25 % of 18 bu.	28. 75 % of 100 ft.	34. $16\frac{2}{3}$ % of 24.
23. $12\frac{1}{2}$ % of 24 qt.	29. 50 % of 12 yr.	35. $12\frac{1}{2}$ % of 48.
24. 20 % of $30.	30. $33\frac{1}{3}$ % of 30 da.	36. 20 % of 35.
25. $33\frac{1}{3}$ % of 15 yd.	31. 25 % of 15 mi.	37. 75 % of 16.
26. $16\frac{2}{3}$ % of 12 rd.	32. $12\frac{1}{2}$ % of $15.	38. 50 % of 98.

82. A RELATION EXPRESSED IN PER CENT

1. A peck is what part of a bushel? What per cent of it?
2. A foot is what part of a yard? What per cent of it?
3. Two are what part of a dozen? What per cent of it?
4. 2 oz. are what part of a pound? What per cent of it?
5. Since $2 \times 3 = 6$, 2 is what part of 6? What per cent?

Give the relation of the first number to the second as a per cent:

6. 4 and 8.	14. 3 and 9.	22. 6 and 36.
7. 5 and 10.	15. 6 and 18.	23. 7 and 42.
8. 5 and 15.	16. 9 and 36.	24. 12 and 16.
9. 6 and 18.	17. 8 and 32.	25. 30 and 40.
10. 4 and 16.	18. 8 and 40.	26. 20 and 30.
11. 5 and 20.	19. 10 and 30.	27. 12 and 18.
12. 16 and 32.	20. 12 and 48.	28. 15 and 45.
13. 17 and 34.	21. 16 and 48.	29. 15 and 60.

Problems in Percentage

1. In selling sugar, a dealer estimates that he will lose 10 % of it in down weight and drying out. From 600 pounds bought, how many pounds does he expect to sell?

2. By paying cash for a piano I got a discount of 5 %. That is, the dealer deducted 5 % of the price of the piano. How much did I save by paying cash for a $400 piano?

3. A dealer sold me a $20 lamp for $18. How much discount was this? The $2 discount was what part of the price? What per cent of it?

4. A 12-lb. ham weighed but 9 lb. when roasted. How many pounds were lost in cooking? What per cent of the first weight was this?

5. I have 10 trees on my lawn. 5 are maples, 3 are elms, and 2 are oaks. What per cent of the trees are maples? What per cent are elms? What per cent are oaks?

6. A certain field produced 40 bu. of corn per acre. It was estimated that careful cultivation would have increased the crop 20 %. What would the yield have been?

7. Last year 100 pupils graduated from the 8th grade of a certain school. This year 120 will graduate. What is the per cent of increase?

8. James bought a $35 sleigh for 20 % below the regular price. How much did it cost him?

9. At a special sale, suits and overcoats were sold 25 % below the regular price. Give the prices you must pay for suits or overcoats marked regularly as follows:

$16; $12; $20; $24; $32; $28.

10. During the annual "August Sale," a furniture dealer gave a 10 % discount on goods. Below are given the regular prices. What are the August prices?

Desk, $20; Dining-room table, $50; Bookcase, $30; Brass bed, $40; Parlor suit, $120; Kitchen cabinet, $10; Morris chair, $15.

11. During an "After Christmas Sale," some articles were marked $33\frac{1}{3}\%$ below the regular price. Give the special prices of the following:

A $30 overcoat;	A $1.50 shirt;
A $45 business suit;	A $4.50 pair of shoes;
A $12 boy's overcoat;	A $9 boy's suit.

12. Some kinds of American hemp lose 40% when manufactured into twine. How many pounds of twine will a ton of such hemp make?

13. With some Italian hemp, the loss is but 20%. How many pounds of twine will 2 tons of such hemp make?

14. After scouring 600 lb. of wool, it weighed but 200 lb. What per cent was lost?

15. Another lot of wool weighing 1000 lb. weighed 450 lb. when scoured. What per cent was lost?

16. A man failing in business pays me 25% of what he owes me. How much do I lose if he owes me $1200? How much if he owes me $1800?

17. A man sold a house and lot that cost him $8500. He made 15% of the cost. How much did he get for them?

18. Four men bought a carload of coal weighing 65,700 lb. One man took 25% of it, another 27% of it, and another 21% of it, and the fourth man, the remainder. How many pounds did each man get?

19. If the total cost of the load of coal described in Problem 18 was $118, how much of it should each man pay?

83. DISCOUNT

When for any reason a deduction is made from a former price, or from the regular price, a **discount** is said to be given.

84. TRADE DISCOUNT

It is the custom among certain wholesale dealers, manufacturers, and publishers, to fix a price, called the **list price**, on their goods and then allow a certain deduction from this price to "the trade," *i.e.* to retail dealers handling their kinds of goods. This deduction, or *discount*, is usually a *per cent* of the *list price.*

Problems in Discount

1. Goods listed at $450 were sold to a dealer at a discount of 15%. Find the net price.

SOLUTION

First method:

$$\begin{array}{r l} \$450 & \\ \underline{0.15} & \\ 2250 & \\ \underline{450} & \\ \$67.50 & = \text{the discount} \end{array}$$

$$\begin{array}{r l} \$450.00 & \\ \underline{67.50} & \\ \$382.50 & = \text{net price} \end{array}$$

Second method:

$$\begin{array}{r l} \$450 & \\ \underline{0.85} & \\ 2250 & \\ \underline{3600} & \\ \$382.50 & = \text{net price} \end{array}$$

EXPLANATION. — In the first method, we found the discount which was 15%, or 0.15 of the list price. Then we subtracted this from the list price.

In the second method, we reasoned as follows: since a discount of 15% is deducted from the list price, the net price is only 85% (100% – 15%) of the list price. Which method do you prefer?

2. During a single month a merchant received goods amounting to $10,650. By paying cash he got an average discount of 4%. Find how much he saved by paying cash.

3. George's father bought a motor boat last October. It being late in the season, he was allowed a discount of 20%. The price earlier in the summer was $475. How much was saved? Find the net cost.

4. Whitney saved money to buy a canoe. He bought a $36 canoe, but the dealer gave him a discount of 10%. How much did it cost him?

5. Sydney bought a tent from Donald. Having used it one year, he gave Sydney a discount of 12½% from what it first cost. It cost Donald $24. How much did it cost Sydney?

6. James bought a new $85 pony cart. His father being a dealer in carts and buggies, James got a discount of 25%. How much did it cost him?

7. When coal was selling for $7.25 a ton, Mr. Harvey got a discount of 10% by having his coal drawn direct from the car to his cellar. How much did he save on 12 tons?

8. A publishing house sold Arithmetics, of which the regular or list price was 40¢, at a discount of 20%. What was the actual price paid for a book? What did 2640 books used in one city cost?

9. A clothing merchant placed a lot of boys' suits on special sale at a reduction of 25% from the regular price. Find the prices paid for suits whose regular prices were:

$12; $8.60; $10; $14.80; $9.20.

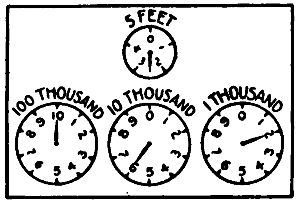
10. A milliner made a special sale of hats at a reduction of 30% from the regular price. Find the price paid for a hat formerly marked $6; a hat formerly marked $10; a hat formerly marked $14; a hat formerly marked $9.20.

11. After Christmas holidays, a merchant marked his toys, etc., to sell at a discount of 40%. Find how much you must pay for a pair of skates whose regular price was

\$1.75. Find how much you must pay for a sled whose regular price was \$2.95.

12. My electric light bill for one month amounted to \$1.98, less a discount of 10%, if paid within 10 days. What did the bill amount to if paid within 10 days?

13. On May 1, the reading on my gas meter was 65,000. On June 1, it was 68,500. The price is \$1.25 per 1000 cubic feet, less 10% discount if paid before June 10. How much is saved by paying before June 10?



14. A certain company advertises gas as follows:

100 to 2000 cu. ft., monthly, \$1.25 per 1000, net.
2000 to 5000 cu. ft., monthly, \$1.25 per 1000, less 10%.
5000 to 10,000 cu. ft., monthly, \$1.25 per 1000, less 20%.
10,000 to 20,000 cu. ft., monthly, \$1.25 per 1000, less 25%.

Find the net cost of the following:

1800 cu. ft.	6300 cu. ft.	9500 cu. ft.
2600 cu. ft.	4500 cu. ft.	12,000 cu. ft.

Drill Table

	List Price	Rate of Discount	Net Price		List Price	Rate of Discount	Net Price
1.	\$ 40	25%		9.	\$ 240	$8\frac{1}{3}$%	
2.	\$ 65	20%		10.	\$ 640	$12\frac{1}{2}$%	
3.	\$150	$33\frac{1}{3}$%		11.	\$ 750	20%	
4.	\$600	$16\frac{2}{3}$%		12.	\$ 800	40%	
5.	\$960	$16\frac{2}{3}$%		13.	\$ 900	$11\frac{1}{9}$%	
6.	\$175	10%		14.	\$ 420	$14\frac{2}{7}$%	
7.	\$450	$16\frac{2}{3}$%		15.	\$1200	$16\frac{2}{3}$%	
8.	\$960	$33\frac{1}{3}$%		16.	\$ 720	25%	

Problems in Finding the Rate of Discount

1. I received a discount of $294 from a bill of goods listed at $840. What was the rate of discount?

SOLUTION

$$\begin{array}{r} 0.35 = 35\,\% \\ 840\overline{)294.00} \\ \underline{252.00} \\ 42.00 \\ \underline{42.00} \end{array}$$

EXPLANATION. — Since it is required to find the rate per cent that one number is of the other, we first find the ratio of $294 to $840, which is $\frac{294}{840}$, and then express it as *hundredths*.

2. I bought goods that were listed at $1150 for $920. What rate of discount was this?

3. Mrs. Baker bought a $12 bed hammock, but as it was slightly damaged, the clerk sold it to her for $10. What was the rate of discount?

4. John's father bought him a suit at a special suit sale for $7.60. The regular price was $9.50. What rate of discount was given?

5. I was given a discount of $238 from a bill of goods listed by a wholesale house at $680. What rate of discount was I allowed?

6. At a fire sale, goods slightly soiled by smoke and water were sold as follows:

ARTICLES	BOYS' SUITS	MEN'S SUITS	SHIRTS	HATS
Regular price	$8.50	$25.00	$1.50	$3.00
Price sold	5.10	16.25	0.60	0.75

Find the rate of discount for each kind of goods.

7. I bought an automobile listed at \$1150 for \$865. What rate of discount did I get?

SOLUTION

\$1150
865
\$285 = Dis.

$$
\begin{array}{r}
0.248^{-} = 24.8^{-}\% \\
1150\overline{)285.00} \\
\underline{230.0} \\
55.00 \\
\underline{46.00} \\
9.000 \\
\underline{9.200}
\end{array}
$$

EXPLANATION. — Here the discount, \$285, did not contain the list price, \$1150, without a remainder when carried to hundredths. It was carried to the nearest thousandth. $0.248^{-} = 24.8^{-}\%$.

8–17. Pianos that had been used were offered for sale by Siegel, Cooper & Co., Oct. 17, 1910, as follows:

INSTRUMENT	ORIGINAL PRICE	SELLING PRICE
Decker & Son	\$375	\$110
J. & C. Fischer	400	125
Emerson	425	125
Krokauer	450	160
Chickering	600	165
Sterling	350	175
Steinway	550	185
Weber	500	240
Eberhardt player piano	650	385
Hasbrouck player piano	700	425

Find the rate of discount on each instrument.

18. Smith, Gray & Co., of New York, on Oct. 24, 1910, advertised the following:

\$40 suits and overcoats, \$27.50.
\$30 suits and overcoats, \$22.50.

Upon which lot is the largest per cent of discount given?

19–24. The Chicago House Wrecking Co. advertised rugs, etc., for sale Oct. 17, 1910, as follows:

Articles	Regular Price	Selling Price
Royal Wilton Rugs	$45.00	$21.35
Wilton Velvet Rugs	32.50	14.65
Axminster Rugs	32.50	15.95
Brussels Rugs	19.00	8.95
Oriental Rugs	155.00	62.50
Brussels Carpets	1.60	0.75

Find the rate of discount on each kind of article.

25–26. W. V. Snyder & Co., Newark, N.J., gave the following special prices on rugs:

$55 Seamless Wilton rugs, 9 × 12, $42.50.
$37.50 Royal Wilton rugs, 8.3 × 10.6, $28.50.

Find the rate of discount on each rug.

Problems in Billing Goods

1. Complete the following bill:

CHICAGO-KENOSHA HOSIERY CO.

MANUFACTURERS OF SEAMLESS AND FULL-FASHIONED HOSIERY

Sold to Messrs. J. R. Doe & Co., Decatur, Ill.

TERMS { 2 per cent 10 days. Net 30 days. July 15, 1911.

2½ doz.	Boys' Stockings @ $1.90				
1½ "	" " @ 2.00				
7½ "	" " @ 2.10				
4 "	" " @ 2.25				

2. The terms "2 per cent 10 days; net 30 days" means that if J. R. Doe & Co. choose to pay the bill within 10 days of the date, a discount of 2% will be allowed. But they need not pay the bill for 30 days. After 10 days they are not allowed a discount.

If Doe & Co. remit before July 25, what should be the amount of the remittance? What must they remit any time between July 25 and Aug. 15?

3. Bill from the same firm to A. L. Dickson & Son, Jan. 4/'11, the following: $3\frac{1}{2}$ doz. pr. hose at $3.25; $7\frac{1}{2}$ doz. pr. children's stockings at $1.80; $12\frac{1}{2}$ doz. pr. men's half hose at $2.50; $8\frac{1}{2}$ doz. pr. boy's stockings at $2.25. What amount will settle the bill within 10 days? Within 30 days?

Make out a bill for the following:

4. Butler Bros., Chicago, Ill., sold to E. M. Rose & Co., Jan. 3, 1911, the following: 240 yd. matting at $17\frac{1}{2}$ cents; 360 yd. matting at 18 cents; 8 rugs at $3; 2 matting rugs at $0.78; 5 couch covers at $3.25; 6 pr. curtains at $150; 12 pr. curtains at $2.75. Terms: 1% in 20 days; net 40 days.

What sum will settle the bill before Jan. 23? Feb. 8?

5. Sidney Shepherd and Co., Buffalo, N.Y., sold to H. M. Murphy & Son, March 13, 1910, the following:

$\frac{1}{4}$ gro. pails at $39.60; $\frac{1}{12}$ gro. steamers at $28.80; $\frac{1}{12}$ gro. steamers at $32.40; all less 10%. Also $1\frac{1}{2}$ doz. drip pans at $4.45 less 60%. Terms: 2% off 10 days; net 60 days.

What sum will settle the bill before March 23? What within 60 days?

6. Rollin & Rollin, importers, New York, sold to J. R. Wilson & Co., St. Paul, Minn., Oct. 20, 1910, the following:

84 boys' suits at $6.25,
20 doz. shirts at $7.00.
Terms: 2% 10 days, net 30 days.

Prepare the bill, and find what sum will settle it if paid before Oct. 30.

NOTE. — It will be found interesting and profitable for pupils to play *going into business*, some acting as wholesale and some as retail firms. Let the retail firms buy bills of goods of the wholesale firms, the latter making out complete bills in the preceding form. The retail firms work through and check their bills before paying them. (See pp. 331, 332.)

General Problems in Percentage

1. 70% of round steak is water. How much water in 8 lb. of steak?

2. If you sleep 8 hr. each day, what per cent of your time do you sleep?

3. If the coal in your coal bin is 6 ft. deep at the beginning of the winter and you use all but 1 ft. in depth, what per cent is left? What per cent have you used?

4. 1000 lb. of potatoes contain 180 lb. of starch. What per cent of potatoes is starch?

5. In 1000 lb. of rice there are 790 lb. of starch. What per cent of rice is starch?

6. Children studying soils find that 325 grams of a certain kind of soil contain 112 grams of water. What per cent of the soil is water?

7. They find that 325 grams of another kind of soil contain 95 grams of water. What per cent of this soil is water?

8. In the year ending June 30, 1909, there were 751,786 immigrants entered the United States. Of these 654,875 were from the countries of Europe. What per cent of the immigrants came from Europe?

9. The population of New York City in 1900 was 3,437,202, and in 1910 it was 4,766,883. What per cent of the population in 1900 was the increase during the ten years?

10. The population of Chicago in 1900 was 1,698,575, and in 1910 it was 2,185,283. What was the per cent of increase?

11. 78% of potatoes is water. How many pounds of water in a bushel of potatoes which weighs 60 lb.?

12. About $77\frac{1}{2}\%$ of a cabbage is water. How much water in a head of cabbage weighing 3 lb.?

13. Of grapes, 58% is water. How much water in a basket of grapes weighing 5 lb.?

14. The following table contains a record of the milk given by one of a farmer's cows in a year. Find the yield of butter fat for the year from this cow.

Month	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.
Lb. milk . .	720	750	710	680	650	620	540	450	420
% butter fat .	4.4	4.4	4.2	4	4.2	4	3.8	3.5	3

15. Some children studying the composition of foods found that 8 oz. of green butter beans weighed only 5.7 oz. when thoroughly dried in an oven. What per cent of green butter beans is water?

16. When the beans were burned, the ash weighed only 0.08 oz. What per cent of the beans is ash?

17. The children found that 12 oz. of fresh beets weighed only 3.6 oz. when thoroughly dried. What per cent of beets is water?

18. They found that 24 oz. of cucumbers weighed only 5 oz. when dried. What per cent of cucumbers is water?

19. 10 oz. of lettuce leaves weighed only 2 oz. when dried. What per cent of lettuce is water?

20. 24 oz. of sweet potatoes weighed only 11 oz. when dried. Find the per cent of water in sweet potatoes.

21. Apples weighing 18 oz. weighed only 7 oz. when dried. Find the per cent of water in apples.

22. A car will carry 1150 bu. of wheat. What per cent of it is yet to be filled when 987 bushels have been loaded?

23. A family wish to furnish a dining room and living room with Mission furniture. It will cost them $385. They can save 10% by waiting for the "June Sale." How much is the saving?

The Woman's Exchange

A woman's exchange is a place where women offer for sale foods that they have prepared.

1. A woman bought at the meat market 15 lb. of meat at 18¢ a pound. She roasted it, and then sold it at the exchange at 40¢ a pound. If the meat lost 30% of its weight in cooking, how much did she make for her work?

2. A woman paid 23¢ a pound for a ham weighing 12 lb. She cooked it, and it was retailed at the exchange at 45¢ a pound. Allowing 35% of the weight for loss in cooking and for bone, how much profit did she make on it?

3. A sirloin beef roast weighing $16\frac{1}{4}$ lb. cost 20¢ a pound. When cooked it was sold for 45¢ a pound. If 33% of its weight was lost in cooking, how much profit was made on it?

4. Leg of lamb cost 18¢ a pound, and when cooked sold for 35¢ a pound. How much profit was made on a leg weighing 6 lb. if 36% of the weight was lost in cooking?

5. Veal roast weighing $7\frac{1}{4}$ lb. cost 22¢ a pound, and when cooked sold at 45¢ a pound. It lost 34% of its weight in cooking. How much was the profit on it?

6. A chicken weighing $3\frac{1}{2}$ lb. cost 20¢ a pound. It lost 40% of the weight in roasting, and was sold at 45¢ a pound. How much profit was made on it?

Problems in Food Values

1. Of chuck roast as purchased, 19.5% on the average is refuse. If I buy a chuck roast weighing 5 lb., how many pounds of it are edible?

2. On the average, 12.8% of sirloin steak is not edible. How much is the edible portion of a sirloin steak weighing 2 lb. 4 oz.?

3. Of the average rib roast, 16.8% is refuse. If you buy a rib roast weighing 5 lb. 8 oz., how many pounds of it are edible?

4. 26.5% of beef tongue is refuse. How much is there in the edible portion of a tongue weighing 3 lb.?

5. 17.4% of leg of lamb is not edible. In $4\frac{1}{2}$ lb. of leg of lamb how much is edible?

6. 23.5% of loin pork chops is waste. If you buy $1\frac{7}{8}$ lb. of pork chops, how much of it is edible?

7. The following table gives the per cent of the constituents of different kinds of animal foods:

Animal Foods	Water Per Cent	Protein Per Cent	Fat Per Cent	Ash Per Cent
Sirloin steak	61.9	18.6	18.5	1.0
Round steak	70.0	21.0	7.9	1.1
Beef tongue	70.8	19.0	9.2	1.0
Veal cutlets	70.7	20.5	7.7	1.1
Leg mutton	67.4	19.1	12.4	1.1
Ham (fresh)	53.9	16.4	28.9	0.8
Chicken (broilers)	74.8	21.6	2.5	1.1
Black bass	76.7	20.4	1.7	1.2

Find the quantity of each of the constituents in 8 lb. of sirloin steak.

Make other problems, using the facts of the table.

SIMPLE INTEREST

85. MEANING OF SIMPLE INTEREST

1. If I pay $60 a month for the *use* of another man's house, how much *rent* do I pay in 3 months? In 6 months? In 1 year?

2. If I pay a livery-stable keeper $1 an hour for the *use* of his horse, how much *horse-hire* do I pay for 3 hours at the same rate?

3. If I pay $5 an hour for the *use* of an automobile, how much do I pay for the use of it for 5 hours?

4. If I pay Mr. Brown $6 a year for the *use* of $100 belonging to him, how much must I pay for the use of it for 2 years?

> *Money paid for the use of money is called* **Interest.** *It is usually a per cent of the sum used.*

Thus, when one says money is worth 5 %, he means that the one *hiring* it, or borrowing it, pays 5% of the amount borrowed for 1 year's use of it.

5. How much must I pay for 1 year's use of $100 at 5%? How much at 6%? How much at 7%?

5% and 6% are very common rates of interest.

Exercises in Interest

1. What is one year's interest of $100 at 6%?

2. At 6%, what is the interest of $200 for 1 year? For 2 years?

3. At 6%, what is the interest of $300 for 2 years? For 3 years?

4. What is 1 year's interest of $300 at 5%? At 7%?

5. At 5%, what is the interest of $500 for 2 years? For 4 years?

6. At 5%, what is the interest of $400 for 1 year? For 6 months?

7. What is the interest of $600 for 6 months at 5%?

8. At 6%, what is a year's interest of $500? 3 years'?

9. At 5%, what is 3 years' interest of $80? 5 years'?

At 6%, what is the interest of:

10. $500 for 6 mo.?
11. $300 for 8 mo.?
12. $400 for $1\frac{1}{2}$ years?
13. $250 for 1 year?
14. $150 for $2\frac{1}{2}$ years?
15. $350 for 1 year?

At 5%, what is the interest of:

16. $800 for 1 yr. 6 mo.?
17. $200 for 2 yr. 6 mo.?
18. $300 for 2 yr. 6 mo.?
19. $600 for 2 yr. 4 mo.?
20. $400 for 2 yr. 3 mo.?
21. $800 for 1 yr. 4 mo.?
22. $700 for 2 years?
23. $90 for 2 yr. 4 mo.?

At 5%, what is the interest of:

24. $340 for 1 year? For 2 years? For 3 years?

25. $80 for 6 mo.? For 3 mo.? For 9 mo.?

26. $90 for 4 mo.? For 8 mo.? For 1 year 4 mo.?

27. $450 for 4 mo.? For 1 yr. 4 mo.? For 1 yr. 8 mo.?

28. $240 for 1 yr.? For 4 mo.? For 1 yr. 4 mo.?

29. $360 for 1 month? For 3 mo.?

At 4%, what is the interest of:

30. $500 for 1 year? For $1\frac{1}{2}$ years? $2\frac{1}{2}$ years?

31. $600 for 1 yr.? For 1 yr. 6 mo.? For 1 yr. 8 mo.?

32. \$ 800 for 6 mo.? For 1 yr. 3 mo.? For $\frac{5}{8}$ yr.?

33. \$ 200 for 2 yr.? For 2 yr. 6 mo.? For $\frac{1}{4}$ yr.?

34. I borrowed \$ 800 at 6 % on May 10, 1907, and kept it until May 10, 1910. How much interest did I pay?

35. I borrowed \$ 540 at 5 % on March 16, 1909, and kept it until June 16, 1910. How much interest did I pay?

36. I borrowed \$ 1250 at 4 % on July 1, 1908, and returned it Dec. 1, 1910. How much interest did I pay?

37. If you loaned \$ 450 at 6 % on Nov. 5, 1908, and it was paid back on Feb. 5, 1910, how much interest would it earn for you?

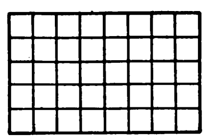
The money upon which the interest is paid is called the **Principal.** *The sum of the principal and interest is called the* **Amount.**

VIII. MENSURATION

86. THE AREA OF A RECTANGLE

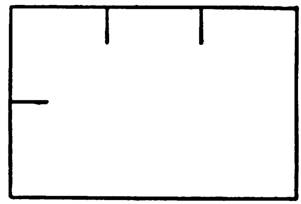
The area of any surface is the **number of units of measure** that it contains, such as square inches, square feet, or square miles.

The unit of measure may be of any size, but it is always a **square**.



How many squares are there in this rectangle? If each of these squares represents a square inch, what is the area of the rectangle? How many squares are in each row? How many such rows of squares are there? How can the whole number of squares in the rectangle be obtained from the number in each row and the number of rows?

1. If one side of a rectangle is 3 ft., and the side adjacent 2 ft., into how many square feet may it be divided? What is the area?



2. Two adjacent sides of a rectangle are sometimes called the **base** and the **altitude**.

If the length of the base is 6 inches, and of the altitude 4 inches, what is the area?

3. Find the area if the base is 20 ft. and the altitude 17 ft.

4. Complete: *The area of a rectangle is the product of* ——.

Problems in Finding Areas

1. A rectangular field is 40 rods long and 35 rods wide. What is its area? How many acres? How much is it worth at $80 per acre?

2. The sidewalk in front of a house is 5 ft. wide and 66 ft. long. How many square feet in it? How much did it cost at 27¢ a square foot?

3. Find the cost of linoleum for a kitchen which is 18 ft. long and 15 ft. wide, at $1.20 a square yard.

4. The area of a rectangle is 320 sq. in., and it is 16 in. wide. How long is it?

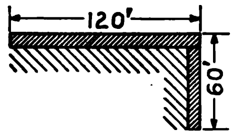
5. One side of a square is 8 yd. What is its area?

6. A bathroom is 6 ft. 8 in. by 10 ft. How many 1-inch square tiles will it take for the floor?

7. A boy in a class in woodwork uses a whitewood board 9 in. wide and 96 in. long. How many square inches in it? How many square feet? How much does it cost him at 9¢ a square foot?

8. A boy uses 6 whitewood boards, each 8 in. wide and 36 in. long. How many square inches in them? How many square feet? Find the cost at 12¢ a square foot.

9. I have a corner lot and wish to make a concrete walk 4 ft. wide across the front and side. The greatest lengths of the walks are 60′ and 120′ as shown in the figure. Find the cost at $1.35 per square yard.



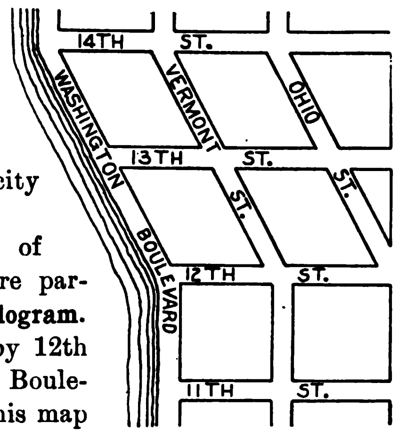
10. How many square feet in the cement floor of a garage 20′ × 26′? How much did it cost at 16½¢ a square foot?

87. THE AREA OF A PARALLELOGRAM

(Review page 113.)

Two straight lines that cannot meet, however far they may be extended, are called **parallel**. Thus, the top and bottom edges of this leaf are parallel. Point out the parallel lines in the room.

What are some other parallel lines of which you can think? What are some parallel lines in this city map?

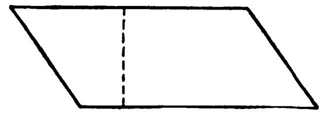


A figure of four sides, of which the opposite sides are parallel, is called a **parallelogram**. Thus, the block bounded by 12th St., 13th St., Washington Boulevard, and Vermont St., in this map is a parallelogram.

How many parallelograms are shown in the map?

1. Is a rectangle a parallelogram? Is a square a parallelogram?

2. Any one of the sides of a parallelogram may be called the **base**. What kind of angles does the dotted line in the figure make with the lower side?



The width of a parallelogram measured at right angles to the base is called the **altitude**.

3. Draw a parallelogram on paper, and cut it out. Draw an altitude of it. Cut the parallelogram into two parts by cutting along the altitude.

4. Now fit the two parts together so that they make a rectangle.

5. Compare the area of the rectangle with that of the parallelogram before it was divided. Compare their bases. Compare their altitudes.

6. How is the area of the rectangle found? Then how may the area of the parallelogram be found without cutting it?

Give the method of finding the area of any parallelogram.

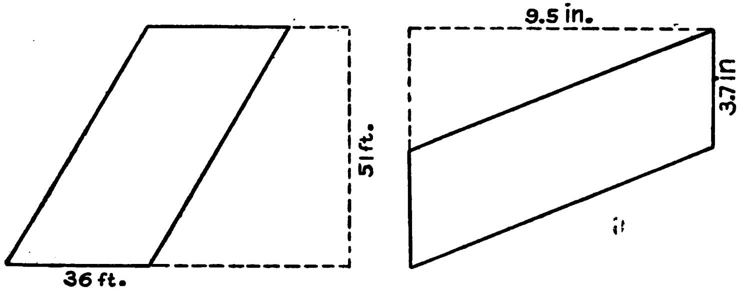
7. Cut out another parallelogram whose base is 4 in. and altitude 3 in. and make it into a rectangle. What is its area?

Problems

1. Find the areas of parallelograms with the following dimensions:

Altitude 12 in., base 16 in.;
Altitude 49 ft., base 28 ft.;
Altitude $6\frac{1}{3}$ yd., base 15 yd.;
Altitude 256 ft., base 1284 ft.

2. Find the area of each of the following parallelograms:



3. In the city map, p. 207, if the length of the block along 12th St., between Washington Boulevard and Vermont St., is 480 ft., and the width of the block from 12th St. to 13th St. is 426 ft., how many square feet in the block? How many square rods? How many acres?

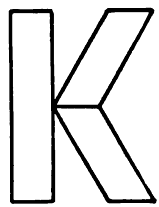
4. If the pavement of 13th St. is 54 ft. wide and the length of the block from Washington Boulevard to Vermont St. is 480 ft., how many square feet of pavement does it take for 13th St. between Washington Boulevard and Vermont St.?

5. If the distance along Vermont St. from 12th St. to 14th St. is 1020 ft. and the width of the street 54 ft., how many square feet of pavement in Vermont St. between 12th St. and 14th St.?

6. Strips are cut bias from a piece of velvet, as shown in the picture. Each strip is 15 in. long and 3 in. wide. How many square inches of velvet does it take for 12 of these bias pieces?

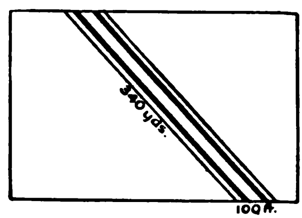
7. A band on a dress is made of six pieces of goods cut bias, each piece being 22 in. long and 2 in. wide. How many square inches of cloth does the band take?

8. A boy makes pennants to sell. The picture shows one made of blue felt, upon which is sewed the letter M in white felt. The letter is made of four pieces, each $1\frac{1}{2}$ in. wide. The height of the letter is 4 in., and each of the two middle pieces is 2 in. long. How many square inches of white felt does the letter take? How many square yards in 6 doz. of these letters?



9. The letter K on a pennant is made of three pieces of white felt, as in the picture. The rectangular piece is 6 in. long and $1\frac{1}{4}$ in. wide. Each of the other pieces is $3\frac{3}{4}$ in. long and $1\frac{1}{4}$ in. wide. How many square inches of felt in the letter? How many square yards in 10 doz. of these letters?

10. The shaded parallelogram in the figure is the land taken by a railroad built across a man's field. The strip of land taken is 100 ft. wide and 340 yd. long. How many acres of land does the road occupy? If the man was paid \$65 an acre for the land, by the railroad company, how much did he receive for it?



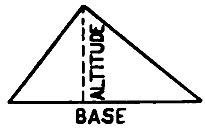
11. Find the area of each of the following parallelograms:

Base, $17\frac{1}{2}$ ft.; altitude, 14 ft.
Base, 24 rd.; altitude, $19\frac{7}{8}$ rd.
Base, $6\frac{3}{4}$ in.; altitude, $3\frac{5}{8}$ in.
Base, $7\frac{3}{8}$ yd.; altitude, 18 ft.
Base, 12 mi.; altitude, 376 rd.

88. THE AREA OF A TRIANGLE

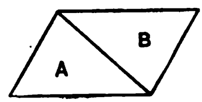
(Review page 114.)

A figure formed by three straight lines, as shown in the drawing, is a **triangle**. How many corners has a triangle? How many angles?



When one side of a triangle is called the **base**, the distance from the opposite corner, or **vertex**, to this side is called the **altitude**. In a right triangle, one of the sides is the altitude.

1. Draw a parallelogram. Draw a line joining two opposite corners, or vertices, thus dividing the parallelogram into two triangles, such as A and B.



2. Cut out the triangles into which your parallelogram is divided. By placing one triangle upon the other, show that they are equal. Then one triangle is what part of the parallelogram?

3. Compare the altitudes and bases of the triangle and the parallelogram.

4. How is the area of the parallelogram found? Then how is the area of the triangle found?

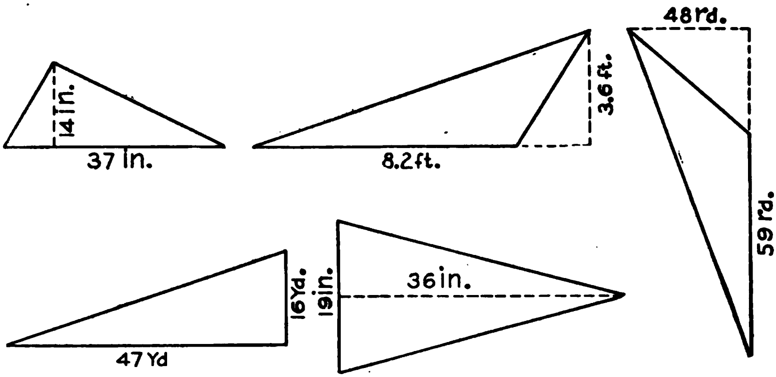
Problems

1. Find the area of a triangle whose base is 7 in. and altitude 6 in. Base 12 in. and altitude 9 in. Base 45 in. and altitude 37 in.

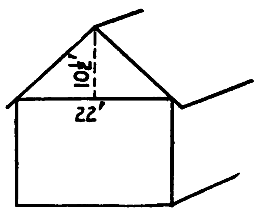
2. One of the short sides of a right triangle is 5 ft. and the other 3 ft. Find the area.

3. The area of a triangle is 56 sq. in., and its altitude 8 in. Find the base. If the area is 27 sq. yd. and base 6 yd., find the altitude.

4. Find the area of the following triangles:

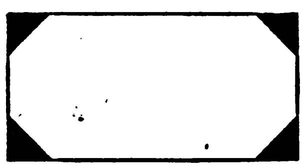


5. The gable of a house is 22 ft. wide and $10\frac{1}{2}$ ft. high. How many square feet of lumber does it take to side it?



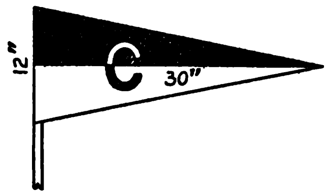
6. How many square feet of lumber does it take to board up the gable of a house if the gable is 30 ft. wide and 14 ft. high?

7. This figure shows a blotter. The corners are bound with triangular pieces of blue leather. Each piece of leather extends along the edge of the blotter 4 in. each way. How many square inches in the top surface of each piece of leather?



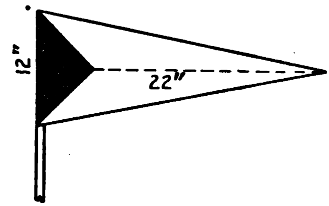
8. On each of the short sides of the leather corner pieces there is a strip 1 in. wide for folding and pasting. How many square inches of leather does it require for the blotters of a class of 24 pupils?

9. This pennant is made of a strip of blue and a strip of white felt. How many square inches of each color does it take?

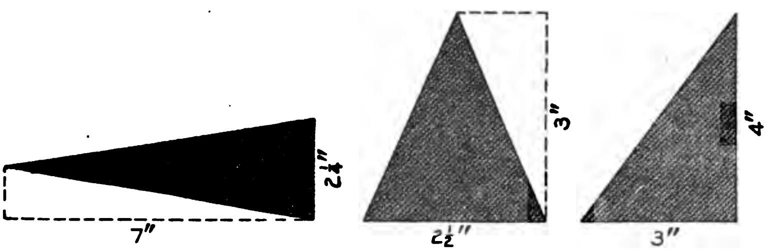


10. How many square yards of each kind of felt would it take to make 3 doz. pennants of this kind?

11. This pennant is made of white and blue felt. Find how many square inches of each kind of felt it takes. How many square yards of each kind of felt will it take to make 3 doz. of these pennants?



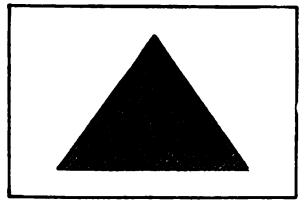
12. In shaping garments, small triangular pieces of cloth are used, called gussets. If two gussets like the one at the left are used in each garment, how many square inches of cloth does it take to make the gussets for 500 garments? Allowing $12\frac{1}{2}$ sq. in. for waste in cutting, how many yards of cloth 27 in. wide does it take?



13. Two gussets like the one in the middle are used in making a shirt. How many square inches of madras does it take to make the gussets for a lot of 1000 shirts? Allowing 48 sq. in. for waste in cutting, how many yards of madras 27 in. wide does it take?

14. How many square inches of cloth in 1000 gussets like the one at the right?

15. Triangular pieces of goods are mounted on cardboard for samples. If the base of each sample is 5 in. and the altitude $3\frac{1}{2}$ in., how many square inches are required for 1000 samples?

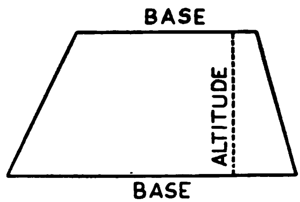


16. If samples of cloth are cut with the base 4 in. and altitude 3 in., how many square inches are required for 1000 samples?

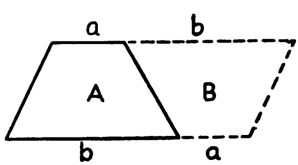
17. I have a yard of cloth 1 yard wide to be cut into samples that are right triangles with each of the short sides $1\frac{1}{2}$ in. There being no waste, how many samples will it make?

89. THE AREA OF A TRAPEZOID

A **trapezoid** has two parallel sides called the **upper base** and the **lower base**, or the **bases**, and two sides that are not parallel. The distance at right angles between the bases is the **altitude**.



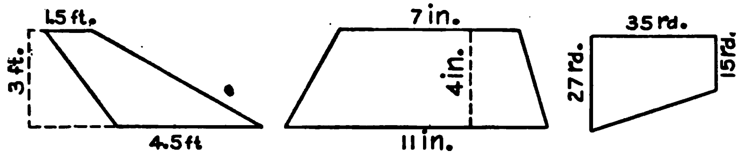
Cut from paper a trapezoid, as A in the figure. Cut out another from this as a pattern. Call it B. Place them as in the figure. What kind of a figure do A and B together form? Compare the altitude of the trapezoid A with that of the parallelogram now formed. Compare the bases of the trapezoid with the base of the parallelogram. Compare the areas of the two.



State the method of finding the area of any trapezoid.

Problems

1. Find the areas of the following trapezoids:



2. Find the area of a trapezoid whose altitude is 6 yd. and bases 12 yd. and 4 yd.

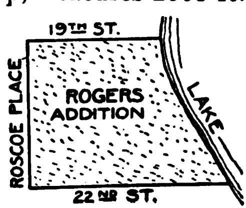
3. Find the area of a trapezoid whose parallel sides are 19.4 rd. and 28.6 rd., and the distance between them 12 rd.

4. By dividing a trapezoid into two triangles, show another way to find its area. Compare the bases of the triangles with the bases of the trapezoid. Compare their altitudes with the altitude of the trapezoid.

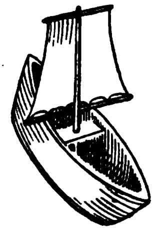
5. A farmer has a field in the form of a trapezoid whose altitude is 40 rd. and parallel sides 60 rd. and 94 rd. How many acres in it? How long does it take to plow it for sowing in wheat, if he has teams that plow 5 acres a day? How much wheat will it yield at 24 bu. an acre?

6. A farmer has a field of corn in the shape of a trapezoid whose altitude is 26 rd. and bases 45 rd. and 57 rd. How many bushels in the crop, if it yields 60 bu. to the acre? How much is it worth at 54¢ a bushel?

7. Rogers Addition, shown on this map, measures 1668 ft. on 19th St.; 2296 ft. on 22nd St.; and 1618 ft. on Roscoe Place. Mr. Rogers paid $650 an acre for the land, and laid it off into 192 town lots, which sold at an average of $210 each. How much did he make?

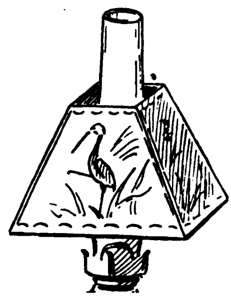


8. A boy made a boat with a sail, as in the picture. The sail was 8 in. wide at the top, 12 in. wide at the bottom, and 12 in. high. How many square inches of canvas in the sail? If he sails it on a pond where the pressure of the wind against the sail is 0.45 lb. per square foot, what is the pressure that moves the boat?



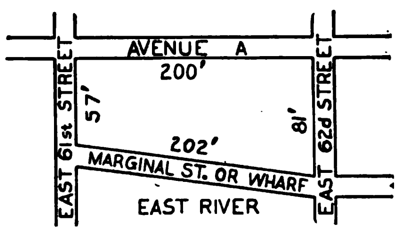
9. If the sail like this on a big ship is 28 ft. wide at the top, 38 ft. wide at the bottom, and 30 ft. high, how many square feet of surface does it have? When the wind is blowing at 20 mi. an hour, the pressure of the wind on the sail is about 2 lb. per square foot. What is the total pressure against the sail?

10. A lamp shade has 4 sides, each a trapezoid with altitude 8 in. and bases 10 in. and 4 in. It is made from one piece of material, by cutting, bending, and pasting it. If it is cut from a sheet of material 16 in. wide and 25 in. long, how much material is wasted?



THIS CITY BLOCK FOR SALE

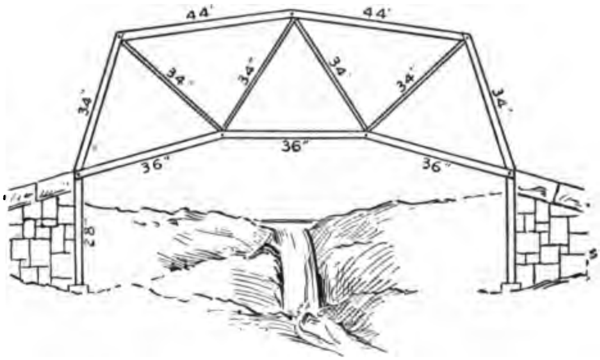
APPLY TO JOS. P. DAY.



11. How much is it worth at $3.25 a square foot?

Measuring Lumber: Sydney Builds a Bridge

1. Sydney builds a "truss" bridge across a brook, such as railroad and highway engineers usually build. The trusses are all triangles, for any other shape would collapse and not sustain the weight. The measures are the distances between bolts. Each piece is 3 in. longer than the measures given here. How many feet in the 13 pieces for one side?



2. How many pieces in both sides are each 44 in. long? How many are each 34 in.? How many are each 36 in.?

3. To avoid waste he cuts the 44-in. (47 in. in all) pieces from a board 8 ft. long and the rest from boards 10 ft. long.

4. How many feet in the total length of the pieces used? Lumber is sold by the "board foot."

> *A* **board foot** *is a piece* 1 *foot long,* 1 *foot wide, and* 1 *inch thick.*

If a board is less than an inch thick, a *square foot* is still called a *board foot*, but if it is more than an inch thick, a *square foot* will have as many board feet as there are inches

in the thickness. Thus, a piece 1 ft. long, 1 ft. wide, and 2 in. thick would contain 2 *board feet.*

5. How many board feet in a piece 1 ft. long, $\frac{1}{2}$ ft. wide, and 2 in. thick?

6. The pieces for the trusses were made out of lumber 6 in. wide and 2 in. thick. The 4 28-in. supports shown in the end view took another piece of the same kind 10 ft. long. How many board feet in all?

7. Find the cost of this lumber at $4\frac{1}{2}$¢ a board foot.

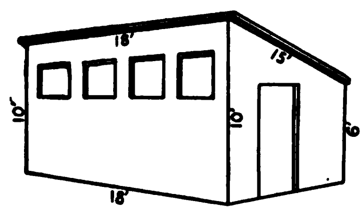
8. The market price of lumber is usually a price per 1000 feet (M). What would be the market price of this?

9. The floor of the bridge is 3 ft. wide and 9 ft. long. The 2 boards upon which the 4 28-in. supports rest are each 6 in. wide and 5 ft. long. For this Sydney buys a board 10 ft. long. The other boards are each 1 ft. wide. How many pieces will it take? You can get this lumber in pieces 8 ft., 10 ft., 12 ft., or 14 ft. in length. What length is best to buy? How many boards 12 ft. long will it take?

10. How much does the flooring cost at 4¢ a board foot?

Problems in Measurement

1. From the dimensions given in the picture find the number of square feet in the siding of this hen house. Do not deduct for the windows. If the lumber is 1 inch thick or less, how many *board feet* is this?



2. The roof is 15 ft. by 18 ft., including the projections. If it takes 9 shingles to cover a square foot, how many shingles will it require?

3. A double row of shingles is usually used for the first row. How many extra shingles are required for this?

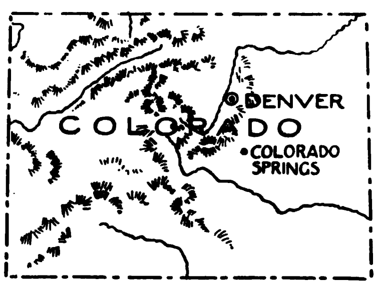
4. Shingles are sold in bunches of 250 each. How many bunches are required? If the shingles are $5 per 1000, how much is that per bunch? How much will all the shingles cost?

5. If the siding costs $25 per 1000 board feet, find the cost of the siding.

6. The house also required 10 rafters 2″ by 4″ by 15′ long; 2 pieces 4″ by 6″ by 12′ long; 2 more of the same kind each 8′ long; 7 pieces 2″ by 4″ by 12′. Find the cost of all this at $28 per 1000′.

7. Suppose the paint cost $4.50, the glass $1.60 for each of the 4 windows, and the hinges and nails 75 cents. Find the total cost of the hen house.

Many of our states are rectangles or nearly so. Find the area of the following, (1) in square miles; (2) in acres. (There are 640 acres in a square mile.)



NOTE. — These measurements are close approximates.

8. Colorado is almost a perfect rectangle 380 miles long and 275 miles wide.

9. Wyoming is 360 miles long and 270 miles wide.

10. The west boundary of Utah is 345 miles, the Colorado boundary 275 miles, the south boundary 270 miles, and the Idaho boundary is 160 miles.

SUGGESTION. — The area is either the sum of two rectangles or the difference between two rectangles. Find it both ways.

11. Oregon is equivalent to a rectangle 205 miles wide and 340 miles long.

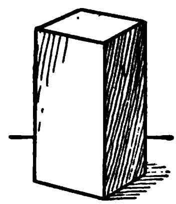
12. South Dakota is equivalent to a rectangle 205 miles by 390 miles.

13. The dimensions of North Dakota are 215 miles and 330 miles.

14. Nevada is equivalent to a rectangle whose dimensions are 330 miles and 215 miles, and a triangle whose dimensions are 330 miles and 255 miles.

90. AREA AND VOLUME OF RIGHT PRISMS

The figure is that of a **right prism.** The surfaces are called **faces** of it, and where they meet, the **edges.** The face upon which this prism appears to stand and the opposite face are the **bases.** The distance between the bases is the **altitude.** The base of a prism may have three or more sides.



How many sides has the base of this prism?

Exercises

1. The base of a right prism is a square whose side is 4 in., and the altitude is 6 in. How many square inches in its whole surface?

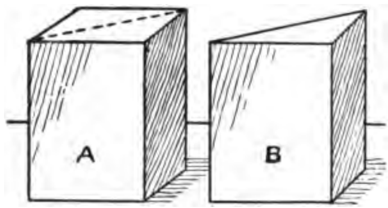
2. How many cubic inches in its volume? Could the prism be built of 1-inch cubes? How many in a layer? How many layers?

3. The base of a right prism is a rectangle 5 in. wide and 9 in. long, and its altitude is 8 in. Find the area of its whole surface.

4. Find the number of cubic inches in its volume.

5. A block of ice is 12 in. square and 22 in. long. Find its entire surface, and its volume.

6. The prism A in this figure is divided by a plane through two opposite edges into two prisms like B, with triangles as bases. Compare the volumes of A and B. Compare their altitudes. Compare their bases. Give a rule for finding the volume of a right prism whose bases are triangles.



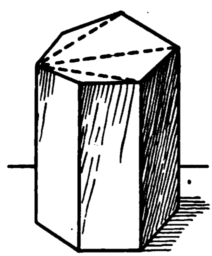
7. If you should construct two prisms of clay like A and B in Problem 6, and weigh them, what relation would you find between their weights?

8. If convenient, let the pupil make clay prisms and compare their volumes by comparing their weights in this way.

9. The altitude of a right prism is 16 in., and the bases are right triangles whose short sides are 9 in. and 14 in. Find the volume.

10. The altitude of a right prism is 24 in., and each base is a triangle whose altitude is 8 in. and base 12 in. Find the volume of the prism.

11. Any right prism may be divided into prisms with triangles for bases, by planes through opposite edges. Into how many prisms with triangular bases is this one divided? What is the sum of their volumes? How is the volume of each found?

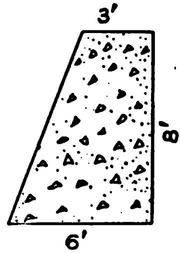


Compare the sum of their bases with the base of the given prism. Compare their altitudes with the altitude of the given prism. Then how may the volume of a right prism with any kind of base be found?

12. Find the volume of a right prism whose base is 25 sq. ft. and altitude 32 ft.

13. A stone pier is 14 ft. high, and has a triangular base whose area is 28 sq. ft. How many cubic feet of stone in it?

14. The figure shows the cross section or end view of a concrete retaining wall that is 50 ft. long. How many cubic feet of concrete in the wall? How many cubic yards?



15. A masonry dam across a river is 82 ft. long, 14 ft. high, 4 ft. wide at the top, and 8 ft. wide at the bottom. How many cubic feet of masonry does it contain? How many cubic yards? If the stone weighs 170 lb. per cubic foot, what is the weight of the whole dam?

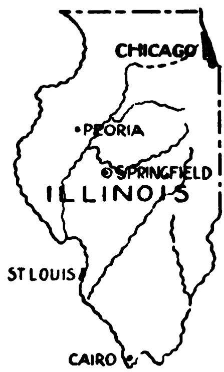
16. In building a railroad, a cut 640 ft. long is made. The cut is 12 ft. deep, 14 ft. wide at the bottom, and 30 ft. wide at the top. How many cubic yards of earth are removed in its construction?

17. An irrigation ditch is 10 ft. deep, 10 ft. wide at the bottom, and 20 ft. wide at the top. If the water flows at a rate of 1 ft. a second, how many cubic feet of water will the ditch deliver in a minute?

91. DRAWING TO SCALE

A **plan** or **map** is a drawing of the same shape as the object or region which it represents. The distance on any part of a plan or map represents a fixed number of miles or other units of measure. If, for example, in a plan or map, 100 mi. is represented by 1 in., it is said to be drawn "to the scale of 100 mi. to 1 in."

Problems



1. In this map of Illinois, a distance of 200 mi. is represented by an inch. That is, the map is constructed "to the scale of 200 mi. to the inch."

The map is $1\frac{5}{8}$ in. long. What is the extreme length of the state?

2. The greatest width of the map is 1.05 in. How many miles wide is the state?

3. Measure on the map the distance from Chicago to Springfield. How many miles from Chicago to Springfield?

4. Find, similarly, the number of miles from Chicago to Peoria. From Chicago to St. Louis, Mo. From Chicago to Cairo.

5. From Chicago to Rock Island is approximately 170 mi. By what distance should it be represented on this map?

6. From Chicago to Danville, Ill., is approximately 135 mi. By what distance should this be represented on this map?

7. On a map drawn to the scale of 200 mi. to the inch, from Chicago to New York, in a straight line, is 3.4 in. How many miles from Chicago to New York?

8. On the same map, from Chicago to New Orleans is 4.1 in. How many miles from Chicago to New Orleans?

9. On a map drawn 240 mi. to the inch the distance from Chicago to Denver is $3\frac{9}{16}$ in. How far does a wireless message between these cities travel?

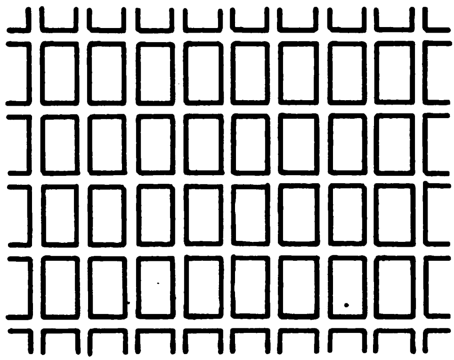
10. Draw the ground plan of a house, on a scale in which 4 feet are represented by an inch.

11. Measure the dimensions of the schoolroom, and draw a plan of it to a scale of 20 ft. to the inch.

12. Let each pupil draw to a convenient scale the floor plan of his home.

13. In the map of a city in which there are 10 blocks to the mile, how large should a block be represented, if the scale is 1000 ft. to the inch?

14. This figure is a map of that part of Chicago lying between Chicago Ave. on the north, Michigan St. on the south, State St. on the east, and Wells St. on the west. The scale is 2640 ft. to the inch. How many blocks to the mile is it north and south? How many blocks east and west?



15. Draw to scale a map of the block upon which your school is situated, and the blocks that adjoin it.

16. Let each pupil draw to scale the plans of one article made in the wood shop, or that he expects to make.

17. On a map drawn to the scale of 110 mi. to the inch, the state of Colorado is represented by approximately a rectangle 3.4 in. long east and west and 2.5 wide north and south. How many square miles in the state?

18. On the same map, the area of Wyoming is represented by approximately a rectangle 3.2 in. long and 2.5 in. wide. How many square inches does it contain?

19. On this same map, the state of Nevada is represented by approximately a trapezoid whose altitude is 2.88 in., and bases 2 in. and 4.16 in. How many square miles does it contain?

20. Tennessee is approximately a trapezoid. On a map, measure its width north and south, and its length along the north and the south borders. Then compute its area in square miles using the scale of the map.

21. Pennsylvania is approximately a rectangle. Arkansas and Missouri are approximately trapezoids. By consulting a map, find their areas.

22. The area of an irregular region may be found approximately by dividing it into parts in the forms of triangles, parallelograms, or trapezoids, and finding the areas of the parts separately and adding.

Thus, this map of Indiana is drawn to the scale of 110 mi. to the inch. It may be divided into approximately a rectangle and two triangles. Find its area.

23. Find the areas of other regions by using your geography in this way.

24. By consulting a map, it is seen that the Mississippi River and its tributaries drain an area approximately a trapezoid. By using the scale to which the map is drawn, compute the area drained by these rivers.

25. The approximate area of the United States occupied by the Appalachian Mountains is that of the parallelogram in the map. Find this area in square miles.

NOTE. — Drill on the interpretation of geographical maps in the school, of architects' drawings that may be obtained, or of plans for the construction of articles in woodwork, may be supplied to supplement these exercises. There are two important kinds of problems in drawing to a scale: finding the actual distances or areas that are represented to a scale in the drawing, and drawing to a given scale a plan or a map from given dimensions.

Some Common Relations in Measurements

1 barrel	**= 31.5 gal. = $4\frac{1}{7}$ cu. ft.**
1 gal.	**= 231 cu. in.**
1 cu. ft.	**= 7.5 gal. or $\frac{7}{29}$ bbl.**
1 bushel	**= 2150.42 cu. in. or about $1\frac{1}{4}$ cu. ft.**
1 ton of coal	**= 35 cu. ft. (approximately)**
1 ton of hay	**= 500 cu. ft. (approximately)**

1. How many gallons in a tank 6 ft. long, $2\frac{1}{2}$ ft. wide, and 2 ft. deep?

2. How many tons of coal can be put into a bin 12 ft. long, 8 ft. wide, and 6 ft. deep?

3. How many tons of hay in a mow 36 ft. long, 30 ft. wide, and 18 ft. deep?

4. How many barrels will a watering tank 12 ft. long, 4 ft. wide, and $2\frac{1}{2}$ ft. deep contain?

5. How many barrels will fill a swimming pool 48 ft. long, 16 ft. wide, and 5 ft. deep?

6. How much coal can I put into a bin 14 ft. long, 7 ft. wide, and 5 ft. deep?

7. A box 54 inches long, 44 inches wide, and 38 inches deep will contain how many bushels?

8. How much hay in a mow 50 ft. long, 40 ft. wide, and 20 ft. deep?

9. How many bushels of oats can I put into a bin 18 ft. long, 14 ft. wide, and 8 ft. deep?

SUGGESTION. — When the dimensions are given in feet, use 1 bu. = $1\frac{1}{4}$ cu. ft., from which we get 1 cu. ft. = $\frac{4}{5}$ bu.

10. How many bushels will a bin 8 ft. by 6 ft. by 4 ft. contain?

Miscellaneous Problems

1. Make a statement for the following bill of goods: $1\frac{1}{2}$ doz. eggs at 27¢ a dozen; 1 lb. of butter at 32¢ a pound; 1 bag of salt at 5¢; $2\frac{3}{4}$ lb. of prunes at $0.18 per pound; 1 one-pound box of cocoa at $0.40; $\frac{1}{2}$ bu. of potatoes at $0.25 per peck; 1 25-lb. sack of flour at $3\frac{3}{5}$¢ a pound.

2. How much would you pay the grocer on the above bill if he gave you 2 per cent off for cash?

3. What is the cost of a pound cake made with 10 eggs at 28¢ a dozen; 1 lb. of flour at 4¢; 2 lb. of sugar at 6¢; 1 lb. of butter at 30¢ per pound; and $\frac{1}{2}$ pt. of milk at 5¢ a pint?

4. What would be the cost to make double the following recipe for a fruit cake: 2 lb. of flour at $4\frac{1}{2}$¢ per lb.; 1 lb. of

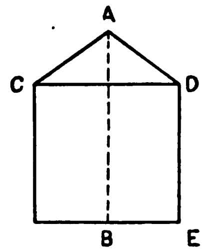
butter at 27¢ per lb.; 1 lb. of sugar at $6\frac{1}{2}$¢ per lb.; 2 lb. of currants at $12\frac{1}{2}$¢ per lb.; $\frac{3}{4}$ lb. of blanched almonds at 50¢ per lb.; 1 lb. citron at 35¢ per lb.; 12 eggs at 28¢ per doz.; and $\frac{1}{2}$ pt. of molasses at 10¢ per pt.?

5. What length of a board 18″ wide will make 2 sq. ft.?

6. What length of a board 16″ wide will make 5 sq. ft.?

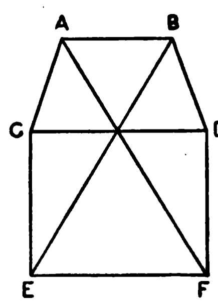
7. If 1 pt. of paint is enough for 6 sq. ft. of surface, how many pints will be required for 7 open flower boxes 45 inches long, 7 inches wide, and 5 inches deep?

8. How many square feet of lumber are there in an open flower box whose sides measure 52 in. long at the top and 46 in. long at the bottom, and whose ends measure $6\frac{1}{4}$ in. wide at the top and 5 in. wide at the bottom, and which is $5\frac{1}{2}$ in. deep?



9. A sheet of leatheret measures 22″ × 28″. How many sheets will supply a class of 42 pupils, for corners of blotter pads, if the corners are cut in the form of a right triangle whose base and altitude measure each 5″?

10. What per cent of the leatheret is waste?



11. A kite measures 14″ from *A* to *B*. *CD* and *DE* are each 10″. If one inch is allowed on each side for lapping, how many square inches of tissue paper are used to cover the kite?

12. A kite in the form of an irregular hexagon has an altitude of 30″. *AB* equals 14″. *CD* and *DF* equal 22″. Allowing 1 in. on each side for lapping, how many square inches of tissue paper are used?

13. From a rectangular piece of manila paper to be made into an envelope, $14\frac{1}{2}'' \times 24''$, $\frac{1}{4}$ sq. ft. is cut away. What per cent remains?

14. A boy lost $\frac{1}{8}$ of his kite string. He then added $24\frac{5}{8}$ ft. The string was then its original length. What was the length of the string?

15. How many coats can be made from $52\frac{1}{2}$ yd. of cloth if one coat requires 3.7 yd.?

16. What part of a sheet of strawboard does a boy use for a book cover $7\frac{1}{2}'' \times 5\frac{3}{4}''$, cut from a sheet $22\frac{1}{2}'' \times 28\frac{5}{8}''$?

17. Compare the surface of a window box $32'' \times 5'' \times 7\frac{1}{2}''$ with that of another window box $28'' \times 6'' \times 5\frac{1}{2}''$.

18. $\frac{1}{20}$ of a hide, containing 16 sq. ft., worth $0.40 per square foot, is waste. Find the cost of what is used.

Cream of Tomato Soup.

$\frac{1}{2}$ *can of tomatoes*	$\frac{1}{8}$ *teaspoonful pepper*
1 *qt. milk*	$\frac{1}{4}$ *cup flour* (1 *oz.*)
1 *slice onion*	$\frac{1}{4}$ *cup butter* (2 *oz.*)
1 *teaspoonful salt*	$\frac{1}{4}$ *teaspoonful soda*

19. The above recipe is to be used for Cream of Tomato Soup in a school lunch room. This amount will serve six. Ninety-six are to be served with the soup. Make out the amount of each ingredient that will have to be used. Give the amount of milk in gallons, of butter in pounds, and of flour in pounds.

20. If milk is worth $0.32 a gallon, butter $0.35 a pound, flour $0.03½ a pound, and tomatoes $0.12 a can, what will be the cost of the soup to serve ninety-six students, allowing $0.02½ for salt, pepper, soda, and onion? What will be the cost per student?

21. If $\frac{2}{5}$ of the cost of the material is added for making and serving, what will be the cost per plate? If the person in charge of the lunch counter makes a profit of $\frac{1}{8}$, what must be charged for each plate of soup?

22. If milk sells for $0.05 a pint bottle, $0.08 a quart, and $0.30 a gallon, how much would be paid for 64 qt. if bought by the pint?

23. How much money and what per cent would be saved if bought by the quart instead of by the pint?

24. If it were bought by the gallon instead of by the quart, how much is saved?

25. Mr. Brown, a grocer, sells tomatoes at $1.60 a bushel. Mr. Smith, another grocer, sells the same grade of tomatoes at $0.45 a peck. Which would be the better place at which to buy? If one were buying 2 bushels, how much money is saved by buying at the cheaper place?

26. W. V. Synder & Co., Newark, N.J., gave the following special prices on rugs:

$55 Seamless Wilton rugs,	9 × 12,	$42.50.
$40 Royal Wilton rugs,	9 × 12,	$31.50.
$37.50 Royal Wilton rugs,	8.3 × 10.6,	$28.50.
$18 Tapestry Brussels rugs,	9 × 12,	$15.

Find the rate of discount on each rug.

27. A room is 18 ft. wide, 21 ft. long, and 9 ft. high. How much does it cost to plaster it at 40¢ a square yard? Allow $\frac{1}{6}$ of the surface of walls for doors, windows, mantel, and baseboards.

28. A box 3 feet wide and $4\frac{1}{3}$ feet long contains 39 cubic feet. How deep is it?

29. An 8-inch cube is what part of a cubic foot?

ADVANCED ARITHMETIC

PART ONE: SEVENTH YEAR

I. DENOMINATE NUMBERS

NOTE. — The tables of denominate numbers have been given incompletely throughout the preceding grade work, as use of them demanded. They are now given completely and consecutively as a final treatment of the subject.

1. MEASUREMENTS

Distance, weight, time, liquids, etc., are measured by certain standard units of measure, with most of which we are now familiar through their use.

Exercises

1. To measure your height, what units of measure would you use?

2. In terms of what unit of measure would you express your weight?

3. By what unit of measure is coal bought and sold? Milk?

4. What is meant by measuring a thing?

5. Name all the units of measure with which you are familiar.

2. DENOMINATE NUMBERS

Any number of the standard units of measure, such as 4 feet, 20 pounds, 8 gallons, is called a **denominate** number.

A denominate number expressed in terms of two or more units of measure, such as 8 bu. 2 pk. 3 qt., is called a **compound denominate** number.

3. LINEAR MEASURE

The measures used in measuring length or distance are called linear measures.

Exercises

1. For convenience, in measuring short and long distances, small and large units of length measure are employed. Name all of the units of length in common use.

2. These units bear definite relations to each other, expressed by the *table of linear measure.* If you do not know it already, learn thoroughly the following:

Table of Linear Measure

12 inches (in.)	= 1 foot (ft.)
3 feet	= 1 yard (yd.)
$5\frac{1}{2}$ yards	= 1 rod (rd.)
320 rods	= 1 mile (mi.)
5280 feet	= 1 mile

3. How many inches in a yard?

4. How many feet in a rod? $5\frac{1}{2} \times 3$ ft. = —— ft.

5. How many yards in a mile? $320 \times 5\frac{1}{2}$ yd. = —— yd.

6. Find, by measuring, the length and the width of the schoolroom, in feet. In yards.

7. Is the room more or less than 3 rd. long?

8. Estimate the length of some object. Then measure it and see how nearly accurate your judgment is.

9. Estimate the length of a rod on the floor, and then measure the distance marked.

10. Estimate the distance of 10 rods out of doors, and then measure it.

11. What building is about a mile from your school?

12. How far do you live from school?

NOTE. — By measuring off a stout cord a rod long, or 50 ft. long, or any other desired length, a convenient means is secured for making long measurements out of doors. An expensive tape line is not necessary.

Pupils should make many estimates and measurements of short and long distances, as suggested in the preceding problems, to develop accuracy in judgment of distances.

Reduction in Linear Measure

Changing any number of units of one denomination to units of another denomination is called **reduction**. Changing a given unit to a smaller unit is called *reduction to a lower denomination*, as changing from feet to inches. Changing a given unit to a larger unit is called *reduction to a higher denomination*, as from feet to yards.

Exercises in Reduction to Lower Denominations

Reduce:

1. 6 ft. to inches; $8\frac{1}{2}$ ft. to inches; $10\frac{3}{4}$ ft. to inches.
2. 3 yd. to inches; 4 yd. to inches; $5\frac{1}{2}$ yd. to inches.
3. 2 rd. to yards; to feet.
4. 10 mi. to rods; to yards. 5. $8\frac{1}{2}$ mi. to feet.
6. State the method of reducing any number of units of one denomination to units of a smaller denomination.

7. Sweet peas are planted in a row 4 in. apart. How many seeds can be planted in a row 5 yd. 2 ft. 8 in. long?

SOLUTION:

5 yd. = 5 × 3 ft. = 15 ft.
15 ft. + 2 ft. = 17 ft.
17 ft. = 17 × 12 in. = 204 in.
204 in. + 8 in. = 212 in.
212 in. ÷ 4 in. = 53.
Hence, there are 53 seeds.

Reduce:

8. 2 ft. 5. in. to inches.

9. 2 yd. 2 ft. to feet.

10. 1 mi. 248 ft. to feet.

11. 2 mi. 18 rd. to rods.

Exercises in Reduction to Higher Denominations

Reduce:

1. 48 in. to feet; 6 in. to feet.

2. 27 ft. to yards; 45 ft. to yards.

3. 144 in. to yards; 288 in. to yards.

4. 33 ft. to rods; 132 ft. to rods.

5. 10,560 ft. to miles; 23,760 ft. to miles.

6. State the method of reducing a number of any denomination to a number of a higher denomination.

7. A flower bed is 195 in. long. How many yards, feet, and inches is this?

SOLUTION:

195 ÷ 12 = 16 with remainder 3.
Hence 195 in. = 16 ft. 3 in.
16 ÷ 3 = 5 with remainder 1.
Hence 16 ft. = 5 yd. 1 ft.
Hence 195 in. = 5 yd. 1 ft. 3 in.

Reduce:

8. 182 in. to yards, feet, and inches.

9. 64 ft. to rods, yards, and feet.

10. 1000 rd. to miles and rods.

11. Express 21 in. as feet, decimally to hundredths.

12. Express 20 ft. as yards, decimally to hundredths.

13. Express 12,000 ft. as miles, decimally to hundredths.

Applications of Linear Measure

1. A class of pupils wished to divide 20 yd. of ribbon into 48 equal parts for class colors. How many inches long must each piece be cut?

SOLUTION: 20 yd. = 20 × 36 in. = 720 in.
$\frac{1}{48}$ of 720 in. = 15 in.
Hence, each piece was 15 in. long.

2. How many badges, each containing 10 in., can be made from 15 yd. of ribbon?

3. How many books can be bound with 45 yd. of tape, if each book requires 9 in. of tape?

4. How many bolts, each 5 in. long, can be cut from a bar of iron 10 ft. long?

5. 60 ft. of a certain kind of wire weighs 1 lb. What is the weight of a mile of it?

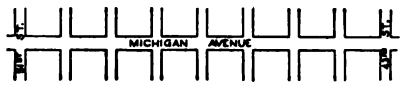
6. Find the cost of constructing a ditch a half mile long at $2.50 a rod.

7. How many feet of wire does a man need to build a fence 6 wires high around a lot 20 yd. wide and 28 yd. long?

8. The rim of the driving wheel on a locomotive is 18 ft. around. How many revolutions does the driver make in going a mile?

9. A tight board fence 120 ft. long is to be made of 9-inch boards, nailed upright. How many boards will it require?

10. In a certain city one block is 440 ft. long. How many blocks are there to the mile?



11. From 43d Street to 51st Street along Michigan Avenue, Chicago, is one mile. If the cross streets are 60 ft. wide, how many feet in one block?

12. In a garden, a strip 4 yd. wide and 7 yd. long is sowed in beets. The rows are 18 in. apart, and run the long way, and the plants are 12 in. apart in the rows. Allowing 9 in. for margins on all four sides, how many rows will there be? How many plants in each row? How many plants in all?

13. Geraniums are to be set 8 in. apart, in a row 6 yd. 1 ft. 8 in. long. The end plants are to be placed 4 in. from the ends. How many plants are required?

4. SQUARE MEASURE

1. Draw upon the blackboard a square foot. Draw lines dividing the square foot into square inches. How many square inches in the square foot?

2. Similarly, by drawing a figure, show how many square feet there are in a square yard.

3. Learn thoroughly the following:

Table of Square Measure

144 square inches (sq. in.)	=	1 square foot (sq. ft.)
9 square feet	=	1 square yard (sq. yd.)
$30\frac{1}{4}$ square yards	=	1 square rod (sq. rd.)
160 square rods	=	1 acre (A.)
640 acres	=	1 square mile (sq. mi.)

4. How many square inches in a square yard?

5. How many square feet in a square rod?

6. A strip of ground 16 rods long and 10 rods wide contains an acre. Why?

7. Give other dimensions of a rectangular piece of ground that would contain an acre.

8. Measure off an acre of ground near the school, and mark the four corners of it.

9. How many square feet in an acre? $160 \times 30\frac{1}{4} \times 9$ sq. ft. = —— sq. ft.

Reduction in Square Measure

1. How many square inches in 5 sq. ft.?

2. How many square feet in 8 sq. yd.?

3. How many square inches in 4 sq. yd.? $4 \times 9 \times 144$ sq. in. = —— sq. in.

4. How many acres in 6 sq. mi.? In 20 sq. mi.? In 36 sq. mi.?

5. How many square rods in 10 sq. mi.?

6. Reduce 4 acres to square rods. To square feet.

7. Reduce 2 sq. yd. 7 sq. ft. 80 sq. in. to square inches.

SOLUTION:

2×9 sq. ft. = 18 sq. ft.
18 sq. ft. + 7 sq. ft. = 25 sq. ft.
25×144 sq. in. = 3600 sq. in.
3600 sq. in. + 80 sq. in. = 3680 sq. in.

Reduce:

8. 5 sq. ft. 30 sq. in. to square inches.

9. 10 sq. yd. 6 sq. ft. to square feet.

10. 40 A. 80 sq. rd. to square rods.

11. 2 sq. mi. 420 A. to acres.

12. 121 sq. yd. to square rods.

13. Express 365 sq. in. as square feet, decimally to hundredths.

14. Express 860 sq. in. as square yards, decimally to hundredths.

15. Reduce 5160 sq. in. to square yards, square feet, and square inches.

SOLUTION: 5160 ÷ 144 = 35 with remainder 120.
Hence, 5160 sq. in. = 35 sq. ft. 120 sq. in.
35 ÷ 9 = 3 with remainder 8.
Hence, 35 sq. ft. = 3 sq. yd. 8 sq. ft.
Hence, 5160 sq. in. = 3 sq. yd. 8 sq. ft. 120 sq. in.

Reduce:

16. 300 sq. in. to square feet and square inches.

17. 40 sq. ft. to square yards and square feet.

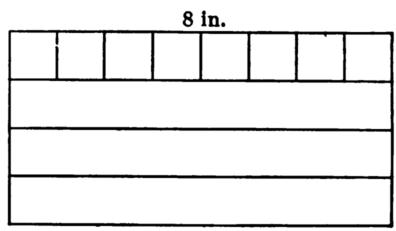
18. 425 sq. rd. to acres and square rods.

Applications of Square Measure

1. What is a rectangle? Name two kinds of rectangles.

2. Find the area, or number of square inches, in a rectangle 8 in. long and 4 in. wide.

SUGGESTION. — How many square inches in a rectangle 8 in. long and 1 in. wide?
A rectangle 4 times as wide will contain how many square inches?



Scale $\frac{1}{4}$

3. State the method of finding the area of a rectangle.

4. How many pieces of sheet copper, each containing 16 sq. in., used in hammering out pin trays, can be cut from a sheet containing 2 sq. yd., there being no waste?

5. How many boards for making notebook covers, each 9 in. by 9 in., can be cut from 5 sq. yd. of cardboard, there being no waste?

6. How many strips of leather, each containing 6 sq. in., can be cut from a skin containing 15 sq. ft., if 30 sq. in. are wasted in cutting?

7. Glass is bought at wholesale by the box. A box contains 50 sq. ft. of glass. Find the number of panes of window glass 6 in. by 8 in. in a box.

8. A printer buys a stock of paper in sheets 3 ft. by $3\frac{1}{8}$ ft. He has to cut this into sheets for letterheads 8 in. by 10 in. How many sheets of letterhead can he cut from a sheet of stock? (No waste.)

9. How many blocks of ice 18 in. wide and 24 in. long can be cut from a square rod of a pond?

10. How many blocks of ice 18 in. by 24 in. can be packed in a layer in an ice house 40 ft. wide and 60 ft. long?

11. In a rain 1.2 gallons of water fall on a square yard of surface. How many gallons fall on an acre?

12. It is a rule that in the construction of modern school buildings the amount of floor space of a room should not be more than about 5 times the amount of window space. If a room contains 8 windows, each 36 in. wide and 80 in. high, and the room is 25 ft. by 32 ft., does it conform to this rule?

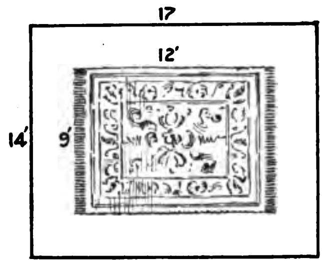
13. If each pupil should have 16 sq. ft. of floor space, how many square yards of floor should there be in a room to accommodate 30 pupils?

14. A man has a lawn 50 ft. by 40 ft. How much will it cost him to have it sodded at 8¢ a square yard?

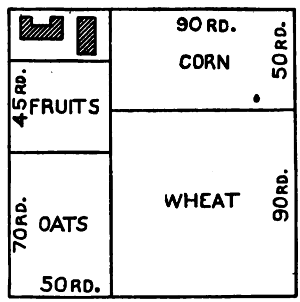
15. In front of a lot a cement sidewalk is constructed 5 ft. wide and 60 ft. long. How much will it cost at $1.25 per square yard?

16. When the pressure of the wind is 0.06 lb. on a square inch, what is the total pressure against the side of a house 18 ft. high and 40 ft. long?

17. In a room 14 ft. by 17 ft., a rug 9 ft. by 12 ft. is placed centrally, and filling at 37¢ a square yard is used to cover the rest of the floor. What is the cost of the filling?



18. The diagram represents a farm 140 rd. square. How many acres in it?



19. How many acres of wheat?

20. How many acres of corn?

21. How many acres of oats?

22. How many acres of fruits?

23. The total forest area in the United States is approximately 677,735,000 acres. How many square miles is this?

24. Where coffee is grown in South America, there are usually 5 coffee trees to every square rod. How many coffee trees are there to an acre?

25. The average yield of a coffee tree is 6 lb. per season. How many pounds will an acre produce? If this coffee brings 12¢ a pound at the plantation, what is the income from an acre?

26. A speculator bought a lot of land measuring 160 rd. by 40 rd. at $2 a square rod, and sold it at 2¢ a square foot. Find his profit.

27. An orange grove contains one tree to 5 sq. rd. How many trees are there to an acre? If each tree yields, on the average, 20 boxes of oranges in a season, what is the crop of an acre? How much is it worth at $1.25 a box?

28. If one shingle covers 16 sq. in. of a roof, how many shingles will it take to cover a roof 15 ft. by 25 ft.?

29. Shingles are sold in bundles of 250. How many bundles will it take for the above roof? How much will they cost at $4.25 per 1000?

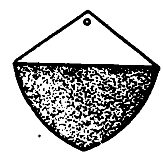
Woodworking Problems

The following table gives the cost, in cents per square foot, of different kinds of wood used in the construction of articles in a manual training shop.

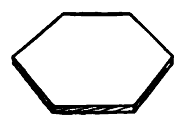
PRICE LIST OF LUMBER

KIND OF WOOD	COST PER SQUARE FOOT						
Thickness (inches) . .	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{2}$	$\frac{3}{4}$	$\frac{7}{8}$	1	$1\frac{1}{4}$
Whitewood	4	4	6	7	7	8	9
Chestnut	4	4	6	7	7	8	9
Red Oak	4	5	5	7	7	9	10
Gum	4	5	5	7	7	9	10
White Pine	$3\frac{1}{2}$	$3\frac{1}{2}$	4	6	6	8	8
Basswood	$3\frac{1}{2}$	$3\frac{1}{2}$	4	6	6	8	8

Find the cost of material in the following:



1. Twenty match scratchers like the drawing, made of basswood. Quantity of stock required for each: $\frac{1}{4}'' \times 4'' \times 5''$.



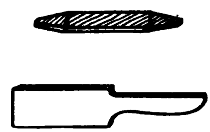
2. Twenty hexagonal mats, like the drawing, made of white pine. Quantity of stock required for each: $1'' \times 8'' \times 8''$.

3. Twenty toothbrush racks made of whitewood. Quantity of stock required for each: $\frac{1}{4}'' \times 2\frac{1}{2}'' \times 1\frac{3}{4}''$, and $\frac{1}{4}'' \times 2\frac{1}{2}'' \times 7\frac{1}{2}''$.

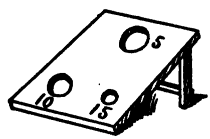
4. Twenty match safes made of whitewood. Quantity of stock required for each: $\frac{1}{4}'' \times 2'' \times 6''$, $\frac{1}{4}'' \times 1'' \times 4''$, and $\frac{1}{4}'' \times 1\frac{1}{2}'' \times 2''$.

5. Twenty pen racks, like drawing, made of basswood. Quantity of stock required for each: $\frac{1}{4}'' \times 2\frac{1}{2}'' \times 10''$.

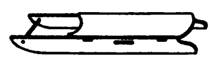
6. Ten games of "tip cat," like drawing. Cats made of basswood. Quantity of stock required for each: $1\frac{1}{4}'' \times 1\frac{1}{4}'' \times 4\frac{1}{2}''$. Bats made of white pine. Quantity of stock required for each: $\frac{1}{2}'' \times 3'' \times 15''$.



7. Five bean-bag games, made of chestnut. Quantity of stock required for each: $\frac{1}{2}'' \times 18'' \times 24''$, and $\frac{1}{2}'' \times 1\frac{1}{2}'' \times 50''$.



8. Twenty sleds, made of white pine and red oak. Quantity of pine stock required for each: $\frac{7}{8}'' \times 9'' \times 37''$, $\frac{7}{8}'' \times 5'' \times 12\frac{1}{2}''$, and $\frac{1}{2}'' \times 13'' \times 30''$. Quantity of red oak required for each: $1'' \times 1'' \times 13''$.



9. Fifteen drawing-boards made of white pine. Quantity of stock required for each: $1'' \times 15'' \times 20''$ and $1'' \times 2'' \times 15''$. Add $\frac{1}{20}$ for waste in making.

10. Fifteen T-squares made of gum-wood. Stock required for each: $\frac{1}{2}'' \times 2'' \times 9''$ and $\frac{1}{4}'' \times 2'' \times 24''$. Four flat-heads occur at 3¢ a dozen.

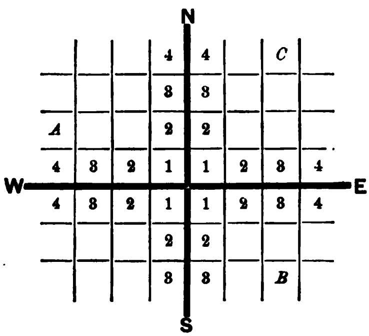
Division of Land

Most of the land in the Central and Western **states** has been surveyed and divided into **townships**, by a system of parallel lines running east and west and another system of parallel lines running north and south.

A township is 6 miles square.

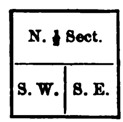
Each township is subdivided into 36 equal squares, called **sections**. The sections are always numbered as in the figure.

The sections are divided into halves and quarters; the quarters into halves and quarters; etc.



6	5	4	3	2	1
7	8	9	10	11	12
18	17	16	15	14	13
19	20	21	22	23	24
30	29	28	27	26	25
31	32	33	34	35	36

Township.



Section 15.

Exercises

1. How many square miles in a township?

2. How many square miles in a section?

3. How many acres in a section?

4. A man's farm is described as the southwest quarter of section 15. Locate his land on the figure. How many acres has he?

5. A farmer owns the north half of section 31. Locate it in the figure. How many acres has he?

6. A man owns all of section 23 and the northwest quarter of section 24. How many acres has he?

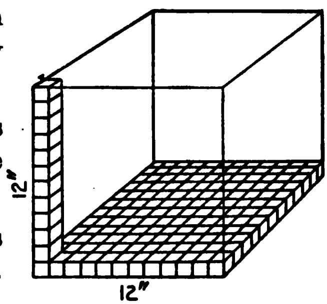
7. How much is the north half of the northeast quarter of section 6 worth at $45 an acre?

8. A ½-mile square is what part of a section? How many acres does it contain?

9. A man owns 40 A. What part of a section does he own?

10. If his farm is square, how far is it along one side? How far around it?

5. CUBIC MEASURE



1. How many cubic inch blocks can be laid in one row 12 in. long?

2. How many of these rows can be placed upon one square foot of surface?

3. How many cubic inches in a layer of such blocks 1 ft. square?

4. How many such layers of blocks are required to make a cubic foot?

5. How many one-inch cubes are there in a cubic foot?

6. How many cubic blocks, each containing one cubic foot, can be laid in a layer 1 yd. square?

7. How many such layers are there in a cubic yard?

8. How many cubic feet in a cubic yard?

9. Learn thoroughly the following:

Table of Cubic Measure

1728 cubic inches (cu. in.)	= 1 cubic foot (cu. ft.)
27 cubic feet	= 1 cubic yard (cu. yd.)
128 cubic feet	= 1 cord
$24\frac{3}{4}$ cubic feet	= 1 perch
1 cubic yard	= 1 load

NOTE. — The *cubic inch*, *cubic foot*, and *cubic yard* are the units of cubic measurement most frequently used in everyday life. The *cord* is used in measuring firewood; the *perch* in measuring stone; and the *load* in measuring earth removed for excavations, etc.

Exercises in Cubic Measure

Reduce:

1. 4 cu. ft. to cubic inches; $12\frac{1}{2}$ cu. ft. to cubic inches.
2. 8 cu. yd. to cubic feet; $3\frac{1}{3}$ cu. yd. to cubic feet.
3. 5 cords to cubic feet. 4. 24 perches to cubic feet.
5. 16 loads to cubic feet; 40 loads to cubic feet.
6. 8.36 cu. yd. to cubic feet; to cubic inches.
7. 5 cu. ft. 56 cu. in. to cubic inches.

SOLUTION: 5 cu. ft. = 5 × 1728 cu. in. = —— cu. in.
—— cu. in. + 56 cu. in. = —— cu. in.

8. 12 cu. ft. 256 cu. in. to cubic inches.
9. 7 cu. yd. 4 cu. ft. to cubic feet.
10. 1 cu. yd. 230 cu. in. to cubic inches.
11. 4 perches 16 cu. ft. to cubic feet.
12. 2 cords 94 cu. ft. to cubic feet.
13. 10,368 cu. in. to cubic feet.
14. 270 cu. ft. to cubic yards.
15. 896 cu. ft. to cords. 16. 297 cu. ft. to perches.

17. 7250 cu. in. to cubic feet, expressed decimally to hundredths.

18. 8 cu. ft. to cubic yards, expressed decimally to hundredths.

19. 103,470 cu. in. to higher denominations.

SOLUTION: 103,470 ÷ 1728 = 59 with remainder 1518.
Hence, 103,470 cu. in. = 59 cu. ft. 1518 cu. in.
59 ÷ 27 = 2 with remainder 5.
Hence, 59 cu. ft. = 2 cu. yd. 5 cu. ft.
Hence, 103,470 cu. in. = 2 cu. yd. 5 cu. ft. 1518 cu. in.

20. 5680 cu. in. to cubic feet and cubic inches.

21. 1692 cu. ft. to cubic yards and cubic feet.

22. 4250 cu. in. to cubic feet, expressed decimally to hundredths.

23. 500 cu. ft. to cubic yards, expressed decimally to hundredths.

24. 124 cu. ft. to perches, expressed decimally to hundredths.

Applications of Cubic Measure

1. What name is given to a solid bounded by 6 rectangles? By 6 squares?

2. How many cubic inches in a rectangular solid 8 in. long, 4 in. wide, and 3 in. thick?

SUGGESTION. — On a rectangular surface 8 in. long and 1 in. wide a row of 8 inch-cubes may be laid; 4 such rows, or a layer, may be laid on a surface 4 in. wide; 3 of these layers will contain 3 × —— inch-cubes, or 96 inch-cubes or cubic inches.

3. Make a statement of the method of finding the contents or the volume of a rectangular solid.

4. What is the volume of a rectangular solid 4 in. high, 5 in. wide, and 6 in. long?

5. How many cubic inches in a brick 2 in. by 4 in. by 8 in.?

6. How many such bricks does it take to make a cubic foot?

7. How many such bricks are required to build a wall 30 ft. long, 12 ft. high, and 1 ft. wide, allowing nothing for mortar?

8. What part of a cubic yard is a stone 1 ft. by 2 ft. by 3 ft.?

9. A cubic foot of granite weighs 165 lb. Find the weight of a granite monument containing $4\frac{1}{4}$ cu. yd.

10. A schoolroom 4 yd. high, 10 yd. wide, and 15 yd. long contains 60 pupils. How many cubic feet of air has each pupil?

11. Authorities agree that each pupil should be supplied with 75 cu. yd. of fresh air per hour. How many cubic feet per hour is this?

12. If a schoolroom is 12 ft. high, 30 ft. wide, and 36 ft. long, and contains 40 pupils, how often would the air have to be completely changed in it in order to supply each pupil with 75 cu. yd. of fresh air per hour?

13. If the average amount of air inhaled by a pupil at one breath is 25 cu. in., in how many breaths will he inhale 1 cu. ft.?

14. How long will it take 40 pupils to breathe over all the air in a room 25 ft. by 40 ft. by 12 ft.?

15. About one fourth of pure air is oxygen. In 1 cu. ft. of pure air, how many cubic inches of oxygen?

16. How many cubic inches of oxygen does a pupil need in an hour?

17. The loam in a box 4 in. by 8 in. by 9 in. is found to weigh 15 lb. Find the weight of a cubic foot of it.

18. Find the weight of a layer of such loam 1 ft. deep on an acre.

19. Steel weighs 0.28 lb. per cubic inch. Find the weight of a steel beam 20 ft. long, 1 ft. wide, and $\frac{1}{4}$ ft. thick.

20. A gallon contains 231 cu. in. How many gallons in a cubic foot?

21. How many gallons does a tank 3 ft. deep, 4 ft. wide, and 6 ft. long hold?

22. How many boxes, each 2 in. by 3 in. by 4 in., can be packed in a case 4 ft. by 3 ft. by 2 ft.?

23. How many baskets of peaches, each 8 in. by 5 in. by 3 in., can be packed in a crate 2 ft. by $1\frac{1}{2}$ ft. by $2\frac{1}{2}$ ft.?

24. A stone wall is 4 ft. high, $1\frac{1}{2}$ ft. thick, and 60 ft. long. Find the number of perches of stone required in its construction.

25. Find the number of loads of earth removed in making an excavation 6 ft. deep, 15 ft. wide, and 40 ft. long. Find the cost at \$0.75 per load.

26. A trench 40 rd. long is dug 2 ft. deep and 18 in. wide. How many loads of earth are removed?

27. A man has a front lawn 60 ft. wide and 40 ft. long. He wishes to fill it to a depth of 4 in. with soil. How many loads of soil must he order? Find the cost at \$1 per load, a price usually paid in Chicago for putting soil on sandy places.

28. A school garden is 120 ft. by 80 ft. How many loads of earth will raise the whole surface 6 in.?

29. A street 1200 ft. long and 40 ft. wide is to be covered with crushed stone to a depth of 4 in. Find the cost of the stone at \$1.25 per cubic yard.

30. One large tank in Hoboken, N.J., holds 12,000 gallons of molasses. One gallon contains 231 cu. in. Find the number of cubic feet in the tank.

31. A cubic foot of water weighs 62.5 lb. Find the weight of water in a tank 90 in. long and 64 in. wide when the water is 52 in. deep.

32. An *acre inch* is the amount of water sufficient to cover an acre of irrigated land 1 in. deep. Allowing $7\frac{1}{2}$ gallons to a cubic foot, find the number of gallons in an acre inch.

33. An irrigation ditch delivers 4200 gallons of water per minute over a 1-acre field. How long will it take to deliver 3 acre inches?

34. Cast-iron weighs 450 lb. per cubic foot. Find the weight of a casting that contains 1260 cu. in.

35. I bought a square lot of land 300 ft. long. I laid out a 40-ft. street through the middle of it in each direction. What was the area of each of the four lots that remained?

36. If you cut a 6-in. square out of each corner of a square yard, what part of the latter remains?

37. What will a sheet of zinc 8 ft. long and 32 in. wide cost at $7\frac{1}{2}$¢ a pound, if every square foot weighs 8 oz.?

38. At 75¢ a yard, what will it cost to put a picket fence halfway around a corner lot measuring 72 by 150 ft.?

6. MEASUREMENT OF WOOD

1. What is a cord of wood?

2. How long a pile of wood, cut 4 ft. long and piled compactly 4 ft. high, does it take to make a cord?

3. Firewood usually is cut in 4-foot lengths. How many cords of firewood in a rick 4 ft. high and 24 ft. long?

4. How many cords of 4-ft. wood in a rick 12 ft. high and 16 ft. long?

5. How many cords of 4-ft. wood in a rick 6 ft. high and 32 ft. long?

6. How much will a pile of wood 10 ft. by 6 ft. by 8 ft. cost at $8 a cord?

7. 4-ft. wood is usually cut into 16 in. lengths for use in stoves. This is called *stove wood.* How many stove lengths can be cut from a stick of wood 4 ft. long?

8. A rick of stove wood 8 ft. long and 4 ft. high is called a *cord* of stove wood. How many cords of stove wood can be cut from a cord of 4-ft. wood?

9. How many cords of stove wood in a rick 16 ft. long, 8 ft. wide, and 6 ft. high?

10. A dealer buys 4-ft. wood at $6 a cord, and has it sawed into stove wood. He then sells the stove wood at $2.75 a cord. How much profit does he make on a cord of 4-ft. wood?

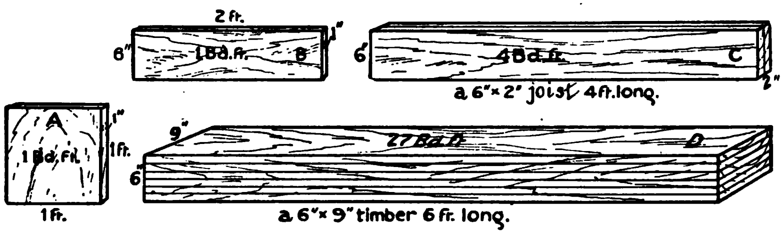
7. MEASUREMENT OF LUMBER

The unit of measure for boards, planks, joists, timber, etc., is generally the **board foot.**

A *board foot* is 1 ft. long, 1 ft. wide, and 1 inch thick.

In studying Section 4 we have learned to find the number of square feet in one face of a board. This number will also represent the number of board feet in a board 1 in. thick.

NOTE.—Lumber less than 1 in. thick is counted as if it were an inch thick. If the lumber is more than an inch thick, the thickness is taken into account.



1. Compare the dimensions and contents of A and B in the illustration.

2. Find the area of the face of C. The number of board feet in C.

SUGGESTION. — $\frac{6}{12} \times 4 =$ ——; $2 \times$ —— $=$ ——.

3. How many 1-inch boards are equal to D? Show that D contains 27 board feet.

4. Make a statement of the method of finding the contents of a plank in board feet.

NOTE. — Without much regard to correctness of language, a lumberman would answer the last question by saying, "Multiply width and thickness in inches and length in feet together, and divide the result by 12." Show where the language here is not correct. He would also speak of a board foot simply as a "foot."

5. How many board feet make a cubic foot?

6. How many board feet in a board 1 ft. wide, 10 ft. long, and 1 in. thick?

7. How many board feet in a board 2 ft. wide, 8 ft. long, and 1 in. thick?

8. Find the number of board feet in a board 4 in. wide, 16 ft. long, and 1 in. thick.

9. Find the number of board feet in a board 2 in. thick, 1 ft. wide, and 14 ft. long. It is equivalent to how many boards 1 in. thick, of this width and length?

Find the contents in board feet of:

10. A board 12 in. wide, 8 ft. long, and 2 in. thick.

11. A board 8 in. wide, 6 ft. long, and 3 in. thick.

12. A board $\frac{1}{2}$ in. thick, 6 in. wide, and 10 ft. long.

13. A board $2\frac{1}{2}$ in. thick, 8 in. wide, and 12 ft. long.

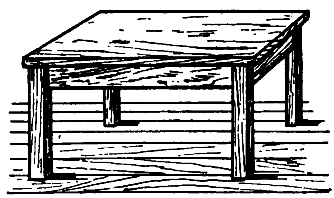
14. A joist 15 ft. long, 4 in. wide, and 3 in. thick. (A $3'' \times 4''$ joist.)

15. A joist 20 ft. long, 5 in. wide, and 4 in. thick.

16. A joist 18 ft. long and 5 in. square.

17. A man measured a stack of lumber and found that it contained 1000 boards, each 1 in. thick, 6 in. wide, and 16 ft. long. How many board feet in the stack?

18. A boy undertook to make a mission table for a Christmas present to his mother. The top was 3 ft. by $2\frac{1}{2}$ ft., and $1\frac{1}{2}$ in. thick. The legs were 27 in. long and 3 in. square. The four pieces mortised into the legs, to which the top was fastened, were each 1 in. thick and 5 in. wide, two of them 28 in. long and two 22 in. long. How many feet of lumber did it take in all? How much did it cost him at 12¢ per board foot?



19. Find the cost of 2500 board feet of flooring at $35 per thousand board feet.

NOTE.—Lumber is sold in quantities for so much per thousand (M) board feet.

Find how much must be paid for:

20. 3000 ft. of spruce at $16 per M.

21. $2\frac{1}{2}$ M white pine at $17.

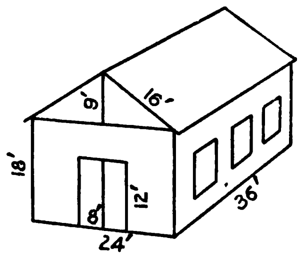
22. 4500 ft. of hard pine at $42 per M.

23. 800 ft. walnut at $80 per M.

24. 750 ft. whitewood at $38 per M.

25. 1600 ft. cherry at $80 per M.

26. The diagram represents a barn. What is the area of the rectangular part of the end?



27. What is the area of the gable, or triangular part?

28. How many feet of 1-inch lumber will it take to cover it? How much will it cost at $20 per M?

29. If the barn is 36 ft. long, what is the area of each side? Of both sides?

30. How many feet of 1-in. lumber will it take to side the barn? How much will it cost at $20 per M?

31. For upright posts in the barn 16 joists are required, each 18 ft. long and 6 in. square. How much will they cost at $25 per M?

32. Also 40 other joists, 14 ft. long, 3 in. thick, and 4 in. wide, are required. How much will they cost at $25 per M?

33. 36 rafters, each 2 in. by 6 in. by 16 ft., are required. Find their cost at $25 per M.

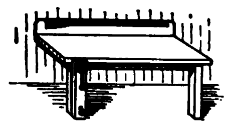
34. A driveway in the barn 12 ft. wide and 24 ft. long is floored with boards 2 in. thick. Find their cost at $30 per M.

35. Find the cost of a board walk 4 ft. wide and 25 ft. long. The cross boards are nailed to three 3″ × 4″ joists and are 2″ thick. The lumber costs $28 per M.

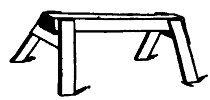
A Boy's Workshop

A boy fitted up a workshop in the attic. He made a work bench which was fastened to the wall, two carpenter's horses, a bench hook, a miter box, a tool cabinet, and a carrying box for tools.

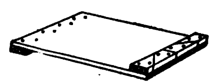
1. For the work bench he bought lumber as follows: one board $2'' \times 4'' \times 48''$, two boards $2'' \times 4'' \times 28''$, two boards $2'' \times 4'' \times 30''$, three boards $2'' \times 10'' \times 72''$, one board $1'' \times 10'' \times 72''$, one board $2'' \times 6'' \times 30''$, and one board $1'' \times 2'' \times 15''$. How many board feet of lumber in the work bench?



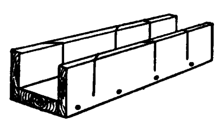
2. He made two carpenter's horses like that in the figure. For the top of each he bought a piece $2'' \times 4'' \times 40''$. For the legs of each he bought a board $1'' \times 4'' \times 120''$. And for two braces of the legs on each he bought a board $1'' \times 8'' \times 24''$. How many board feet in the two horses?



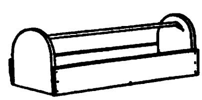
3. He made a bench hook, as shown in the figure. This required a board $1'' \times 10'' \times 12''$, and two $1'' \times 2'' \times 10''$. How many board feet in the bench hook?



4. He made a miter box for cutting miters. For this he required two boards each $\frac{7}{8}'' \times 6'' \times 20''$, and one board $1'' \times 4'' \times 20''$. How many board feet did it take?



5. He made a carrying box for tools. This required two pieces each $1'' \times 8'' \times 27''$, one piece $1'' \times 25'' \times 8''$, and two pieces each $1'' \times 8'' \times 16''$. How many board feet did it take?



8. LIQUID MEASURE

NOTE.—The school should have a set of liquid measures. These should be actually used by the pupils until the knowledge of their sizes and relations becomes a matter of experience.

1. Get a vessel of water, and by filling a gill measure and emptying the contents into a pint measure, see how many gills it takes to make a pint.

2. Find, in the same way, how many pints in a quart, and how many quarts in a gallon.

3. Learn thoroughly the following:

Table of Liquid Measure

4 gills (gi.)	= 1 pint (pt.)
2 pints	= 1 quart (qt.)
4 quarts	= 1 gallon (gal.)
$31\frac{1}{2}$ gallons	= 1 barrel (bbl.)
1 gallon	= 231 cubic inches

4. How many gills in a quart? In a gallon?

5. How many pints in a gallon?

6. How many cubic inches in a quart? In a pint?

7. Estimate the number of gallons of water in some vessel. Then measure it with a gallon measure, and see how nearly accurate your judgment is.

8. Practice estimating in this way the quantity of liquids in vessels until you can estimate accurately at sight any given quantity of liquid.

Exercises in Liquid Measure

1. How many quart bottles can be filled from a 5-gallon can of milk?

Reduce:

2. $3\frac{1}{2}$ gal. to quarts; to pints.

3. $5\frac{1}{2}$ qt. to pints; to gills.

4. 6 gal. 3 qt. to quarts.

5. 2 qt. 1 pt. to pints.

6. 126 gal. to barrels; 1200 gal. to barrels.

7. 21 pt. to quarts and pints.

8. 35 qt. to gallons and quarts.

9. 18 gal. to barrels, expressed decimally to hundredths.

Applications of Liquid Measure

1. In cooking, a measure called a *cup* is sometimes used. It is equal to $\frac{1}{2}$ pt. How many cups in a pint? How many gills in a cup?

2. Milk is shipped in 10-gal. cans from farms to dairy stations in cities. There it is bottled by bottling machines and distributed to customers. How many quarts in one can?

3. How many pint bottles would one 10-gal. can fill?

4. The milk is hauled from the railroad station to the dairy in loads of 24 cans each. How many gallons in one load? How many quart bottles does one load fill?

5. This milk retails at 8¢ a quart. How much do the consumers pay for one 10-gal. can of milk?

6. If a family uses 2 qt. of milk, how many families would one 10-gal. can supply?

7. When milk is bottled, the bottles are placed in crates. There are 12 bottles to a crate. How many crates of quart bottles will a 10-gal. can fill?

8. If a retail wagon carries 8 crates of quart bottles and 12 crates of pint bottles, what is the retail value at 4¢ a pint?

9. A grocer pays 20¢ a gallon for milk, and retails it at 8¢ a quart. How much is his profit on a gallon? If he handles 30 gallons a day, how much is his profit?

10. How many $\frac{1}{2}$-pt. bottles of mineral water can be filled from a tank holding 650 gallons?

11. A ship with 1200 persons aboard carries a supply of 12,000 gal. of fresh water. If a person, on the average, uses 2 qt. of water a day, how many days will the supply last?

12. A tank of a gasoline stove holds 2 qt. 1 pt. of gasoline. How many times can it be filled from a 5-gal. can of gasoline?

13. How many ink bottles, each holding $\frac{1}{2}$ gi., can be filled with 1 gal. of the fluid?

14. Some children made 3 gal. of lemonade. They sold it in glasses holding $\frac{3}{8}$ pt. each. How many glasses were there? How much did they receive from the sale of it at 5¢ a glass?

15. A tank holds 16 gal. How many cubic inches in it?

16. How many cubic inches in a 2-qt. bottle?

17. A cistern holds 50 bbl. of water. How many gallons is this?

18. A woman made 15 glasses of crab-apple jelly and 15 glasses of grape jelly. Each glass held $\frac{1}{2}$ pt. How many pints of jelly did she make? Quarts? Gallons?

9. DRY MEASURE

NOTE. — The school should have a set of dry measures, to be used by the pupils as suggested under liquid measure.

1. For what is dry measure used?

2. By filling a pint measure with beans, or other material easily obtained, and emptying the contents into a quart measure, show how many pints, dry measure, make a quart.

3. In a similar manner, show the number of quarts in a peck, and the number of pecks in a bushel.

4. Learn thoroughly the following:

Table of Dry Measure

2 pints (pt.)	= 1 quart (qt.)
8 quarts	= 1 peck (pk.)
4 pecks	= 1 bushel (bu.)

NOTE. — The dry quart is larger than the liquid quart. The dry quart contains 67.2 cu. in., while the liquid quart contains only 57.75 cu. in.

Exercises in Dry Measure

Reduce:

1. 3 bu. to pecks; to quarts. 12 bu. to pecks; to quarts.
2. 5 pk. to quarts; to pints. $7\frac{1}{2}$ pk. to quarts; to pints.
3. 16 qt. to pecks; 41 qt. to pecks.
4. 21 pt. to pints and quarts. 5. 336 cu. in. to quarts.
6. 63,008 cu. in. to bushels.

Applications of Dry Measure

1. How many quart boxes will 2 bu. 3 pk. 1 qt. fill?

2. What is the cost of 2 pk. 3 qt. of nuts at 15¢ a quart?

3. If a half-peck basket of peaches sells for 25¢, how much is that per bushel?

4. A grocer buys onions at 80¢ a bushel. He then retails them at 5¢ a quart measure. How much profit does he make on a bushel?

5. A fruit dealer buys peaches at wholesale for $2.25 a bushel. He then puts them in baskets holding 2 qt. each,

and sells them at 25¢ a basket, keeping the basket. How much is his profit on a bushel? If the baskets cost him 1¢ apiece, and he lets them go with the fruit, how much is his profit per bushel?

6. A fruit dealer buys pears at wholesale for $2 a barrel. The barrel contains $2\frac{1}{2}$ bu. He puts them in half-peck baskets costing him 2¢ each, and retails them at 20¢ a basket. How much does he make on a bushel?

7. A gardener sold on one day 2 qt. of green beans to each of 22 customers, and 3 qt. to each of 18 customers. How many quarts did he sell in all? How many bushels?

8. The gardener received 10¢ a quart for the beans. How much was that per bushel? How much did he receive from the day's sales?

9. He sold also 2 bu. 3 pk. 2 qt. of sweet potatoes at 16¢ a quart. How much did he get for them?

10. A gardener sows peas in rows 90 ft. long, using 1 pt. of seed to 100 ft. How many quarts of seed does he need for 20 rows? How much does it cost him at 25¢ a quart?

11. A grocer bought 20 bu. of potatoes at 55¢ a bushel, plus cartage. The cartage was $2. 1 bu. 2 pk. spoiled. He sold the rest at 30¢ a peck. How much was his profit on the lot?

12. Four people went on a trip and brought back a bushel and a half of crab apples, which they divided equally among them. How many quarts did each have?

13. A vegetable peddler started out with 3 bu. of beans. He sold a peck to one customer, a half peck to another, 3 qt. to another, and 2 qt. to another. How many had he left?

10. WEIGHT MEASURES

The system of weights used in ordinary measurements is called the system of *Avoirdupois weight.*

Table of Avoirdupois Weight Measure

16 ounces (oz.)	= 1 pound (lb.)
100 pounds	= 1 hundredweight (cwt.)
2000 pounds	= 1 ton (T.)

A ton of 2000 pounds is sometimes called a *short* ton, in distinction from a ton of 2240 lb., called a *long ton*, which is used in measuring some mining products in wholesale transactions.

NOTE. — In addition to the above table of weights, there are other tables of weights used by people engaged in certain technical pursuits.

Goldsmiths, in weighing the precious metals and stones, use the system of *Troy weight:*

24 grains (gr.)	= 1 pennyweight (pwt.)
20 pennyweights	= 1 ounce (oz.)
12 ounces	= 1 pound (lb.)

Physicians and druggists, in prescriptions, use the system of *Apothecaries' weight:*

20 grains (gr.)	= 1 scruple (℈)
3 scruples	= 1 dram (ʒ)
8 drams	= 1 ounce (℥)
12 ounces	= 1 pound (lb.)

The pound of Troy and Apothecaries' weight is less than the pound Avoirdupois.

The learning of these two tables is part of a technical education.

Exercises in Weight Measure

Reduce:

1. $3\frac{1}{2}$ T. to pounds; $8\frac{1}{4}$ T. to pounds; 1.75 T. to pounds.
2. 1 T. to hundredweight; 7 T. to hundredweight; $2\frac{1}{2}$ T. to hundredweight.

3. 48 oz. to pounds; 144 oz. to pounds; 320 oz. to pounds.

4. 6000 lb. to tons; 18,000 lb. to tons; 3550 lb. to tons.

5. How many pounds in 4480 long tons?

6. How many short tons in 100 long tons?

7. How many long tons in 56 short tons?

Applications of Weight Measure

1. A family pays 30¢ for 100 lb. of ice. How much is that per 1000 lb.? How much per ton?

2. An ice company stores 300 tons of ice from a pond in winter. How many families would this supply, if each family used, on the average, 1500 lb. a season?

3. A refrigerator in a meat market holds 4 tons of ice. When 2860 lb. have been put in, how many more pounds are required to fill it?

4. A market man has 7850 lb. of ice put into his refrigerator at one time. How much does it cost at $3.90 a ton?

5. A market man has his refrigerator filled with ice once a week. If 6500 lb. are put in each time, on the average, how many tons will be used in a year (52 weeks)? How much will the ice cost for a year at $4 a ton?

6. A can of mustard contains 4 oz. How many pounds in 4 doz. cans?

7. In a recent year one large manufacturing company imported 30,000 tons of tin, costing $500,000. How much was this a pound?

8. Mrs. Hastings bought $1\frac{1}{2}$ lb. of veal at 28¢ a pound. Mrs. Webb bought a 3-lb. chicken at 18¢ a pound. Allowing 2 oz. for waste of the veal, and 12 oz. for waste of the chicken, which was the more economical purchase?

United States Postal Rates

Domestic mail matter is classified as first, second, third, and fourth class. The postage differs for the different classes of mail matter. The postal rates to foreign countries are in general different from the domestic rates.

1. The postage on first-class domestic mail matter is 2¢ an ounce. How much is the postage per pound?

2. Find the postage on each of the following first-class pieces of mail: a package weighing $2\frac{1}{2}$ lb.; one weighing $1\frac{3}{4}$ lb.; one weighing 1 lb. 8 oz.; one weighing 2 lb. 6 oz.

3. All classes of mail matter may be registered for safety at a cost of 10¢ in addition to the regular postage. How much is the postage on a first-class registered package weighing 1 lb. 3 oz.?

4. I received a package of first-class matter through the mail that had 52¢ postage on it. How many ounces did it weigh? How many pounds and ounces?

5. The rate of postage on second-class matter, when sent by the publisher, or by a news agency to regular subscribers, or to another news agency, is 1¢ a pound or fraction thereof. How much is the postage on a piece of second-class matter weighing 2 lb. 5 oz.?

6. If second-class matter is sent by persons other than the publishers or news agencies, the rate is 1¢ for each 4 oz. or fraction thereof. What is the rate per pound?

7. If you mail a bundle of newspapers (second class) weighing 2 lb. 8 oz., how much is the postage?

8. The rate of postage on third-class mail matter (books, photographs, etc.) is 1¢ for each 2 oz. or fraction thereof. How much is the postage on a package of third-class matter weighing 3 lb.? One weighing $2\frac{1}{4}$ lb.?

Problems in Buying and Selling Coal

Coal is found in commercial quantities in 27 states and territories of the United States and in Alaska.

1. The total land surface of the states and territories, including Alaska, is 3,547,746 sq. mi. How many acres?

2. Of this, 280,397 sq. mi. are coal fields. How many acres?

3. Illinois has the largest area of coal fields of all the states. There are 42,900 sq. mi. of coal fields in the state. What decimal part of the coal-field area of the United States is this?

4. In a recent year the output from the coal mines of Illinois was 41,480,104 short tons. How many carloads would this make, allowing 40,000 lb. to the load?

5. If a man can mine an average of 4 tons 500 lb. a day, how many miners, working 300 days of the year, would it require to mine the coal in Illinois?

6. Coal dealers buy coal by the short ton from the mines. It is shipped to coal yards in the cities, and then retailed to the consumers. A dealer received one day a car containing 48,000 lb. of coal, one containing 42,800 lb., one containing 46,600 lb., and one containing 39,900 lb. How much did it cost him at $2.75 a ton?

7. A man ordered 10 T. of coal from a coal dealer. It was delivered in loads as follows: 1 T. 850 lb., 1 T. 950 lb., 1 T. 1700 lb., 2 T. 100 lb., 1 T. 900 lb., 1 T. 1500 lb. Did he receive 10 T.?

8. A coal company delivered to a customer one load of coal of 2150 lb., one of 1950 lb., one of 2200 lb., and one of 2350 lb. How much was the bill at $6.50 a ton?

The Products of a Farm

A farmer raises wheat, corn, oats, and hay, which he hauls to market and sells each year. He also raises hogs and cattle, which he ships to the city market.

1. He hauls his wheat, when threshed, and stores it in a large building called an *elevator*, which is situated by a railroad track. From the elevator it is reshipped later. The capacity of a certain elevator is 2,400,000 bu. Wheat weighs 60 lb. to the bushel. Find the capacity of the elevator in tons.

2. How many carloads, of 60,000 lb. each, does the elevator hold?

3. In one day the man delivers four loads of wheat at the elevator, weighing as follows: 2400 lb., 2700 lb., 2550 lb., and 2350 lb. How many bushels is this?

4. He hauls in all 90,000 lb. of wheat to the elevator. How many bushels?

5. He raises also 1000 bu. of oats. Of this he sells 12 loads of 1 T. each. One bushel of oats weighs 32 lb. How much does he get for the oats at 40¢ a bushel? How many bushels does he have left?

6. He sells 44,890 lb. of timothy hay at $20 per ton, and 46,880 lb. of clover hay at $16 per ton. How much does he get for the hay?

7. He sells 40 head of cattle, the total weight of which is 36,500 lb., at $8.40 per hundredweight. How much does he receive for them?

8. He sells hogs, the total weight of which is 19,240 lb., at $9.75 per hundredweight. How much does he receive for them?

11. MEASUREMENT OF TIME

Table of Time Measure

60 seconds (sec.) = 1 minute (min.)	12 months or 365 days = 1 year (yr.) common
60 minutes = 1 hour (hr.)	366 days = 1 leap year
24 hours = 1 day (da.)	100 years = 1 century
7 days = 1 week (wk.)	"Thirty days have September, April, June, and November."
30 (31, 28, 29) days = 1 month (mo.)	

All the other months have 31 days except February, which has 28.

A true or solar year does not have exactly 365 days, but 365 da. 5 hr. 48 min. 49.7 sec., or nearly 365¼ da. Since 365 da. is not exact, in every fourth year, except the century years that are not divisible by 400, one day is added, making 366 da. or a **leap year**. The extra day in leap years is added to the month of February, making it 29 days long.

Two weeks are called a **fortnight**, 3 months or 13 weeks a **quarter**, and 10 years a **decade**.

Problems in the Measurement of Time

1. How many decades have passed since July 4, 1776?

2. Except century years not divisible by 400, years divisible by 4 are leap years. Which of the following were leap years: 1728, 1776, 1812, 1860, 1886, 1900, 1908?

3. Find the time from March 16, 1895, to July 25, 1910.

SOLUTION: Count by years as far as possible, then by calendar months, then by days.

From March 16, 1895, to March 16, 1910, is 15 yr.
From March 16, 1910, to June 16, 1910, is 3 mo.
From June 16 to July 5 is 19 da.
Hence the time is 15 yr. 3 mo. 19 da.

Find the time from:

4. April 12, 1904, to July 28, 1910.

5. August 10, 1890, to June 18, 1902.

6. February 6, 1897, to January 10, 1900.

7. January 1, 1903, to December 21, 1909.

8. October 8, 1907, to May 3, 1910.

9. Find your exact age to-day in years, months, and days.

10. Abraham Lincoln was born February 12, 1809. How many years, months, and days ago to-day was that?

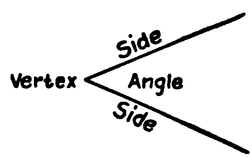
11. The Pennsylvania train, No. 22, which leaves Chicago at 10.30 A.M. (30 minutes after 10 in the morning), reaches Washington, D.C., at 8.40 A.M. the next day. How many hours and minutes are required for the trip?

12. The Big Four Train, No. 34, which leaves Chicago at 9.05 P.M., reaches Cleveland at 12.30 P.M. the next day. How many hours and minutes is that?

13. A train leaving Cincinnati at 9.10 P.M. arrives at Chicago at 7.10 A.M. the next day. The distance is 305 miles. How many miles an hour does the train run, on the average?

12. ANGLE AND ARC MEASURE

1. Draw two straight lines from the same point. These lines form an **angle**. The lines are its **sides**, and the point its **vertex**.

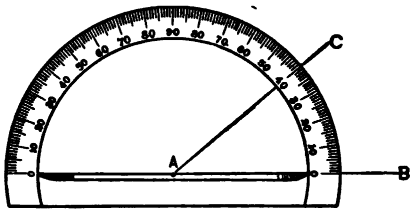


2. What is a right angle? Draw a right angle.

3. If a right angle is divided into 90 equal parts, each part is called a **degree**. Hence a right angle contains 90 degrees (90°). How many degrees in $\frac{1}{2}$ of a right angle? In $\frac{1}{6}$ of a right angle? In 4 right angles?

4. Draw two lines crossing each other at right angles. How many right angles do they form? How many degrees in all of the angles?

5. If these angles were divided into any number of smaller angles, how many degrees would there be in all of the angles together? How many degrees in all of the angles around a point?



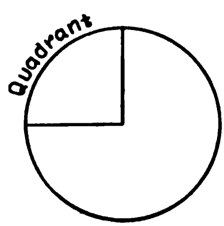
6. The number of degrees in an angle is found by means of a **protractor**, shown in the figure.

7. For accurate measurements in surveying, astronomy, etc., where angles are used, a degree is divided into 60 smaller units of measure, called minutes, and a minute into 60 smaller units, called seconds.

Table of Angle Measure

60 seconds (60″) = 1 minute (1′)
60 minutes = 1 degree (1°)
90 degrees = 1 right angle

8. Draw a circle. Any part of the circumference is called an **arc**. Draw a right angle with its vertex at the center of the circle. The arc of the circumference cut off by the sides of the angle is called a **quadrant**. If the lines were drawn dividing the right angle into degrees, they would also divide the quadrant into how many equal parts? Each of these is called



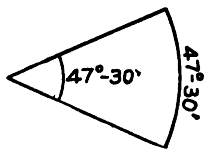
a **degree of arc.** How many degrees of arc in a quadrant? How many in the whole circumference?

9. A degree of arc, as well as a degree of angle, is divided into minutes and seconds.

Table of Arc Measure

60 seconds (60″)	= 1 minute (1′)
60 minutes	= 1 degree (1°)
90 degrees	= 1 quadrant
360 degrees	= 1 circumference

NOTE.—It is evident that if the two radii are drawn joining the ends of an arc to the center of the circle, there will be just as many degrees, minutes, and seconds of angle in the angle formed as there are degrees, minutes, and seconds of arc in the arc.



Problems in Angle and Arc Measurement

1. How many degrees in $\frac{1}{2}$ of a circumference? In $\frac{1}{6}$ of a circumference?

2. Reduce 42,895″ to degrees, minutes, and seconds.

3. How many degrees between the hands of a clock at 1 o'clock? At 2 o'clock? At 4 o'clock? At 6 o'clock?

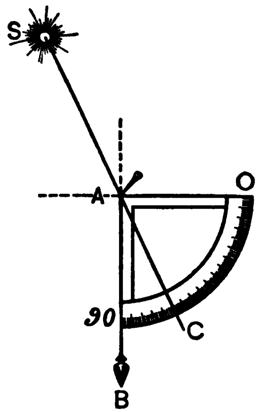
4. When a star is straight overhead, how many degrees is it from the western horizon?

5. What does it mean to say that two stars are 20° apart?

6. If a star is 25° from zenith (from straight overhead), how far is it above horizon?

7. A *sextant*, an instrument used for measuring angular distances, and especially by sailors for measuring the latitude and longitude of a ship at sea, has an arm bearing a scale in

degrees that is $\frac{1}{6}$ of a circumference long. How many degrees does it contain? (An illustration may be found in the dictionary.)



8. The angle of elevation of the sun above the horizon may be found by use of a simple instrument like the figure, called a *quadrant*, that the pupil can easily make out of cardboard, a pin, a string, and a weight. Cut out $\frac{1}{4}$ of a circle from cardboard, using a radius of about 8 in. Mark the arc in degrees. From a pin put through the cardboard at the center A, suspend a plumb line made of string and a small weight. To find the elevation of the sun, hold the instrument upright so that the plumb line is over the 90° mark. Note the point C when the shadow of the pin A made by the sun falls on the scale. The number of degrees from O to C is the elevation of the sun.

How may the same instrument be used to find the elevation of stars, by pointing the edge of AB towards the star?

NOTE. — The making of this quadrant has been found to be an interesting and valuable piece of hand work.

13. MEASUREMENT OF PAPER

Paper is bought and sold by the *sheet*, *quire*, and *ream*.

Table of Paper Measure

24 sheets = 1 quire	
20 quires = 1 ream	

NOTE. — The quire has practically gone out of use now, except in selling letter paper in boxes. When paper is bought of a wholesale house, it is usually put up in reams containing 500 sheets. Any quantity less than a ream is computed by sheets.

1. How many sheets in $\frac{1}{2}$ quire? In 2 quires? In 5 quires?

2. How many quires in $\frac{1}{2}$ ream? In 5 reams? In 10 reams?

3. How many quires in 48 sheets? In 96 sheets?

4. A man bought a box of letter paper containing 2 quires for 48¢. How much was that a sheet?

5. I paid 36¢ for a box of paper containing 1 quire. How much was that per sheet?

6. Which is cheaper, to buy a ream of 500 sheets at a wholesale house for $2, or to buy 20 quires put up in boxes at 10¢ a quire? Why?

7. A man bought blotting paper at 72¢ a quire, and sold it at 5¢ a sheet. How much was his profit?

8. Letter paper is generally bought of a wholesale house by a printer in sheets 17 in. by 22 in. How many letter heads $8\frac{1}{2}$ in. by 11 in. can be cut from one sheet? Explain how. How many letter heads of this size will a ream of 500 sheets make?

9. A man wants 8000 letter heads $8\frac{1}{2}$ in. by 11 in. How many reams (500 sheets) of paper $17'' \times 22''$ must the printer order from the wholesale house for them?

10. Book paper may be bought in sheets 24 in. by 36 in., 25 in. by 38 in., or 28 by 42 in. A book is to be made with pages $7\frac{1}{2}$ in. by $5\frac{1}{2}$ in. From which of these sizes of stock should the paper be cut for the book, to have the least waste?

11. A stationer buys 500 sheets of blotting paper $19'' \times 24''$. He cuts it into blotters $4'' \times 9''$. How many blotters will he have? What per cent of each sheet is wasted?

12. If he bought at 3¢ a sheet and sold at 5¢ a dozen blotters, how much did he gain?

14. UNITS IN COUNTING

Table

12 things = 1 dozen	12 gross = 1 great gross
12 dozen = 1 gross	20 things = 1 score

1. How many things in a gross? In a great gross?

2. What does it mean to say that one lived to the age of threescore and ten?

3. How many pencils in 2 gross?

4. How many gross in 1000 pencils?

5. Find the cost of 1200 dozen penholders at $2.50 a gross.

15. ADDITION, SUBTRACTION, MULTIPLICATION, AND DIVISION OF COMPOUND NUMBERS

The principles and methods of simple numbers apply equally to the fundamental processes in compound numbers.

Problems in Addition

1. The lengths of the blackboards in a schoolroom are 4 yd. 1 ft. 8 in., 3 yd. 9 in., and 2 yd. 2 ft. 5 in., respectively. Find the total length of the three boards.

WORK

4 yd.	1 ft.	8 in.
3	0	9
2	2	5
10 yd.	1 ft.	10 in.

EXPLANATION. — The like units are written in columns. The sum of the column of inches is 22 in. Reduced to feet and inches, this is 1 ft. 10 in. Write 10 in. and add 1 ft. to the next column. Similarly, reduce the sum of the column of feet to yards and feet, etc.

Add:

2.

3 yd.	1 ft.	6 in.
2	2	4
6	1	7

3.

2 hr.	13 min.	27 sec.
4	25	32
1	27	10

4.

4 lb.	9 oz.
3	6
5	12

6.

45°	18′	12″
32	43	51
28	24	8

5.

2 bu.	3 pk.	5 qt.
4	2	7
5	1	3

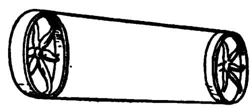
7.

1 gal.	3 qt.	1 pt.
3		1
2	2	

8. At the remnant counter a girl bought one piece of ribbon containing 1 yd. 16 in.; another containing 2 ft. 20 in.; and a third containing 1 yd. 2 ft. What was the total length of the three pieces?

9. A room is 4 yd. 2 ft. 6 in. wide and 5 yd. 1 ft. long. How much border is required to go around it?

10. A belt is to run over two pulleys whose centers are 6 yd. 1 ft. apart. The distance around the rim of each pulley is 2 yd. 2 ft. 2 in. How long must the belt be cut?



Problems in Subtraction

1. A piece of cloth contains 3 yd. 1 ft. 9 in. A piece 1 yd. 2 ft. 6 in. is used. How much remains?

WORK

3 yd.	1 ft.	9 in.
1	2	6
1 yd.	2 ft.	3 in.

EXPLANATION. — Like units are written in columns. Since 1 ft. is less than 2 ft., change 1 yd. to 3 ft. and add the 1 ft. This gives 2 yd. 4 ft. Then 4 ft. − 2 ft. = 2 ft., and 2 yd. − 1 yd. = 1 yd.

Subtract:

2.

4 yd.	2 ft.	6 in.
2	1	8

4.

10 gal.	2 qt.	1 pt.
6	3	1

3.

5 bu.	3 pk.	4 qt.
2	1	7

5.

7 sq. yd.	4 sq. ft.	80 sq. in.
2	7	96

6. When a boy was 10 yr. old, he was 1 yd. 11 in. tall. When he was 12 yr. old, he was 1 yd. 1 ft. 7 in. tall. How much did he grow in the two years?

7. Measure the height of some of the tallest pupils in the class, in yards, feet, and inches. Let each pupil find the difference between his height and that of the tallest in the class.

8. A board is 2 yd. 1 ft. long. A boy cuts off a piece 2 ft. 8 in. long. How much is left?

9. From an iron rod 3 yd. 2 ft. 6 in. long a blacksmith cuts off a piece 1 yd. 1 ft. 10 in. long. How much is left?

10. An eclipse of the sun began at 9 hr. 51 min. 39 sec. A.M., and ended at 10 hr. 9 min. 24 sec. A.M. How long did it last?

Time between Dates

The time between two dates may be found by subtraction, using the numbers of the years and the numbers of the months named, as illustrated in the following problem.

1. Find the time in years, months, and days, from May 12, 1904, to March 3, 1910.

SOLUTION: May is the 5th month and March the 3d month of the year. Count 30 days to a month.

1910 yr.	3 mo.	3 da.
1904	5	12
5 yr.	9 mo.	21 da.

EXPLANATION. —

33 da. – 12 da. = 21 da.

14 mo. – 5 mo. = 9 mo.

2. The war between Russia and Japan began February 7, 1904. How many years, months, and days ago to-day was that?

3. The North Pole was reached by Commander Robert E. Peary, April 6, 1909. How long ago was that to-day?

Find the time from:

4. April 27, 1875, to January 16, 1901.

5. August 4, 1901, to November 1, 1905.

6. January 24, 1889, to December 3, 1900.

7. June 2, 1906, to March 10, 1912.

8. Find the exact number of days between Jan. 15 and Aug. 17, 1910. Test your work by reference to Table, p. 406.

9. How many days between Feb. 10, 1912, and Jan. 7, 1913?

10. How does this method of finding the time between two dates differ from that at the bottom of p. 265?

Problems in Multiplication

1. A wire fence 5 wires high is 34 yd. 2 ft. 8 in. long. How much wire does it take to build it?

WORK

34 yd.	2 ft.	8 in.
		5
174 yd.	1 ft.	4 in.

EXPLANATION. — 5 × 8 in. = 40 in.; 40 in. = 3 ft. 4 in. Write the 4 in.; 5 × 2 ft. = 10 ft. 10 ft. + 3 ft. = 13 ft.; 13 ft. = 4 yd. 1 ft. Write the 1 ft. 5 × 34 yd. = 170 yd.; 170 yd. + 4 yd. = 174 yd.

Multiply:

2. 3 yd. 2 ft. 5 in. by 3.

3. 3 gal. 2 qt. 1 pt. by 4.

4. 2 hr. 13 min. 10 sec. by 6.

5. 3 sq. ft. 27 sq. in. by 8.

6. The desks in a school room are placed 2 ft. 10 in. apart. How long a row is required for 5 desks?

7. The side of a square lot is 24 yd. 2 ft. 4 in. What is the perimeter?

8. How much belting does it take to drive 20 machines in a factory, if it takes 21 ft. 5 in. for each machine?

9. In the wood shop a boy constructs a table 2 ft. 4 in. high. How long a piece of timber does he need for the four legs?

10. A machinist desires to cut 4 bars, each 3 ft. 2 in. long. How long must the bar be from which they are cut?

11. Find the cost of 24 sash curtain rods 3 ft. 10 in. long at 8¢ a foot. Allow 1 ft. 9 in. waste in cutting.

Problems in Division

1. A man has a lot of which the frontage is 26 yd. 2 ft. He wishes to set 5 shade trees along the lot, one at each end. How far apart must they be placed?

FORM

$$\begin{array}{r} \text{6 yd. 2 ft.} \\ 4\overline{)\text{26 yd. 2 ft.}} \end{array}$$

EXPLANATION. — 26 yd. ÷ 4 = 6 yd., with remainder 2 yd. Write the 6 yd. Reduce 2 yd. to feet, giving 6 ft. 6 ft. + 2 ft. = 8 ft. 8 ft. ÷ 4 = 2 ft.

Divide:

2. $3\overline{)\text{17 lb. 4 oz.}}$

3. $5\overline{)\text{11 hr. 21 min. 10 sec.}}$

4. $10\overline{)\text{13 bu. 1 pk. 6 qt.}}$

5. $4\overline{)\text{9 sq. ft. 120 sq. in.}}$

6. A coat rack is 15 ft. 8 in. long. It is to contain 9 hooks. The outer hooks are to be 2 in. from the ends. How far apart must the hooks be placed?

7. A fence 30 yd. 2 ft. 6 in. long is to be made with 12 posts. How far apart must the posts be set?

8. Three boys went nutting and gathered 2 pk. 3 qt. 1 pt. of nuts. They wished to divide the nuts equally among them. How many should each boy have?

16. LATITUDE AND LONGITUDE

NOTE. — A map should be used in teaching this topic. Practice should be given in finding the latitude and longitude of places, and of locating places in given latitude and longitude.

In the study of geography it is seen that the location of any point of the earth's surface is known by learning its distance north or south of the equator, called its **latitude**, and

its distance east or west of a given meridian, called its **longitude.** In measuring longitude, the meridian commonly used, called the **prime meridian,** is that of Greenwich, near London, England. The meridian of Washington is sometimes used.

Since the earth is practically a large sphere, distances on its surface are measured along arcs of circles, and hence are expressed in degrees, minutes, and seconds of arc.

To say that a place is in latitude 38° N. means that it is 38° north of the equator. And to say that a place is in longitude 117° E. means that it is 117° east of the meridian of Greenwich.

Exercises

1. By looking at a map, find the latitude and the longitude of each of the following: New Orleans; New York; Charleston, S.C.; Denver; Pekin, China; Lima, Peru; Rio Janeiro, Brazil; Cape Town, Africa.

2. The following table gives the longitudes, to the nearest minute, of twelve cities.

How far is it in longitude between Boston and San Francisco?

Place	Longitude	Place	Longitude
Berlin, Germany	13° 24′ E.	Tokio, Japan	139° 42′ E.
Boston	71° 3′ W.	London	6′ W.
Calcutta	88° 20′ E.	Paris, France	2° 20′ E.
Chicago	87° 37′ W.	St. Petersburg	30° 16′ E.
Cincinnati	84° 26′ W.	Manila, Philippines	120° 58′ E.
Cleveland	81° 40′ W.	San Francisco	122° 26′ W.

3. Find the difference in longitude between Berlin and St. Petersburg.

4. Find the difference in longitude between Chicago and Paris.

5. Find the difference in longitude between Boston and Manila.

6. How far is Cleveland west of London?

7. How far is Tokio east of Calcutta?

8. Find the differences in longitude between other cities given in the above table.

Longitude and Time

1. How often does the earth turn on its axis? In what direction does it turn?

2. Make a point on a ball or globe, turn the globe on its axis, and discover the figure described by the point when the globe makes a complete revolution.

3. Through how many degrees does the point move? Through how many degrees of longitude does a point on the earth's surface move in 24 hours?

Table

The earth turns through	
360° longitude in	24 hr. time
15° longitude in	1 hr. time
1° longitude in	4 min. time
15′ longitude in	1 min. time
1′ longitude in	4 sec. time

4. Since a point on the earth's surface moves through 360° in 24 hours, through how many degrees does it move in 1 hour?

Exercises

1. If your city is just passing under the sun, *i.e.* if it is just noon, has a point 15° west of you yet passed under the sun? When will it do so?

2. When you have noon, what time is it 15° west of you? What time 15° east of you?

3. When you have noon, what time is it 30° west of you? What time 30° east of you?

4. What time is it $7\frac{1}{2}°$ west of you when you have noon?

5. When it is 10 A.M. in your city, what time is it 10° west of you? 40° east of you? 90° east of you?

6. When it is 1 P.M. in your city, what time is it 60° west of you? 105° west of you? 90° east of you?

7. When it is noon at Chicago, what time is it at Boston?

WORK

87° 37′	16 × 4 min.		= 64 min.
71° 3′	34 × 4 sec. = 136 sec.	=	2 min. 16 sec.
16° 34′			66 min. 16 sec.

Hence it is 6 min. 16 sec. after 1 P.M. at Boston.

8. When it is noon at Chicago, find the time at San Francisco.

9. When it is 10 A.M. at Boston, find the time at Paris.

10. When it is midnight at Chicago, what time is it at Manila?

11. A boat race occurs on the Thames at 5 P.M. The result is known in Philadelphia at 1 P.M. Account for this.

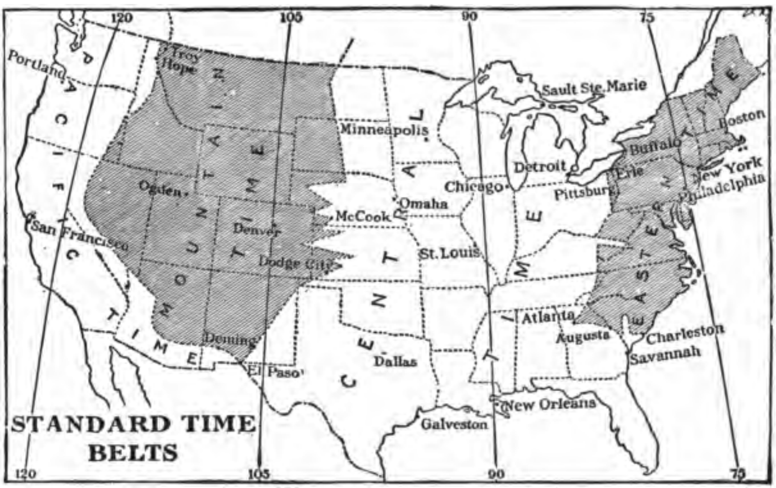
12. When it is 5 A.M., October 22, at Manila, what time and what date is it at Boston?

17. STANDARD TIME

The time considered above is called **local time.** The fact that all places east of a given point have later local time, and all places west of it earlier local time, caused confusion in railroad travel. Hence, in 1883, the railroads of the United States adopted for their own convenience what is

known as **standard time.** At any particular place the time is reckoned as the local time of some specified meridian rather than the meridian through that place. The meridians used are 75°, 90°, 105°, and 120° west of Greenwich. Thus places within about $7\frac{1}{2}°$ of any of these meridians use the local time of that meridian.

The line of division between standard meridians is not a straight line midway between them, but depends upon important railroad terminals. It was fixed by the roads. This is shown in the following map:



Exercises

1. From what meridian do you reckon time in your town or city? Has the meridian (a north and south line) through your place passed under the sun before or after this meridian does?

2. Do you live east or west of the meridian from which you reckon time?

3. How many degrees is your meridian from this meridian? Compare your local time with the standard time.

4. Suppose one lives 7° east of the 90th meridian. Is standard time fast or slow? How much?

The local time of the 75th meridian is **Eastern Time**, of the 90th, **Central Time**, of the 105th, **Mountain Time**, of the 120th, **Pacific Time**.

5. Which time do you use?

6. When it is 8 A.M. Eastern Time, what is it Pacific Time?

7. When it is 4 P.M. Mountain Time, what is it Eastern Time?

8. At noon by Central Time, what is it by each of the others?

9. If one is traveling from New York to Chicago, how will he change his watch as he changes into Central Time?

10. When it is noon in Chicago, what time is it in New York? Charleston? New Orleans?

11. When it is 3 P.M. in San Francisco, what time is it in St. Louis? Detroit? Boston?

12. Make and solve other problems about standard time.

18. FOREIGN MONEY

The money used by people of other nations is different from that used in the United States. Americans who buy goods in a foreign country, or who travel in a foreign country, must know the value of the different denominations of money used in that country.

The following tables give the denominations of money used in some countries of Europe, together with their equivalents in the money of the United States.

Table of English Money

1 pound (£)	= 20 shillings (*s.*)	= $4.8665
1 shilling	= 12 pence (*d.*)	= $0.243+

We think ordinarily of the pound as *about* $5, of the shilling as *about* 25 cents, and of the penny as *about* 2 cents.

Canada uses the same denominations as the United States, although the coins have not quite the same values as those of our money.

Table of French Money

1 franc (fr.) = 100 centimes (c.) = $0.193

We think ordinarily of the franc as *about* 20 cents.

This system is used also by some other countries of Europe, as Belgium and Switzerland. Italy uses the *lira*, Spain the *peseta*, and Greece the *drachma*, all of which have the same value as the franc.

Table of German Money

1 mark (M.) = 100 pfennigs (pf.) = $0.238

We think ordinarily of the mark as *about* 25 cents, and of 4 pf. as 1 cent.

Find the values in dollars and cents of the following:

1. £140	6. 1250 fr.	11. 100 M.
2. £500	7. 50 c.	12. 546 M.
3. £8 12*s.*	8. 5 fr. 10 c.	13. 60 pf.
4. £5 8*s.* 6*d.*	9. 120 fr. 50 c.	14. 20 M. 50 pf.
5. 10*s.* 8*d.*	10. 34 fr. 15 c.	15. 1642 M. 25 pf.

16. Marshall Field & Co., Chicago, buy a bill of goods amounting to £ 1250 15*s*. How much American money do they pay in settlement of the bill?

17. John Wanamaker, New York, buys goods in Paris amounting to 1054 fr. 50 c. How much American money does it take to pay the bill?

18. An American firm buys toys made in Germany amounting to 2544 M. 25 pf. How much American money is paid in settlement of the bill?

19. Make and solve other problems about the purchase of goods in foreign countries by merchants in your own city.

20. Visit some merchant who deals in imported goods, and get from him statements of his bills of goods that he has actually bought. Let the class compute the cost to him, in American money, of these goods.

NOTE. — Part of the class may assume that they are English firms, French firms, etc., and the rest that they are American merchants buying goods from these firms.

A Summer Trip Abroad

One summer Robert Ellis spent his vacation on a trip with his parents to Europe. They went to New York, and from there sailed on a British steamer to Liverpool.

1. During the voyage his father paid out £ 4 8*s*. for fees on the ship. How many dollars was this?

2. When they landed at Liverpool, they took a train directly to London. The three tickets cost £ 3 5*s*. How many dollars was this?

3. They remained 10 days in London, stopping at a hotel that cost them £ 2 10*s*. a day. Find the amount of the hotel bill in American money.

4. They spent £1 12*s*. 6*d*. for cab hire during their stay in London. How much was that in American money?

5. When they left London, they went first to Paris, France. They remained there 12 days. In Paris they paid out 30 fr. 25 c. for cab hire, and 480 fr. for hotel bill. How much was this in our money?

6. While in Paris, Robert's mother went shopping, and spent 325 fr. 50 c. for different things. To how much American money was this equivalent?

7. From Paris they went to Germany, stopping at a number of places. They spent four days in Berlin. There their average daily expenses for hotel bills, etc., amounted to 38 M. 50 pf. Find the amount of this for the four days. How much was it in United States money?

8. Before leaving Berlin, Robert's father bought him some drawing instruments that cost in all 23 M. 75 pf. Find their cost in our money.

9. Their other expenses while in Germany amounted to 385 M. 20 pf. How much was this in our money?

10. They spent a few days in Switzerland, and then went to Italy. At Naples Robert's father bought a painting for 492 lira. He had it packed for 6 lira, and shipped it home. What was the entire cost of the painting in American money?

11. They sailed from Italy to New York on an Italian steamship. The tickets for the three cost 1100 lira. Find their cost in American money.

12. When they started from New York to Liverpool, Robert set his watch according to New York time. When they reached London, he found that his watch did not agree with the clocks there. Was his watch ahead or behind, and how much?

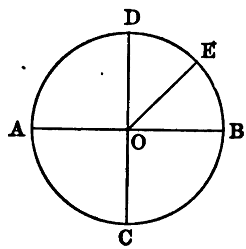
II. MENSURATION

19. THE CIRCLE

1. A curved line, all points of which are equally distant from a point within, called the *center*, is a **circle.**

Sometimes the surface inclosed by the curved line is called the *circle*, and the line itself is called the *circumference.*

2. The distance from the center to the curve is the **radius.** In the figure how many radii are drawn? Name them.



3. Any straight line through the *center* terminating in the *circumference* is a **diameter.** Is a diameter shown in the figure? Which lines are diameters?

4. Into how many equal parts does a diameter divide a circle? *One half* of a circle is called a **semicircle.**

5. Into how many equal parts do two perpendicular diameters divide the circumference? Such parts are called **quadrants.**

6. Any part of a circumference is called an **arc.** The figure bounded by two radii and an arc is a **sector.** Name some sectors in the figure.

7. For convenience in measuring arcs, every circumference, whether large or small, is divided into 360 equal parts, called degrees (360°). How many degrees in a semicircumference? In a quadrant?

8. Each degree is divided into 60 minutes (60′), and each minute into 60 seconds (60″). (See p. 268.)

20. THE RATIO OF CIRCUMFERENCE TO DIAMETER

1. Cut circles of different sizes from stiff cardboard, or bring to the class several circular objects : plates, rings, covers, wheels, or coins. Measure accurately the diameter and circumference of each.

NOTE. — To get an accurate measurement of a circumference, two pupils can work together to an advantage. Take two rulers, one standing edgewise on the other as a guide for the circular object. Mark a point on the circumference and roll through one complete revolution, noticing the distance passed over on the bottom ruler, and holding against the second ruler to get a straight path.

2. *Make a table of your measurements as follows :*

TABLE

	CIRCUMFERENCE	DIAMETER	RATIO OF CIRCUMFERENCE TO DIAMETER
1.	$25\frac{1}{8}$ in.	8 in.	3.140625
2.			
3.			

3. In each case divide the circumference by the diameter, carrying the result to several decimal places.

4. Take all the results that are about alike and find the average.

5. If you have measured and divided accurately, the quotient will be 3.1416 *nearly*. What does this show?

6. If the diameter of a circle is 10 ft., what is the circumference?

> 3.1416 *is the ratio of the circumference to the diameter. It is represented by the Greek letter* π *(pī).*

7. If $D =$ diameter, $R =$ radius, $C =$ circumference, read the following:

$$C = \pi \times D;\ D = C \div \pi, \text{ or } D = \frac{C}{\pi};\ C = 2\,R\,\pi, \text{ or } 2\,\pi R.$$

Find the diameter or circumference or radius. Forecast the result.

8. $D = 20$ ft.; $C = ?$

9. $C = 205$ ft.; $D = ?$

10. $R = 90$ ft.; $C = ?$

11. $C = 4$ ft. 8 in.; $D = ?$

12. $D = 16\frac{1}{4}$ in.; $C = ?$

13. $C = 2$ yd. $1\frac{1}{2}$ ft.; $R = ?$

14. The diameter of a circle is 15 in. Find the circumference.

15. The circumference of a circle is 25 ft. Find to hundredths of a foot the length of the diameter.

Problems with Circles

1. Florence made a circular button bag. She cut a paper pattern, using a pair of compasses. Setting the legs of the compasses 5 in. apart, what was the diameter of the pattern?

2. For finishing, she turned down a half-inch hem, all around. What was the diameter of the finished bag?

3. How much narrow ribbon would she have needed for a binding if she had bound the bag instead of hemming it?

4. If she uses 5 rings on the edge of the bag, as shown in the picture, *about* how far apart are they?

5. Later she made this same style of bag as a collar bag for her father's suit case. She made the radius of her circle 10 inches. How many rings did she need, sewing them *about* 4″ apart?

6. How much lace is needed for the edge of a 27″ circular centerpiece, allowing 23″ for fullness?

(A 27″ centerpiece is one 27 in. in diameter.)

7. How much will the lace cost at 36¢ a yard?

8. If your bicycle wheel is 28 in. in diameter, how far around it? How far does it go when it turns once (makes a *revolution*)?

9. How many revolutions will it make in going a mile?

10. A circus ring is 135 ft. across. How far is it around? Except for three entrances, each 12 ft. wide, the seats extend entirely around it. How long is the highest row of seats? Allowing 24 in. to each person, how many seats in the row?

11. Some children are making a circular flower bed 10 ft. in diameter. It is to have tulips planted in three rings, one 2 ft. from the center, another 3 ft., and another 4 ft. They have 100 bulbs. They want to plant the bulbs 8 in. apart and have 15 for a group in the center. Have they enough for the bed?

12. A birthday cake 9 in. in diameter is to be divided into 15 equal slices. How thick must each slice be cut?

13. How much bead fringe must I buy to go around the border of a circular lamp shade that is 16 in. in diameter?

14. The diameter of the top of a barrel is $17\frac{1}{2}$ in. How long must the strip of sheet iron be cut to make the top of the barrel hoop, allowing 2 in. for lap?

15. A wagon wheel is 46 in. in diameter. How long must the iron bar be cut to make the tire, allowing $\frac{1}{2}$ in. for lap in welding?

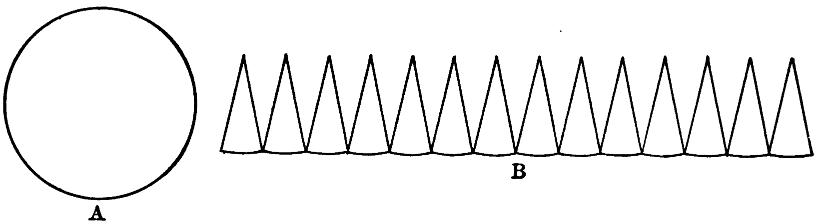
16. An overshot wheel is 8 ft. in diameter. It is to have 28 buckets on its rim. How far apart must the buckets be placed?

17. A semicircular arch 12 ft. in diameter is to be built of 21 equal stones. To what width must the circular edge of each stone be cut?

21. THE AREA OF A CIRCLE

By the **area of a circle** is meant the area of the surface inclosed by the circle.

1. Draw a circle on stiff cardboard and cut it out. Draw two diameters at right angles. Divide each part of the cardboard thus formed into two or more equal parts by cutting along radii.



2. Arrange the parts as in *B*. What figures that you have studied do they most resemble? If these *sectors* were triangles, how would you find the area of each? Of all? Compare the sum of the bases of all the sectors with the circumference of the circle.

3. If these sectors were triangles, the area of all would equal that of one triangle whose altitude is the radius and the base equal to what?

4. While the sectors are not triangles, yet the smaller they are made, the more like triangles they seem, and it is

proved in geometry that the area of a circle is actually the same as you have found by supposing the sectors to be triangles; that is,

> *The area of a circle is the same as that of a triangle having a base equal to the circumference and an altitude equal to the radius.*

5. Cut a circle out of thick cardboard or sheet metal. Out of the same material cut a triangle whose base is equal to the circumference and altitude equal to the radius of the circle. Weigh them carefully, and compare the weights.

State a rule for computing the area of any circle.

NOTE. — In the following problems, R stands for radius, D for diameter, and C for circumference.

6. Find the area of a circle whose diameter is 10 ft.

SOLUTION: $C = 3.1416 \times 10$ ft. $= 31.416$ ft.
$R = \frac{1}{2}$ of 10 ft. $= 5$ ft.
5×31.416 sq. ft. $= 157.080$ sq. ft.
$\frac{1}{2}$ of 157.080 sq. ft. $= 78.54$ sq. ft.

NOTE. — We must use C and R as abstract numbers representing the number of units in performing the computation. The product will then be an abstract number showing the number of units in the area.

Find the area of circles:

7. $R = 6$ ft., $C =$ ——.
9. $C = 100$ ft., $D =$ ——.
8. $D = 12$ ft., $C =$ ——.
10. $C = 50$ ft., $R =$ ——.

Since area $= \dfrac{C \times R}{2}$ and $C = 2 \times R \times \pi$, we may multiply by $2 \times \pi \times R$ instead of C. Then

$$\text{area} = \frac{2 \times \pi \times R \times R}{2} = \pi R^2.$$

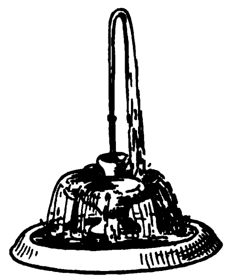
This is the *formula* generally used when the radius or diameter is known.

11. Cut from thick cardboard or sheet metal a circle whose radius is 4 in. Cut from the same material a square whose side is 4 in. Weigh both, and see if the weight of the circle is not approximately 3.1416 times the weight of the square. Show that the result agrees with the rule that the area of a circle equals πR^2.

Find the areas of circles:

12. $R = 15$ ft.	15. $R = 22\frac{1}{2}$ rd.	18. $C = 108$ ft.
13. $D = 42$ ft.	16. $R = 16\frac{1}{2}$ rd.	19. $C = 1$ mi.
14. $C = 400$ in.	17. $D = 80$ yd.	20. $D = 38$ rd.

Problems involving Circles



1. The circular basin of a fountain is 20 feet in diameter. How many square feet in the bottom? What will it cost to cement the bottom at \$1.75 per square yard?

2. Around the fountain basin is a 4-ft. paved walk. How many square feet of paving are needed?

SUGGESTION. — You are to find the difference between the circles. What is the radius of each?

3. The earth is nearly 8000 miles in diameter. What is the approximate length of the equator?

4. From the result of Exercise 3, find at what velocity a person who is at the equator revolves through space.

5. A boy gets $\frac{1}{2}$¢ a square yard for cleaning the snow from a circular skating pond 300 ft. in diameter. How much does he earn?

6. What is the diameter of a circular sailor hat that requires 1 yd. 14 in. of velvet to bind it?

7. What part of a piece of buckram 10 in. square is left after cutting a circular piece 10 in. in diameter from it?

8. How many yards of lace will be required to trim 1 doz. plate doilies 6 in. in diameter, adding $\frac{1}{5}$ extra for fulling?

9. How many square feet of asbestos are needed for a table mat for a 54-in. circular table?

10. A square foot of copper, gauge 19, weighs 1 lb. 14 oz. What is the weight of a circular tray 16 in. across made from this material?

11. A hose will throw water 40 yd. in every direction from the hydrant. Over what area will it sprinkle?

12. How many square feet of ice can be cut from a pond 168 ft. in diameter?

13. If a circular piece of boiler plate 34 in. in diameter is cut from a sheet 3 ft. square, how many square inches of the metal are wasted?

14. A belt runs over a pulley 28 in. in diameter, revolving at a speed of 216 revolutions a minute. How far does a point on the belt travel in a minute?

15. If you can ride around 20 times on a merry-go-round for 5¢, and your path is 20 ft. in diameter, how far do you ride for 5¢?

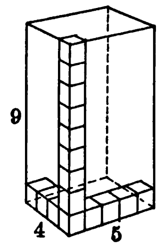
22. RECTANGULAR PRISMS AND THEIR VOLUMES

1. Describe a **right prism.** (See p. 220.)

2. Prisms are named from their bases: *square, rectangular, triangular, hexagonal,* etc. Name some objects that are square prisms; some that are rectangular.

3. Is a square prism also rectangular? What kind of prism is a cube?

4. To find the volume of a rectangular prism whose dimensions are 4 in., 5 in., and 9 in.



(1) First find the number of cubic inches in one row or a square prism 1 in. by 1 in. by 9 in.

(2) Then find the number of such rows in one tier or layer 4 in. wide.

(3) Then find the number of such layers in the prism, and explain the statement 5 × 4 × 9 cu. in. = —— cu. in.

5. How many inch cubes may be put into a box 10 in. long, 8 in. wide, and 5 in. deep?

6. A trunk measures 3 ft. by 20 in. by 18 in. Find its volume. Why multiply by 36 instead of 3?

Find the volumes of rectangular prisms of these dimensions:

	Length	Width	Height		Length	Breadth	Depth
7.	16 ft.	10 ft.	8 ft.	**10.**	$42\frac{1}{2}$ ft.	20 ft.	$13\frac{1}{3}$ ft.
8.	1 yd.	2 ft.	9 in.	**11.**	$12\frac{1}{2}$ yd.	10 ft.	16 in.
9.	24 ft.	16 ft.	4 yd.	**12.**	$20\frac{1}{2}$ ft.	$17\frac{1}{3}$ ft.	6 in.

13. If the base of a rectangular prism contains 30 sq. in., how many cubic inches in the bottom layer? If the prism is 10 in. high, how many cubic inches in the prism?

Statement.—10 × 30 cubic inches = —— cubic inches.

14. The floor of a cellar contains 36 sq. yd. If the cellar is 8 ft. deep, find its cubical contents.

15. A square prism is 16 in. wide and 4 ft. long. Find the volume.

16. A 5-in. cube is cut from the corner of a 20-in. cube. What part remains?

17. A cellar is 40 ft. long, 30 ft. wide, and $7\frac{1}{2}$ ft. deep. How many cubic feet of earth were removed in its construction? How many loads (cubic yards)?

18. A freight car 36 ft. long and $8\frac{1}{2}$ ft. wide is loaded 5 ft. deep with wheat. How many bushels does it contain if 1 cu. ft. = 0.8 bu.?

19. How many tons of ice can be packed in an ice house $40' \times 24' \times 16'$, allowing $2'$ on each side and above and below for sawdust? (1 cu. ft. of ice weighs $56\frac{1}{4}$ lb.)

20. An ice pond is 260 ft. wide and 426 ft. long. If the ice is frozen to a depth of 14 in, how many tons can be cut from the pond?

21. My ice box will hold a piece of ice $32'' \times 24'' \times 14''$. How much will such a piece weigh?

22. A coal bin 12 ft. long and 8 ft. wide is filled to the depth of 5 ft. Allowing 80 lb. to a cubic foot, how many tons are in the bin?

23. A coal car $7\frac{1}{2}$ ft. wide and 30 ft. long is loaded to an average depth of 4 ft. How many tons of coal on the car?

23. THE SURFACES OF RECTANGULAR PRISMS

1. How many surfaces has a rectangular prism?

2. What name is given to a rectangular prism with equal faces?

3. Find the entire surface of a 5-in. cube.

Explain the statement: $6 \times 5 \times 5$ sq. in. = —— sq. in.

4. The entire surface of a cube is 150 sq. in. How long is it?

Find the entire surface of: 5. A 9-in. cube.

6. A cube 10 in. long. 7. A 16-in. cube.

How long a cube has

8. An entire surface of 384 sq. in.?

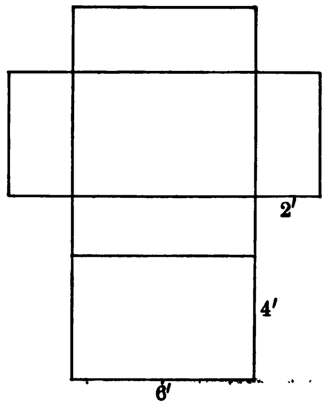
9. An entire surface of 600 sq. ft.?

10. An entire surface of 294 sq. in.?

11. Compare the ends of a square prism with each other.

12. Compare its four sides.

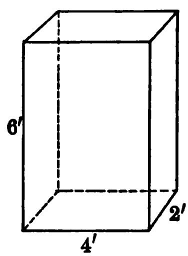
13. Find the entire surface of a square prism 8 in. long and 3 in. wide. (What is the area of each square? How many? Of each rectangle? How many?)



14. Compare the opposite faces of *any* rectangular prism.

15. Give the dimensions in inches of each face in the figure at the right. What is its entire surface in square inches?

Find the entire surfaces of prisms:



16. 10 in. by 6 in. by 4 in.

17. 12 ft. long, 9 ft. wide, 6 ft. high.

18. 20 ft. long, 4 ft. wide, 8 ft. high.

19. How many square yards of lath does it take for the walls and ceiling of a room 14 ft. wide, 18 ft. long, and 9 ft. high?

20. How many square feet of sheathing does it take for the two sides and two ends of a box car 8 ft. wide, 8 ft. high, and 34 ft. long?

21. How many square feet of lumber 1 in. thick does it take to make a box with a lid, the length being 46 in., width 32 in., and depth 20 in., outside measurement?

22. An open rectangular tank 16 in. wide, 24 in. long, and 20 in. deep is to be lined with copper. How many square inches of sheet copper does it take, not allowing for overlapping?

23. How many square yards of goods does it take to line a dress box 42 in. long, 26 in. wide, and 18 in. high?

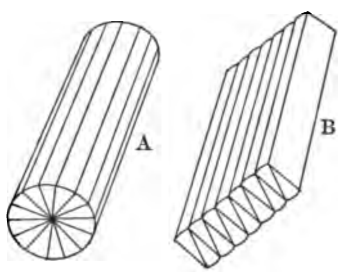
24. How many 6-inch cubes will this box hold?

24. THE CYLINDER

A round object such as a log, a round iron rod, etc., is a **cylinder.** The two ends which are equal circles in parallel planes are called the **bases.**

1. Mention some other common objects that are cylinders.

2. If a cylinder is cut as in Fig. A and arranged as in Fig. B, what does it nearly become?



3. If the circular end is the base, it has been changed into one nearly like a rectangle. Has the area of the base changed in B? Has the length of the cylinder changed?

4. In B, if the base were a rectangle and its area known, how would its volume be found?

5. If the radius of A is 1 in., what is the area of its base or the base of B? If the length of B is 8 in., what is the volume?

6. The volume that we have found by considering that the cylinder is nearly a prism with an equal base and the same height is the volume that is found by geometry to be the true one, viz.:

The volume of a cylinder is the same as that of a prism having a base of the same area and having the same height.

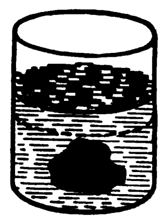
7. Make from some convenient material a cylinder and a prism of the same height, the base of the prism being a rectangle whose base is half of the circumference and whose altitude is the radius of the base of the cylinder. Weigh them, and compare the weights.

8. Find the volume of a cylinder whose radius is 8 in. and whose height is 15 in.

Explain the statement: $(15 \times 3.1416 \times 8 \times 8)$ cu. in. = —— cu. in.

9. A cylindrical pail 6 in. in diameter inside and 12 in. deep contains how many cubic inches?

Explain the statement: $(12 \times 3.1416 \times 3 \times 3)$ cu. in. = —— cu. in.



10. The volume of an irregular shaped object, as a stone, may be found by submerging it in a cylindrical vessel of water. When a stone is submerged in a cylindrical vessel 8 in. in diameter, the water rises 4 in. Find the volume of the stone.

11. Find the volume of 5 irregular bodies by the method of Problem 10.

12. How much will a 2-in. (diameter) iron bar 10 ft. long weigh? (1 cu. ft. weighs 480 lb.)

13. A cylindrical tank for supplying water to a locomotive is 24 ft. in diameter and 16 ft. high. How many gallons will it hold? (1 cu. ft. = $7\frac{1}{2}$ gal.)

14. The tank on a sprinkling wagon is 10 ft. long and 4 ft. in diameter. How many gallons of water does it hold?

15. The cylinder of a steam pump is 14 in. in diameter, and the stroke of the piston is 18 in. How many gallons of water does it discharge at each stroke? If it makes 40 strokes a minute, how many gallons does it discharge in a minute? In an hour?

16. A gas tank is 250 ft. in diameter and 40 ft. high. How many cubic feet of gas does it hold?

17. Water runs through a sewer pipe 3 ft. in diameter at a speed of 60 ft. a minute. How many gallons does the sewer discharge in a minute?

18. A granite lawn-roller is 3 ft. long and 20 in. in diameter. What is its weight, the weight of granite being 165 lb. to the cubic foot?

The Surface of a Cylinder

1. In form, the ends or bases of a cylinder are equal to what? The rest of the surface is called the **convex surface.**

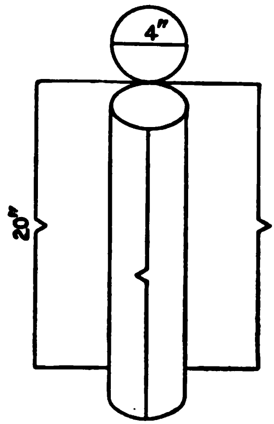
2. Suppose the diameter of a cylinder to be 4 in. Its circumference equals what?

3. The circumference of a cylinder is 8 in.; its diameter is what?

4. A cylinder is 4 in. in diameter. Find the area of its two ends or bases.

5. Roll an oblong paper to form a cylinder. Give the length and circumference of the cylinder thus made.

6. Unroll the paper and give the dimensions of a rectangle equivalent to the convex surface of the cylinder. Explain the diagram at the right.



7. The convex surface is equal to $C \times L$ square units, where C = circumference and L = length. Why is this?

8. A cylinder is 25 in. long, 4 in. in diameter. Find its convex surface. Explain: $(3.1416 \times 4) \times 25$ sq. in. = —— sq. in.

9. A cylinder is 20 in. long, 5 in. in diameter. Find the area of the *entire* surface, or of the convex surface plus the surface of the ends.

10. A tin can is to be made 5 in. high and with a diameter of 4 in. Allowing $\frac{1}{4}$ in. for a seam, find the size of the rectangular sheet of tin from which the convex surface is to be made.

11. What is the area of the entire cooling surface of an ice-cream freezer 6 in. in diameter and 9 in. high?

12. How many square inches of sheet iron does it take to make a joint of stove pipe 6 in. in diameter and 18 in. long? How many to make a dozen joints?

13. How much does it cost to paint a standpipe 16 ft. in diameter and 120 ft. high at 3¢ a square foot?

14. How many bricks does it take to line a well 30 ft. deep and 4 ft. in diameter, if each brick is placed with the surface 2 in. by 8 in. exposed?

III. PROPORTION

25. MEANING OF PROPORTION

When two ratios are equal they form a **proportion.**

Thus, the ratio of the cost of two similar pieces of cloth is the same as the ratio of their length. That is, if 10 yards cost \$4, 15 yards will cost \$6, or

$\frac{\$4}{\$6}=\frac{10\text{ yd.}}{15\text{ yd.}}$. It is read \$4 is to \$6 as 10 yd. is to 15 yd.

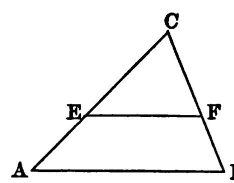
When quantities are in proportion they are said to be **proportional.**

26. SIMILAR FIGURES

1. Similar figures have exactly the same shape. That is, *their corresponding sides are proportional.*

2. Make two similar triangles. Are their corresponding angles equal?

3. Draw a triangle, ABC, as in the figure. Draw EF parallel to AB. By the use of a protractor compare the angles FEC and BAC. Also angles CFE and CBA.



4. Are triangles ABC and EFC similar; that is, do they have the same shape?

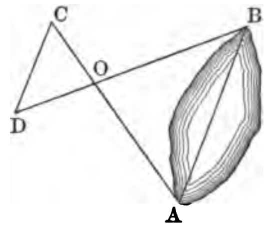
5. Make a triangle in which AC is 6 inches and CB 3 inches. Mark off CE equal to 4 inches and CF 2 inches. Are the two triangles similar?

6. What is the ratio of CA to CE? Of CB to CF?

7. Cut similar triangles from cardboard. Measure their sides and prove the statement on the next page.

> *In similar triangles the ratios of the corresponding sides are equal, and the ratio of any two sides of one is equal to the ratio of the corresponding sides of the other.*

8. Inaccessible distances may be found by the principle of similar triangles. Suppose we are to find the distance AB across a small lake. By measuring from A to C, and from B through O to D, making the ratio of OC to OA the same as of OD to OB, we have similar triangles.



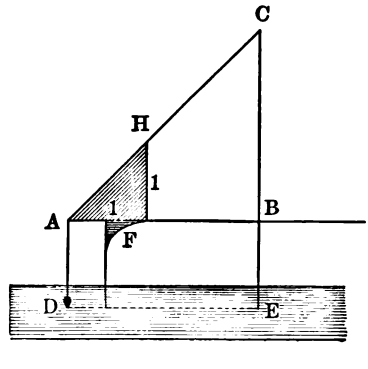
If $OC = \frac{1}{2}\ OA$, and CD measures 20 rods, what is AB?

NOTE.—Make OC any convenient part of OA, and then OD the same part of OB. In the figure $OC = \frac{1}{2}\ OA$, and $OD = \frac{1}{2}\ OB$.

9. I wish to find the distance AB. AC is 15 rods and OC is 5. I measure from B through O to D. If BO is 8 rods, what shall I make OD? Why?

I find DC to be $7\frac{1}{2}$ rods. How far from A to B?

10. A boy, wishing to find the height of a pole CE, made a piece of apparatus which he called his "surveying instrument." It consisted of a right triangle whose two legs were equal. It stood 3 feet from the ground. He moved it along until the point C could just be seen along the hypotenuse of the triangle



when the base of the triangle AF was parallel to the ground. A line with a weight (a plumb line) hung from A. If DE was 27 feet, how high was the pole? (Triangles AFH and ABC are similar. Why?)

11. If HF had been twice AF, and DE had been 40 feet, what would CE have been?

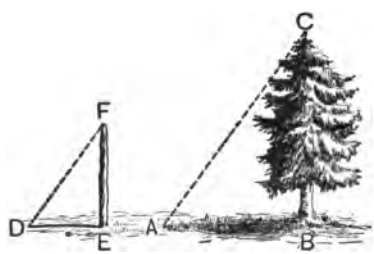
12. Make such an instrument, and find the height of trees, telegraph poles, etc.

13. Make one with the triangle having one leg twice the other, and measure the same heights. Do your results check?

14. Two triangles are similar. One has sides 4, 5, and 7 inches, respectively. The long side of the other is 21 inches. What are the other sides? What if the short side of the latter were 2 inches?

15. In this way measure distances on your school lot.

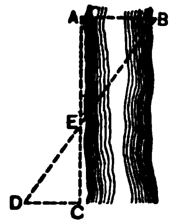
16. Thales, a Greek philosopher and mathematician, about 600 B.C., is said to have amazed the Egyptians by measuring the heights of the pyramids by the length of the shadows which they cast. Measure the height of a tree as follows:



Hold a stick, whose length is known, in a vertical position, and mark the end of its shadow. Measure the length of the shadow of the stick and also of the shadow of the tree. From these measurements compute the height of the tree.

17. When a vertical rod 6 feet high casts a shadow 9 feet long, a tree casts a shadow of 150 feet. How high is the tree?

18. The distance across a stream may be estimated as follows: Find two points A and B, directly opposite each other on the two banks of the stream. Measure off a line AC, several yards long, along the bank (at right angles to AB). At C measure off a distance CD at right angles to AC. By sighting from D to B, locate a stake at a point E of AC in line with D and B. Get the lengths of AE and EC. Triangles ABE and DEC are similar right triangles. Why? From the measurements of AE, EC, and CD, the length of AB may be computed.



Suppose that AE is 100 yd., EC 80 yd., and CD 40 yd. How wide is the stream?

With a tape line, measure some distance in the neighborhood in this way.

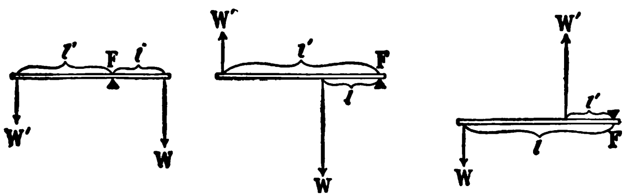
19. Sailors and others use the following method of estimating the distance DA to an object A. With the left eye closed, the finger is pointed, at arm's length, towards A. Then the right eye is closed and the left eye opened, when the object appears to have moved through the distance AB. The distance AB, being transverse to the line of sight, is estimated. The distance CD between the eyes is about one tenth of the distance from the eye to the end of the outstretched finger. If AB is 500 ft., what is the distance of A? If AB is apparently 12 ft.? Estimate in this way the distances to objects about you.

20. If the distance from the eyes to the end is 8 times the distance between the eyes, how would you find the distance to an object by the method of Problem 19?

27. LEVERS

In every simple machine there are two forces involved: the **resistance**, or force to be overcome, and the **effort**, or force necessary to overcome the resistance. The relation between the resistance and effort depends upon the nature of the machine and upon the dimensions of its parts.

In the **lever**, the resistance W and the effort W' are applied at different points of a rigid bar which revolves freely about a point of support called the **fulcrum.** There are three classes of the lever as shown in the figures. The distances l and l'



from the fulcrum to the points of the lever where the resistance and effort, respectively, are applied are called the *arms* of the lever. It is shown that

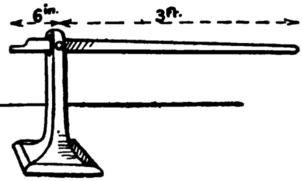
The resistance and effort are inversely proportional to their distances from the fulcrum; that is, $\frac{W}{W'} = \frac{l'}{l}$.

Thus, if two children are playing at "see-saw," the *heavier* child must sit *nearer* the point of support. If John weighs 60 lb. and Harry 80 lb., John weighs $\frac{3}{4}$ as much as Harry; hence he must sit $\frac{4}{3}$, or $1\frac{1}{3}$, times as far from the point of support.

The beam balance, common steelyard, scissors, pincers, crowbar, etc., are familiar examples of levers.

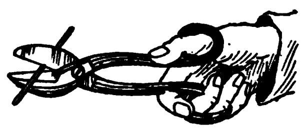
Problems

1. With a "jack," as shown in the figure, a man wishes to raise 1200 lb. How much weight must he use on the long end of the lever?



SOLUTION: Since the long arm is 6 times the short one, but $\frac{1}{6}$ of the weight is needed. Hence, $\frac{1}{6}$ of 1200 lb., or 200 lb., is needed.

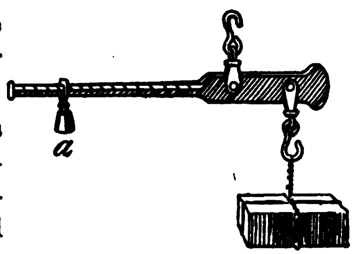
2. The figure shows heavy shears used in cutting sheet metal or heavy wire. When the wire is 2″ from the fulcrum (the point where the two parts are joined) and the hand 8″ from this point, what is the cutting force of a squeeze of 10 lb.?



3. Will the wire be more easily cut if it is moved toward the tip of the blade or toward the fulcrum? Why?

4. For cutting heavy wire would you choose shears with long or short arms (the handles grasped by the hand)? Why?

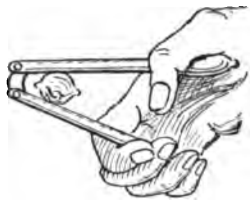
5. This figure shows a pair of steelyards, used in weighing. Would a heavy package to be weighed require the weight *a* to be far from or near to the fulcrum?



6. Suppose the weight *a* is 1 lb. How far from the fulcrum must it be placed to balance a weight of 4 lb. placed 4 in. from the fulcrum?

7. Suppose *a* weighs $2\frac{1}{2}$ lbs., 10 in. from the fulcrum. What weight 2 in. from the fulcrum will it balance?

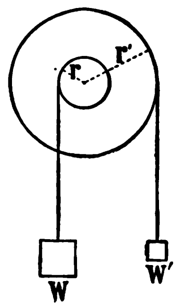
8. In this nut-cracker the hand is 6″ from the fulcrum and the nut but 1″. A pressure of 5 lb. with the hands gives what pressure upon the nut?



9. Two boys are carrying a weight of 150 lb. suspended upon a 9-foot pole between them. The weight is 6 ft. from one boy and 3 ft. from the other. How much does each boy carry?

28. THE WHEEL AND AXLE

The **wheel and axle** consists of a wheel fastened rigidly to a cylinder (axle) so that both turn together about the same axis. The power, or effort, is applied to the rim of the wheel. The weight, or resistance, is applied to the cylinder by means of a rope or cable.

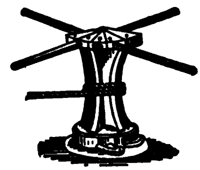


It is found that this simple machine gives the same advantage as the lever. That is, if the radius of the wheel is two times that of the axle, 1 lb. applied at the rim of the wheel will lift 2 lb. How much will 5 lb. lift? If the radius of the wheel is 5 times that of the axle, how much will 1 lb. lift? How much will 10 lb. lift? 20 lb.?

> *The resistance and effort are inversely proportional to the radii of the axle and the wheel, respectively.*

Problems

1. The crank to the windlass of a well is 18 in. long, and the cylinder upon which the rope is wound 9 in. in diameter. What force is needed to raise 48 pounds of water?



2. Two men working at a capstan walk in a circle 8 ft. in diameter, and each exerts a force of 60 lb. The diameter of the axle is 9 inches. What pull is exerted along the rope?

3. A horse walking in a circle 15 ft. in diameter moves a house by means of a capstan 18 in. in diameter. The horse exerts a pull of 1200 lb. What is the resistance of the house?

29. THE STATEMENT OF A PROPORTION

Sometimes a proportion is stated with colons (:) instead of in the fractional form. Thus the fact that "3 is to 4 as 9 is to 12" is stated "$3:4=9:12$."

In such a statement the first and last terms (3 and 12) are called the **extremes**. The other two (4 and 9) are called the **means**. Observe that in this proportion $3 \times 12 = 4 \times 9$.

Exercises

1. Are these ratios equal? Do they form a proportion?

$2:4=4:8$	$4:12=3:9$	$3:9=6:18$
$3:6=6:12$	$5:10=2:4$	$5:15=4:12$

2. In each of the above exercises, see whether the product of the means equals the product of the extremes.

In any proportion the product of the means equals the product of the extremes.

3. $x:6=9:12$. Find the value of x.

SUGGESTION: This means that $12 \times$ some number $= 6 \times 9$. Hence the number is $6 \times 9 \div 12 = 4\frac{1}{2}$. This is usually stated $12x = 54$; $x = \frac{1}{12}$ of $54 = 4\frac{1}{2}$.

Find the missing term:

4. $x : 5 = 12 : 4$.	**10.** $x : 5 = 80 : 10$.	**16.** $\frac{2}{3} : x = 8 : 9$.
5. $x : 9 = 3 : 4$.	**11.** $x : \frac{1}{2} = \frac{3}{4} : \frac{7}{8}$.	**17.** $2\frac{1}{2} : 5 = x : 9$.
6. $x : 12 = 2 : 18$.	**12.** $x : \frac{1}{4} = \frac{1}{3} : \frac{1}{9}$.	**18.** $\frac{1}{12} : x = 8 : 16$.
7. $x : 40 = 5 : 8$.	**13.** $x : \frac{2}{5} = \frac{2}{3} : \frac{1}{2}$.	**19.** $9\frac{1}{2} : 16\frac{1}{4} = 38 : x$.
8. $x : 75 = 6 : 150$.	**14.** $x : \frac{2}{3} = \frac{7}{8} : \frac{5}{12}$.	**20.** $7.25 : 0.5 = x : 3$.
9. $x : 3 = 90 : 6$.	**15.** $x : \frac{3}{4} = \frac{5}{6} : \frac{2}{3}$.	**21.** $x : 2\frac{1}{4} = 1\frac{1}{3} : 7$.

IV. PERCENTAGE

30. GENERAL PROBLEMS IN PERCENTAGE

Per cent means *per hundred*. Thus, if 5% of the words in the spelling lesson were missed, 5 *words of every hundred* were missed.

If a man makes 6% on an investment, he makes \$6 *on every* \$100 *invested*.

To find 25% of 600 is to find 0.25×600, which is 150.

Drill Table

Find:

1. 6% of 150.	**11.** 20% of 140.	**21.** $6\frac{1}{4}$% of 800.
2. 8% of 200.	**12.** 10% of 2000.	**22.** $8\frac{1}{2}$% of 900.
3. 7% of 500.	**13.** 12% of 3000.	**23.** $10\frac{1}{2}$% of 700.
4. 4% of 120.	**14.** 15% of 6000.	**24.** $9\frac{1}{2}$% of 600.
5. 15% of 300.	**15.** $7\frac{1}{2}$% of 800.	**25.** $7\frac{1}{2}$% of 420.
6. 15% of 400.	**16.** $3\frac{1}{2}$% of 600.	**26.** $5\frac{1}{2}$% of 320.
7. 12% of 500.	**17.** $8\frac{1}{3}$% of 300.	**27.** $8\frac{1}{3}$% of 630.
8. 9% of 120.	**18.** $5\frac{1}{2}$% of 1000.	**28.** $12\frac{1}{2}$% of 800.
9. 10% of 750.	**19.** $6\frac{1}{2}$% of 1200.	**29.** 20% of 750.
10. 15% of 150.	**20.** $7\frac{1}{2}$% of 2000.	**30.** 20% of 950.

Problems in Percentage

1. My gas bill for August is \$4.60. I can save 10 % by paying it before September 10. How much will my gas bill for August cost me, if I pay the bill before September 10?

2. George bought a \$35 canoe. By paying cash he got a discount of 10%. How much did he save by paying cash? What was the cost of the canoe, by paying cash?

3. If a man's salary was \$64 a week, and then was increased $12\frac{1}{2}$ %, how much was it after the increase?

4. A grocer buys butter at wholesale, at 28¢ a pound, and retails it at a profit of 25 %. What is his price?

5. If the living expenses of a family were \$80 a month two years ago, what are they now, if the cost of living has increased 20 %?

6. A firm advertises its blankets as 60 % wool and the rest cotton. How much of each in 5-lb. blankets?

7. A firm sold \$7000 worth of goods one week. If 5 % is lost in bad debts, how much will be collected?

8. At a special sale all shoes were sold 20 % below the regular price. Give the sale prices of shoes, the regular prices of which were: \$3.50; \$4; \$4.50; \$5; \$5.25; \$5.75.

9. By paying cash for goods I get a reduction of 5 % from the regular price. How much can I save by paying cash for goods, the regular price of which is \$250?

10. At the "August Sale," Revell's gave a reduction of 10 % from the regular prices. Find how much is saved by buying any of the following in August:

(*a*) \$65 dining-room table;
(*b*) \$185 parlor suite;
(*c*) \$56 set of dining-room chairs;
(*d*) \$24 rocker;
(*e*) \$42 sofa;
(*f*) \$98 bed-room suite;
(*g*) \$36 library table;
(*h*) \$24 settle.

11. A boy had \$80 in the bank. He withdrew 40 % of it to buy a canoe. How much remained?

12. The flooring for a hall cost \$250. The cost of labor to lay and dress it added 60 % more. Find the whole cost.

13. A farmer raised 1500 bu. of wheat one year. If fertilizing will increase the crop 40 %, how much will he then raise?

14. A salesman received $7\frac{1}{2}$ % of his sales as his pay. One month he sold \$6200 worth of goods. How much did he earn?

15. A man's house and lot cost him \$11,500. He has paid 70 % of it. How much remains unpaid?

16. A man delivered 2500 pounds of milk to the creamery. It tested 3.8 % butter fat. He was paid at the rate of 24¢ a pound for the butter fat. How much was this?

17. A dealer pays \$1 per bushel for onions. For how much a peck must he sell them to make a profit of 20 %?

18. For how much a $\frac{1}{2}$ peck must a dealer sell peaches costing \$3 per bushel to make a profit of 20 %?

19. If a grocer's average profits are 30 % of his sales, what are his weekly profits from sales of \$960 per week?

20. A dealer bought hats at \$24 per dozen, and marked them to sell at a profit of 50 %. At what price was each hat marked?

21. 45 % of fresh chestnuts is water. How much water in a bag of chestnuts weighing 28 lb.?

22. 78.3 % of potatoes is water. How much water in a bushel of potatoes weighing 60 lb.?

23. A crate of live chickens weighs 76 lb. The empty crate weighs 12 lb. If chickens shrink 29 % on being dressed, what will be the dressed weight of the lot?

24. A dealer pays 13¢ a pound for live chickens weighing 85 lb. The chickens are dressed, and sold at 22¢ a pound. Allowing 27 % for shrinkage in weight when dressed, find his profit on the lot.

25. I place 360 eggs in an incubator. If 75 % of them hatch, how many chickens shall I get?

26. At the same rate, how many chickens should I get from 840 eggs?

31. FRACTIONAL EQUIVALENTS OF PER CENT

1. 50 % of anything is what part of it?

$$50\,\% = 0.50 = \tfrac{50}{100} = \tfrac{1}{2}.$$

2. 25 % of anything is what part of it?

3. $12\frac{1}{2}\,\% = \frac{1}{2}$ of 25 %. Then $12\frac{1}{2}\,\%$ of anything is what part of it?

4. $6\frac{1}{4}\,\% = \frac{1}{2}$ of $12\frac{1}{2}\,\%$. Then $6\frac{1}{4}\,\%$ of anything is what part of it?

5. $3 \times 33\frac{1}{3}\,\% = 100\,\%$. Then $33\frac{1}{3}\,\%$ of anything is what part of it?

6. $66\frac{2}{3}\,\% = 2 \times 33\frac{1}{3}\,\%$. Then $66\frac{2}{3}\,\%$ of anything is what part of it?

7. $16\frac{2}{3}\,\% = \frac{1}{2}$ of $33\frac{1}{3}\,\%$. Express $16\frac{2}{3}\,\%$ as a fraction.

8. $5 \times 20\,\% = 100\,\%$. Express 20 % as a fraction.

9. $40\,\% = 2 \times 20\,\%$. Express 40 % as a fraction.

10. Express 60 % as a fraction; express 80 % in fractional form.

11. What part of anything is 25 % of it? 75 % of it?

A Table of Equivalents

To be memorized:

$\frac{1}{2} = 50\%$	$\frac{1}{3} = 33\frac{1}{3}\%$	$\frac{1}{5} = 20\%$	$\frac{4}{5} = 80\%$
$\frac{1}{4} = 25\%$	$\frac{2}{3} = 66\frac{2}{3}\%$	$\frac{2}{5} = 40\%$	$\frac{3}{4} = 75\%$
$\frac{1}{8} = 12\frac{1}{2}\%$	$\frac{1}{6} = 16\frac{2}{3}\%$	$\frac{3}{5} = 60\%$	$\frac{3}{8} = 37\frac{1}{2}\%$

Other Equivalents Less Important Are:

$\frac{5}{6} = 83\frac{1}{3}\%$	$\frac{5}{8} = 62\frac{1}{2}\%$	$\frac{1}{12} = 8\frac{1}{3}\%$
$\frac{1}{7} = 14\frac{2}{7}\%$	$\frac{7}{8} = 87\frac{1}{2}\%$	$\frac{1}{16} = 6\frac{1}{4}\%$

Drill Exercises

Find:

1. $12\frac{1}{2}\%$ of 96 lb.
2. 20% of 90 mi.
3. 50% of 2000 lb.
4. 25% of $60.
5. 75% of 400 yd.
6. $33\frac{1}{3}\%$ of 24 hr.
7. 30% of 200 A.
8. $66\frac{2}{3}\%$ of 300 pupils.
9. 25% of 1200 men.
10. $16\frac{2}{3}\%$ of $3000.
11. 50% of $6200.
12. 25% of $5200.
13. 75% of $1600.
14. $\frac{1}{2}\%$ of 6200.
15. $\frac{1}{4}\%$ of 5200.
16. $\frac{3}{4}\%$ of 1600.
17. $37\frac{1}{2}\%$ of $2400.
18. 60% of $1500.
19. $83\frac{1}{3}\%$ of $3660.
20. $14\frac{2}{7}\%$ of $2100.

Problems

1. A man borrows $1200, and at the end of the year repays it, together with 6% of it for the use of it. How much did he pay?

2. A man invests $9660 in a business enterprise that makes a profit of $8\frac{1}{3}\%$ of his investment. How much is his profit?

3. In some of the states in the southwest men pay as high as $16\frac{2}{3}\%$ for the use of borrowed money. At this rate, how much would a man pay for the use of $4290?

4. In Idaho, where land is increasing rapidly in value, a man paid $29.50 an acre for 40 acres of land that was not under cultivation. After holding it one year, he sold it at an increase of $66\frac{2}{3}\%$ of what it cost him. At what price did he sell it?

5. A farmer had a piece of wet land which produced a crop worth $25 an acre. By tiling it so as to drain the water off, he increased the value of the crop $83\frac{1}{3}\%$. What was the value of the crop per acre after the land was tiled?

6. A real estate dealer claimed that if I invested $6000 in some Western land, it would yield me a profit of $33\frac{1}{3}\%$ in two years. How much would the profit be?

7. Of the 840 bu. of apples taken from an orchard, $14\frac{2}{7}\%$ were defective. How many bushels were marketable?

8. If $12\frac{1}{2}\%$ of the apples put into a large storage house spoil, and 1240 bu. are stored, how many bushels spoil?

9. If $33\frac{1}{3}\%$ of the weight of meat is lost in shrinkage when cooked, what ought a ham weighing 12 lb. when raw to weigh when baked?

10. If $37\frac{1}{2}\%$ of a chicken is waste, how much edible matter in a chicken weighing 4 lb. 8 oz.?

11. If $62\frac{1}{2}\%$ of a certain soil is fine sand, how much sand in a load containing 32 cu. ft.?

12. In a load of soil containing 28 cu. ft., $14\frac{2}{7}\%$ is fine sand, 25% coarse sand, and $12\frac{1}{2}\%$ gravel. Find the number of cubic feet of each in the load.

13. The percentage of nutritive matter in butternuts is $12\frac{1}{2}\%$. Find the amount of food in 12 lb. of butternuts.

14. Alfalfa hay contains about $14\frac{2}{7}\%$ protein, and red clover hay about $12\frac{1}{2}\%$ protein. How much more protein (the real nutritive part of the hay) in a ton of alfalfa than in a ton of red clover?

32. A RELATION EXPRESSED AS PER CENT

A *per cent*, like any other fraction, expresses a relation between two quantities. Likewise, the relation between two numbers, or their *ratio*, is often expressed in terms of *per cent.* Thus, instead of saying that $\frac{1}{4}$ of the cost was gained in selling an article, one may say that 25% of the cost was gained.

Exercises and Problems

1. Compare 2 and 4; thus, 2 is $\frac{1}{2}$ or 50% of 4; 4 is 2 times or 200% of 2.

2. Compare 3 and 6; 2 and 8; 3 and 12.

3. 5 is what part of 20? What per cent of it?

4. 16 is what part or per cent of 48? $24 is what part of $36? What per cent?

5. 12 ounces is what part of a pound? What per cent?

6. 800 lb. is what part of a ton? How many 100ths of a ton, or per cent, is it?

7. 16 is what per cent of 40?

8. 48 is what per cent of 64?

9. 90 is what per cent of 144?

10. 35 is what per cent of 105?

11. 20 is what per cent of 40? Of 60? Of 120? Of 400?

12. 45 is what per cent of 90? Of 180? Of 450? Of 900?

13. 60 is what per cent of 180? Of 300? Of 1200?

14. 30 is what per cent of 90? Of 120? Of 150? Of 210? Of 240?

15. George bought a $32 canoe for $28. What per cent of discount did he get?

16. When a man sells bananas for 20¢ a dozen that cost him 15¢, what per cent of the cost is he making?

17. When the cost of porterhouse steak is increased from 24¢ to 28¢ a pound, what is the per cent of increase?

DRILL TABLE

What per cent of:

1. 10 is 5?	**9.** 90 is 30?	**17.** 25 is $8\frac{1}{3}$?
2. 20 is 5?	**10.** 28 is 7?	**18.** 100 is 30?
3. 24 is 8?	**11.** 56 is 7?	**19.** 200 is 150?
4. 32 is 8?	**12.** 60 is 40?	**20.** 300 is 24?
5. 50 is 10?	**13.** 90 is 45?	**21.** 300 is 75?
6. 60 is 20?	**14.** 25 is $12\frac{1}{2}$?	**22.** 400 is 40?
7. 75 is 25?	**15.** $37\frac{1}{2}$ is $12\frac{1}{2}$?	**23.** 400 is 60?
8. 80 is 40?	**16.** $16\frac{2}{3}$ is $8\frac{1}{3}$?	**24.** 400 is 160?

Miscellaneous Problems in Percentage

1. A merchant sold a pair of shoes for $5 that cost him $3.50. What was his gain per cent?

SOLUTION

$$
\begin{array}{r}
\$5.00 \\
\underline{\$3.50} \\
\$3.50)\$1.50(0.42\tfrac{6}{7} \\
\underline{1.400} \\
1000 \\
\underline{700} \\
\dfrac{300}{350} = \dfrac{6}{7}.
\end{array}
$$

EXPLANATION. — The ratio of the gain to the cost is found by dividing. Since the result is wanted in *per cent*, the quotient is carried to *hundredths*, and the remainder is expressed as a common fraction. Then the gain is $42\frac{6}{7}$%. The division might have been carried to four decimal places thus, 0.4286−

Then 0.4286− = 42.86−%.

2. The wages of the men in a certain factory are to be raised 20% from their present scale. How much will a man get after the raise who is getting \$2.25 before the raise?

3. The Tariff Act of 1909 places a duty of 40% upon the value of imported kitchen utensils. What is the duty upon a bill valued at \$980?

4. A mason estimates that the material for a certain cement walk will cost \$75, and that the labor will cost 25% more than the material. At what price must he take the job to make 20% of the total cost of material and labor?

5. To find the number of "board feet" of lumber in a 3-in. oak flooring, $33\frac{1}{3}$% of the area to be covered is added to allow for the waste in the "tongue-and-groove." Find how much flooring is required for the two floors of a building $36' \times 48'$.

6. From a sheet of straw board $22\frac{1}{2}'' \times 28''$ a boy cuts book covers $7\frac{1}{2}'' \times 5''$. How many can be cut from 1 sheet? What per cent of each sheet is waste?

SUGGESTION. — Compare the area wasted with the area of the sheet.

7. What per cent is waste when $5'' \times 7''$ covers are cut from sheets $21'' \times 28''$?

8. In making a hammered brass article, a piece of brass containing 17 sq. in. is cut from a sheet 4 in. wide and 5 in. long. What per cent of the material is wasted?

9. A copper ornament containing $8\frac{1}{2}$ sq. in. is cut from a sheet of copper 2 in. by 5 in. What per cent of the metal is wasted?

10. A class in clay work was provided with 15 lb. of plastina for use in modeling. When the work was finished and the plastina put away, it weighed only $12\frac{1}{2}$ lb. What per cent of it was wasted?

11. In his examinations in arithmetic a boy had 10 problems out of 12 right. His grade was what per cent?

12. In his history examination he answered 6 questions out of 8 correctly. What per cent was his grade in history?

13. In spelling, he had 27 words right out of 30. What per cent was his grade in spelling?

14. In a school of 320 children, 145 were boys. What per cent were girls?

15. In this school the average daily attendance was 305. What was the per cent of attendance?

16. The following table gives the number of pupils attending the different kinds of schools in the United States in 1908. What per cent of the number attending public schools were in each kind of school? What per cent of those in private schools were in each kind of school?

SCHOOLS	NUMBER OF PUPILS		
	PUBLIC	PRIVATE	TOTAL
Elementary	16,291,506	1,230,805	17,522,311
High Schools and Academies	790,912	163,808	954,720
Colleges and Universities	60,258	102,660	162,918
Professional Schools	12,110	52,654	64,764
Normal Schools	64,066	7,801	71,867

17. Find the total number of pupils attending all kinds of schools in 1908, then find what per cent of them were in each kind of school.

18. The amount of raw silk produced by the whole world in 1907 was 48,634,000 lb. Of this, Japan produced 14,043,000 lb. What per cent of the world's product did Japan produce?

19. In 1908 the amount of rice grown in the United States was 21,889,620 bu. Of this, Louisiana produced 11,550,000 bu. What per cent of the whole crop did Louisiana produce?

20. In 1908–1909 the total amount of cane sugar produced by the world was 7,342,854 tons. Of this, the United States produced 1,045,000 tons. What per cent of the world's product did the United States produce?

21. The total amount of beet sugar produced by the world the same season was 6,889,218 tons. The United States produced 380,000 tons of this. What per cent of the beet sugar did the United States produce?

22. In 1905 there were 3610 cheese factories in the United States. Of these, 1454 were in Wisconsin. What per cent of all the cheese factories were in Wisconsin?

23. At the same time Wisconsin had 902 out of the 5235 butter factories in the United States. What per cent of these were in Wisconsin?

24. According to the Payne-Aldrich Tariff Law, which was passed in 1909, the duty on all decorated chinaware bought abroad by people of the United States is 60% of its value. If Siegel & Cooper buy a set of china dishes in Europe for $28, what duty must they pay on them to get them admitted to the United States? If they add $10 to cover their expenses and profits, how much must you pay for that set of dishes?

25. The duty on undecorated chinaware is 55%. A set of dishes of this kind bought in Europe for $18 costs a merchant how much duty to get it into the United States? Allowing $8.50 for profit, etc., how much must you pay him for these dishes?

26. The duty on opera glasses, telescopes, etc., is 45 % of their value. What would be the duty on a pair of opera glasses costing $2 in Europe?

27. The duty on imported automobiles is 45 %. If your father bought an automobile in Paris for $1650, what duty would he have to pay to get it into the United States?

28. The duty on pocket knives valued at more than $1.25 a dozen and not more than $3 is 10 ¢ each and 40 % of their cost abroad. Allowing the local merchant 10 ¢ for profit, etc., how much must you pay him for a knife that cost 25 ¢ in Dublin?

29. For a double-barreled shotgun valued at $8 in Germany the duty is $4 and 15 % of the value. Allowing the merchant of whom you buy it $3 for handling it, how much must you pay for the gun?

30. The duty on lace made of cotton, flax, or other vegetable fiber, is 60 %. What is the duty on lace that costs 80 ¢ a yard in Ireland?

31. The duty on toys is 35 %. If a mechanical toy costs 20 ¢ in Germany, and you allow the merchant of whom you buy it 8 ¢ for profit, how much must you pay for it?

32. Find the saving in per cent by buying during this sale.

Parlor Suites

Saturday, Last Day
Of The Sale

One more chance left you to buy a beautiful, substantial, up-to-date parlor suite of three or five pieces away under the usual price.

$55 Parlor Suites for	$35	$97 Parlor Suites for	$58
$60 Parlor Suites for	$38	$128 Parlor Suites for	$75
$68 Parlor Suites for	$40	$161 Parlor Suites for	$100
$94 Parlor Suites for	$55	$204 Parlor Suites for	$125

33. The accompanying table gives the average weights of boys and girls at different ages, found from thousands of measurements. Find the per cent of increase in weight from year to year of both boys and girls.

Age	Weight in Pounds	
	Boys	Girls
9	50.0	47.1
11	59.8	56.6
13	75.8	72.7
15	96.4	89.0
17	116.6	104.4

34. Find the per cent of increase in the population of each of the following cities from 1900 to 1910:

City	Population in 1900	Population in 1910
Cleveland, O.	381,768	560,663
New Orleans	287,104	339,075
Lowell, Mass.	94,969	106,294
Wilkesbarre, Pa.	51,721	67,105
Terre Haute, Ind.	36,673	58,157

33. PROFIT AND LOSS

It is common in any business transaction to compute the *profit or loss* as a certain per cent of the cost of the articles sold.

Thus, if a merchant makes 25% of the cost of a suit for which he paid $16, his profit is $4.

If he sold a suit for $20 that cost him $16, he made $4, which is $\frac{1}{4}$ of the cost, or his gain is 25%.

Problems in Profit and Loss

1. A merchant bought a suit for $12, and sold it at a gain of 30%. How much did he gain? For how much did he sell it?

2. A dealer gained \$3 upon goods that cost him \$15. His gain was what part of the cost? What per cent of it?

3. A book cost the dealer \$2, and he sold it at a gain of 40%. How much did he gain? For how much did he sell it?

4. If a dealer buys a book for \$3 and sells it for \$4.20, how much does he gain? What part of the cost is this? What per cent of it?

5. By selling a suit for \$24, a dealer made \$8. What did the suit cost? The gain was what part of the cost? What per cent of it?

6. A house and lot that cost \$8000 were sold for \$10,000. What per cent was gained?

7. A man sold a house and lot for \$6000, which was \$1000 more than it cost him. What per cent did he make?

8. By selling a stock of goods for \$8000 a merchant lost \$2000. What per cent did he lose?

9. A grocer paid 80¢ a bushel for potatoes and sold them for 30¢ a peck. What was his per cent of gain?

10. An article costing \$18.50 was sold at a gain of 20%. Find the gain and the selling price.

11. Mr. Ward bought a city lot for \$1600, but when he sold it he could get only \$1400 for it. What per cent did he lose?

12. If a dealer pays \$38.40 per dozen pairs for shoes, for how much a pair must he sell them to make 25% of the cost?

13. A merchant buys coal at \$6.25 a ton, including the cost of delivery, and sells it at \$7.50. If he loses 2% in bad debts, what is his net rate of gain?

14. A merchant's sales for the year amounted to \$75,000. His average gross gain was 12½% of the sales. Clerk hire

was $2400, and sundry expenses were $1250. Find the net gain.

15. A grocer bought 2000 bu. of potatoes at 45¢ a bushel. $\frac{3}{4}$ of them were sold at a gain of 20 %; the remainder at a loss of $11\frac{1}{9}$ %. Find the average rate of gain upon all.

16. A dealer bought 25 boxes of oranges of 144 each at $3.50 per box. In all, he found 120 decayed ones. At what price per dozen must he sell the remainder to gain 25 % of the total cost?

17. A grocer bought 250 pounds of raw coffee beans at $14\frac{1}{4}$ cents a pound. In roasting, it lost 25 % of its weight. He sold the roasted coffee at 22 cents a pound. How much did he make? What per cent of the cost did he make?

18. A merchant bought 2000 pounds of sugar at $4\frac{1}{2}$¢. If the loss in drying out, down weights, etc., was 10 % of the amount bought, what per cent did he make by selling at 6¢?

19. A merchant bought coffee at 22 cents a pound and sold it for 35 cents. What per cent does he make if he loses 5 % in bad debts?

20. One year a merchant sold goods that cost him $1500 at an average advance of $33\frac{1}{3}$ % of the cost, but lost 2 % from bad debts. Find his net rate of gain.

21. After paying rent $600, clerk hire $2100, lighting and heating $320, and sundries $340, a man has a net profit of $3900 from his business. If his sales for the year were $64,000, find the average gross gain per cent upon the selling price. Find the net gain per cent upon the cost.

22. During the year a merchant's sales amounted to $24,650. The average gain was 20 % of the sales. His rent was $480, clerk hire $1800, and sundries $540. The net gain was what per cent of the cost of the goods?

23. A man bought some city lots for \$12,600. He paid taxes on them for two years, amounting to \$302.40. Then during a money panic he had to sell them for \$11,000. What per cent was his loss?

24. The owner of a delicatessen shop buys 8 lb. of beef at 22¢ a pound. When he roasts it, it shrinks 30% of its weight. At what price must he sell it to gain 20%?

25. He buys a ham weighing 14 lb. at 20¢ a pound. When he bakes it, it loses 35% of its weight. At how much a pound must he sell it to make a profit of 25%?

26. Let each pupil make and give to the class an original practical problem in profit and loss.

By inspection, find the loss or gain:

	Cost	Rate of Profit or Loss	Profit or Loss		Cost	Rate of Profit or Loss	Profit or Loss
1.	\$3000	20%		**19.**	\$4200	$16\frac{2}{3}\%$	
2.	\$6500	40%		**20.**	\$10,800	$11\frac{1}{9}\%$	
3.	\$8200	10%		**21.**	\$6500	50%	
4.	\$9600	$12\frac{1}{2}\%$		**22.**	\$7800	20%	
5.	\$1250	30%		**23.**	\$1800	25%	
6.	\$1800	$16\frac{2}{3}\%$		**24.**	\$4200	$16\frac{2}{3}\%$	
7.	\$3690	$33\frac{1}{3}\%$		**25.**	\$600	25%	
8.	\$2400	$66\frac{2}{3}\%$		**26.**	\$800	$12\frac{1}{2}\%$	
9.	\$3200	$12\frac{1}{2}\%$		**27.**	\$450	20%	
10.	\$2400	$37\frac{1}{2}\%$		**28.**	\$500	30%	
11.	\$3200	$62\frac{1}{2}\%$		**29.**	\$600	15%	
12.	\$2700	$66\frac{2}{3}\%$		**30.**	\$1200	25%	
13.	\$8400	$8\frac{1}{3}\%$		**31.**	\$1500	20%	
14.	\$4800	$6\frac{1}{4}\%$		**32.**	\$1600	10%	
15.	\$2800	$2\frac{1}{2}\%$		**33.**	\$2100	20%	
16.	\$1600	15%		**34.**	\$2500	8%	
17.	\$9600	25%		**35.**	\$3200	$6\frac{1}{4}\%$	
18.	\$3500	$14\frac{2}{7}\%$		**36.**	\$2400	$12\frac{1}{2}\%$	

By inspection, find the rate of loss or gain:

	Cost	Profit or Loss	Rate		Cost	Profit or Loss	Rate
1.	$10,000	$1000		**17.**	$7500	$3750	
2.	$1500	$500		**18.**	$9000	$1500	
3.	$2400	$600		**19.**	$8000	$2000	
4.	$1500	$750		**20.**	$1200	$300	
5.	$1200	$600		**21.**	$100	$21	
6.	$3600	$600		**22.**	$200	$38	
7.	$5000	$600		**23.**	$400	$36	
8.	$5000	$750		**24.**	$1000	$120	
9.	$9600	$1600		**25.**	$1200	$120	
10.	$6000	$1500		**26.**	$2000	$600	
11.	$9000	$4500		**27.**	$1500	$450	
12.	$2800	$400		**28.**	$750	$150	
13.	$6400	$800		**29.**	$600	$150	
14.	$7200	$800		**30.**	$800	$160	
15.	$6300	$900		**31.**	$900	$72	
16.	$7500	$2500		**32.**	$1800	$360	

34. COMMISSION

The money paid by one person, called the *principal*, to another, called the *agent*, for transacting business, is called **commission.**

The commission is usually reckoned as a per cent of the money involved. Thus, an agent gets a per cent of the money received for goods sold, or of the cost of goods bought, or of the amount of money collected.

Commission is sometimes reckoned upon the quantity sold, as so much per bushel, car, pound, etc.

A merchant who sells and *handles* the goods sold on commission is called a **commission merchant.** If an agent merely arranges for purchases or sales of goods for another, *without actually receiving and delivering them*, he is called a **broker.**

Thus, if a commission merchant of Chicago sells for an apple grower in Colorado a carload of apples to a fruit firm in Chicago, he receives the apples and delivers them to that firm. But a Chicago *broker* will sell a carload of wheat for a St. Paul firm to another broker who represents a firm in Cleveland, and the wheat will be shipped directly to Cleveland, from St. Paul.

Problems and Exercises in Commission

1. A salesman sold \$75,500 worth of goods one year. His commission was $7\frac{1}{2}$ % of his sales. What did he earn?

2. A salesman sold \$60,000 worth of goods one year, and received a commission of \$3600. What was his rate of commission?

3. What is the yearly income of a salesman who sells \$60,000 worth of goods at $7\frac{1}{2}$ % commission? If his expenses are \$150 per month, what is his net income?

4. A salesboy was offered his choice of the following: \$8.50 a week; \$5 a week and 1 % of his sales; or 4 % of his sales. He chose the last. His sales averaged \$265 per week. How much better than \$8.50 a week is this? How much better is this for the year, if he worked 50 weeks?

DRILL TABLE

No.	Sales	Rate	Com.	No.	Sales	Rate	Com.
5.	\$85,000	4%		10.	\$345,000	2%	
6.	\$75,000	$8\frac{1}{3}$%		11.	\$95,000	$12\frac{1}{2}$%	
7.	\$96,000	$6\frac{1}{4}$%		12.	\$42,000	5%	
8.	\$24,000	25%		13.	\$65,000	15%	
9.	\$36,000	20%		14.	\$72,000	8%	

15. An agent bought 500 barrels of apples for me at $1.20 per barrel. Find his commission at 3%. How much per barrel did it add to the cost?

16. My agent bought a carload (720 bu.) of potatoes for me at 40 cents a bushel, commission $2\frac{1}{2}$%. What was his commission? How much per bushel did it add to the cost? If freight, drayage, and other expenses are 9 cents a bushel, for how much a bushel must I sell them to make 30%?

17. A Chicago broker sold for a shipper in Minneapolis 8500 bu. of Minnesota wheat to a miller in Toledo. His commission or brokerage was $\frac{1}{8}$¢ a bushel. How much was his commission on the sale?

18. He sold for a shipper in Kansas City 7600 bu. of corn to a firm in Cincinnati at a commission of $\frac{1}{8}$¢ a bushel. What was his commission on this sale?

19. A commission merchant sold a consignment of Colorado apples to a firm for $350, and charged 5%. What was his commission?

20. A real estate agent sold my house for me at $2\frac{1}{2}$% commission. What was his fee, if the house sold for $9750?

21. During August and September, Jenkins & Marshall, real estate brokers, sold 5 houses for the following sums: $9500; $7750; $11,500; $8750; $6750. How much did they make at $2\frac{1}{2}$% commission?

22. A man took orders for a grocery firm at $12\frac{1}{2}$% of his sales. One month his orders amounted to $1436. How much did he earn? His orders averaged $1020 per month for the year. How much did he make per year?

23. In Oklahoma a man drives through the country and small towns, and takes orders for coffee, tea, spices, etc., for a firm in Minneapolis, Minn. He receives 35% of his sales

for commission. One week his orders amounted to \$128. What was his commission?

24. If his sales amounted to an average of \$120 a week, what would he earn in a year of 52 weeks?

25. A lawyer charges $2\frac{1}{2}$ % for collecting money from a debtor for a client. What would be his fee for collecting a debt of \$7000?

26. What would be his fee if he charged 3 % for collecting a debt of \$1260?

27. For settling an estate a lawyer charges 2 % of the amount of money involved in the estate. What would be his fee for settling an estate amounting to \$65,278?

35. "MARKING DOWN" GOODS

Sometimes goods are marked to sell at a certain per cent of profit, but for some reason must be sold at a reduction from the marked price. The problem in such cases is to find the net loss or gain.

Problems in "Marking Down" Goods

1. Goods costing \$250 were marked to sell at a gain of 40 %, but were sold at 25 % less than marked. Find the gain.

2. Shoes costing \$30 per dozen pairs were marked to sell at a gain of 50 %, but were sold for "a quarter off," that is, at a reduction of 25 % from the marked price. What was the gain or loss on each pair?

3. Boys' suits costing \$60 per dozen were marked to sell at a gain of 40 %, and sold at "a quarter off." What was gained?

4. An article costing \$3.50 was marked at an advance of 20 % and marked down 10 %. Find the gain.

5. Goods costing \$80 were marked to sell at 40 % gain, but were marked down 10 %. What per cent of the cost was gained?

6. Goods costing \$350 were marked at 20 % above cost and then sold at 10 % off. What per cent was gained?

7. How much is lost by marking an overcoat that cost \$24 at 50 % above cost, then selling it at 40 % off?

8. What per cent is gained by marking a suit that cost \$18 at 40 % above cost and then selling it at 20 % off?

9. A merchant made \$1.50 when selling an \$18 overcoat at "a quarter off." At what per cent of profit had he marked it?

10. After marking down a suit 25 %, a dealer asked \$15 for it. Being unable to sell it at this price, he gave another reduction of 15 % (on the \$15) and still make 75 cents. At what per cent above cost was it first marked?

11. What reduction can a dealer give on a piano costing him \$180, but marked to gain 50 %, in order to get back the cost of the piano?

12. A dealer received an invoice of \$3500 worth of goods which he marked $33\frac{1}{3}$ % above cost. He then marked them down 10 % for "a bargain sale." What was his profit? What per cent of profit did he make?

13. Smith and Foster sold a set of table-ware marked at \$120 at 25 % less than the marked price and still made \$18. Required the cost price.

14. If the marked price of the table-ware had been \$150, what would the per cent above cost have been?

DRILL TABLE

Fill out this table:

No.	Cost Price	Rate of Gain	Profit	Marked Price	Rate of Reduction	Selling Price	Net Loss or Gain
15.	$ 3.50	20%			10%		
16.	$ 7.60	25%			12½%		
17.	$10.50	33⅓%			2½%		
18.	$ 3.60	33⅓%			10%		
19.	$ 9.60	16⅔%			5%		
20.	$16.40	20%			8%		
21.	$24.80	40%			25%		
22.	$16.50	45%			16⅔%		
23.	$17.20	50%			12½%		

24. What per cent profit do I make by marking goods at 50% above cost and then discounting the marked price 20%?

SOLUTION

$$\begin{array}{r} 5\overline{)150\%} \\ \underline{30\%} \\ 120\% \\ \underline{100\%} \\ 20\% \end{array}$$

EXPLANATION. — The goods were marked to sell at 150% of the cost. They were marked down 20%, or ⅕ of the marked price. ⅕ of 150% of the cost is 30% of the cost. Hence, the goods sold for 120% of the cost, or at a gain of 20%.

25. Do I make or lose by marking goods to sell at a profit of 50% and selling at "one third off"?

26. Some goods were marked 60% above cost. At a bargain sale they were sold at "a quarter off." What per cent did the dealer make?

27. If a merchant marks his goods to sell at a gain of 30%, but has to sell them at 30% from the marked price, what per cent does he lose?

DRILL TABLE

Give the rate of loss or gain:

No.	Marked Rate of Gain	Rate of Reduction	Rate of Loss or Gain	No.	Marked Rate of Gain	Rate of Reduction	Rate of Loss or Gain
1.	50%	20%		**10.**	30%	20%	
2.	40%	10%		**11.**	50%	$33\frac{1}{3}\%$	
3.	40%	$16\frac{2}{3}\%$		**12.**	40%	25%	
4.	50%	$16\frac{2}{3}\%$		**13.**	30%	25%	
5.	$33\frac{1}{3}\%$	10%		**14.**	20%	8%	
6.	20%	$12\frac{1}{2}\%$		**15.**	25%	$16\frac{2}{3}\%$	
7.	40%	$12\frac{1}{2}\%$		**16.**	$33\frac{1}{3}\%$	20%	
8.	50%	8%		**17.**	60%	40%	
9.	60%	35%		**18.**	60%	25%	

36. TRADE DISCOUNT

It is a custom among certain wholesale dealers, manufacturers, and publishers, to fix a price on their goods, called the *list price*, and then to allow a certain deduction from this price to purchasers. The list prices are generally printed in catalogues. A deduction is often made from a bill of goods if paid within a limited number of days. This deduction is called *trade discount*. The price after the amount of the discount is deducted is called the *net price*.

Problems in Trade Discount

1. What will a bill of goods listed at $90 cost, if the discount is 20%?

2. Goods listed at $150 cost $120. What was the rate of discount?

3. A bill of $1600 was discounted at 30%. What was the net price?

4. What is the rate of discount when a piano listed at \$800 sells for \$600?

5. By paying cash a merchant may save 5%. How much is this on a bill of \$2400?

6. I buy a bill of goods amounting to \$68.50. By paying the bill within 10 days I am allowed a discount of 2%. How much do I pay, if I pay within this time?

7. A bill for erasers and crayon bought by a school amounted to \$12.40. If a discount of 12% was allowed, what was the net price?

8. The ice company sells a 1000-pound ticket for \$3.50. If you pay for it in advance, you get it for \$3. What rate of discount do you get by paying in advance?

9. A bill of goods listed at \$375 was sold at 22% off. Find the net price.

10. I got a discount of \$294 from a bill of goods listed at \$840. What was the rate of discount?

11. I bought goods that were listed at \$1150 for \$920. What rate of discount was this?

DRILL TABLE *A*

Find the net price:

	List Price	Rate of Discount	Net Price		List Price	Rate of Discount	Net Price
1.	\$40	25%		9.	\$240	$8\frac{1}{3}$%	
2.	\$65	20%		10.	\$640	$12\frac{1}{2}$%	
3.	\$150	$33\frac{1}{3}$%		11.	\$750	20%	
4.	\$600	$16\frac{2}{3}$%		12.	\$800	40%	
5.	\$960	$16\frac{2}{3}$%		13.	\$900	$11\frac{1}{9}$%	
6.	\$175	10%		14.	\$420	$14\frac{2}{7}$%	
7.	\$450	$16\frac{2}{3}$%		15.	\$1200	$16\frac{2}{3}$%	
8.	\$960	$33\frac{1}{3}$%		16.	\$720	25%	

DRILL TABLE *B*

Fill out the table:

	List Price	Rate of Discount	Discount	Net Price		List Price	Rate of Discount	Discount	Net Price
1.	$465	20%			8.	$950		$96	
2.	$970	$33\frac{1}{3}$%			9.	$1050		$175	
3.	$1640	40%			10.	$1260			$1050
4.	$1830	5%			11.	$940		$23.50	
5.	$1960	$2\frac{1}{2}$%			12.	$630			$598.50
6.	$3470	5%			13.	$126			$108
7.	$1698	$16\frac{2}{3}$%			14.			$236.25	$708.75

37. BILLING GOODS TO "THE TRADE"

The following problems show the forms of bills sent to purchasers where discounts are deducted.

Problems

1. Check the following bill:

CHICAGO, ILL., May 1, 1910.

W. D. Williams & Co.,
Urbana, Ill.

Bought of A. G. SPAULDING & BROS.
ATHLETIC GOODS, 147 WABASH AVE., CHICAGO.

TERMS: NET CASH.

3	Doz. Tennis Rackets	$18.00	54.00	
6	Doz. Tennis Balls	3.25	19.50	
$\frac{1}{2}$	Doz. Tennis Nets	17.60	8.80	
			82.30	
		Less 10%	8.23	
				74.07

2. Complete the following:

INDIANAPOLIS, IND., July 6, 1911.

Mr. R. H. Brabb, Springfield, Mo.

Bought of HOLLWEG & REESE

IMPORTERS OF CHINA, GLASS, AND QUEENSWARE

TERMS: 60 days or 2% off 10 days.

6	Doz. 5274 Plates	$2.25	*	*		
6	" " Teas	2.50	*	*		
$\frac{7}{12}$	" " Coffee Cup Only	2.40	*	*		
6	" " Fruit	1.50	*	*		
2	" " Deep Coup. Soup	2.00	*	*		
1	Only " Covd. Dish	1.65	*	*		
3	" " Casseroles	1.65	*	*		
			*	*		
	Less 10%		*	*		
					*	*
2	Doz. 19,783 H. & Co. Fruit	3.19	*	*		
2	Only " Baker	1.50	*	*		
2	" " Lobster Salad	1.69	*	*		
			*	*		
	Less 25%		*	*		
					*	*
	Pkg.			50	*	*
					*	*

Make out bills for the following, using the name "Board of Education" of some town, and the name of some dealer. Supply dates. Acting as clerk for the dealer you may receipt the bill in proper form.

3. Permanent equipment for a class of 42 pupils in a primary school, discount 25 %:

42 pr. scissors	@	$3.00 per doz.
42 compasses	@	1.20 per doz.
42 rulers	@	2.00 per 100
42 triangles	@	0.09
3 punches	@	0.40
paper cutter		3.00
boxes, etc.		4.20

4. Supplies for the same class, discount $12\frac{1}{2}\%$:

$\frac{1}{2}$ ream (500 sheets) cardboard	22 × 28	@ 90¢ per 100 sheets
400 sheets cover-paper		@ $2.50 per 100 sheets
100 sheets strawboard	22 × 24	@ $2 per 100
42 sheets pressboard	24 × 32	@ 70¢ per doz.
$\frac{1}{4}$ ream manila tag	22 × 28	@ $7.50 per ream
1 roll leatheret	30 yd.	@ 25¢
50 tubes glue		@ 7¢
2 doz. tubes photo paste		@ $1.00

5. Bill the following supplies at a discount of 20%:

4 doz. Le Page's glue, 4-oz. cans		@ $1.30
10 pkg. folding paper	4 × 4	@ 6¢
8 pkg. colored paper	6 × 9	@ 20¢
2 pkg. cartridge paper, assorted	14 × 18	@ 60¢
8 pkg. manila drawing paper, gray	8 × $10\frac{1}{2}$	@ 15¢
4 doz. rulers, 12-in. brass edge		@ 40¢
5 Higgins's liquid paste, 14-oz. jars		@ $28\frac{1}{2}$¢

6. Equipment for a class of 30 pupils in Sloyd work, discount 20%:

30 Sloyd knives	@	$4.05 per doz.
30 6-in. try-squares	@	1.58 per doz.
30 compasses	@	1.00 per doz.
30 T-squares	@	63¢ per doz.
30 45-degree triangles	@	50¢ per doz.
30 30–60-degree triangles	@	50¢ per doz.
30 rulers	@	12¢ per doz.
30 pencils	@	24¢ per doz.
30 trays	@	$8.10 per doz.

7. The equipment for a class of 36 pupils in bent-iron work, discount $33\frac{1}{3}\%$:

36 flat-nose pliers, No. 5	@ $2.00 per doz.
36 round-nose pliers, No. 5	@ 2.10 per doz.
1 cutting machine	4.75
36 rulers	0.36

8. Supplies for a class of 36 pupils in bent-iron work, discount $33\frac{1}{3}\%$:

3500 binders	@ 10¢ per 100
15 50-ft. coils $\frac{1}{4}$ inch iron	@ 20¢
3 tubes black paint	@ 25¢
50 pencils	@ 2¢

Workshop Problems

1. Make out a bill for the following set of tools, with 5% discount for cash:

1 work bench	@	$9.00	1 2-foot rule	@	$0.35
1 No. 5 jack plane	@	2.09	1 7″ try-square	@	.40
1 No. 2 smoothing plane	@	1.66	1 8″ bevel	@	.44
1 26″ ripsaw	@	1.40	1 pair 6″ dividers	@	.35
1 24″ compass saw	@	.90	1 nail set	@	.10
1 set chisels, $\frac{1}{4}$″, $\frac{1}{2}$″, 1″	@	1.44	1 marking gauge	@	.30
1 $\frac{1}{2}$″ gouge	@	.37	1 rubber mallet	@	.65
1 hammer	@	.65	1 coping saw	@	.20
1 hatchet	@	.56	1 oil stone	@	.35
1 ratchet brace	@	1.76	2 No. 2 hand screws, each	@	.30
1 set auger bits $\frac{1}{4}$″, $\frac{3}{8}$″, $\frac{1}{2}$″	@	1.38	1 can glue	@	.25
2 gimlet bits, each	@	.10	1 drawknife	@	.55
1 counter sink	@	.23	1 sq. reamer	@	.20
1 5″ screwdriver	@	.35	1 coes wrench	@	.50
1 plumb and level	@	.90	1 file	@	.27

2. The above set can be purchased complete for $24. What per cent is saved in buying the complete set instead of one tool at a time?

3. The following is the cost of Le Page's liquid glue :

½ pt.	$0.30	1 qt.	$0.80	1 gal.	$ 2.60
1 pt.	0.50	½ gal.	1.50	5 gal.	12.50

If large quantities of glue are used, what per cent is saved in buying in 1-pt. lots instead of ½-pt. lots? In ½-gal. lots instead of 1-pt. lots? In gallon lots instead of pint lots? In gallon lots instead of quart lots? In 5-gal. lots instead of quart lots? In 5-gal. lots instead of gallon lots?

4. No. 1 sandpaper costs $ 3.38 per ream, $ 0.20 per quire, $ 0.10 per doz. Where large quantities of sandpaper are used, how much is saved per ream in buying in ream lots instead of quire lots? In quire lots instead of dozen lots? What is the percent of saving in each case?

NOTE. — 24 sheets = 1 quire; 20 quires = 1 ream.

5. Emery paper costs $ 4.63 per ream; emery cloth costs $ 13.25 per ream. What is the difference in cost? What is the per cent of difference?

6. A No. 605 bed-rock plane costs $ 2.09. The bed of a No. 605 plane costs $ 1.50. In case the bed of the plane is broken, what per cent is saved in buying a new bed instead of having to buy an entire new plane?

7. A "frog" of a No. 605 plane costs $ 0.30. If the frog should break, what per cent is saved in being able to buy a new frog instead of having to buy an entire new plane?

8. A 1½-lb. rubber mallet costs $ 0.65. A 3-lb. rubber mallet costs $ 1. What is the difference of cost in 12 of each? What per cent of difference?

9. 2-ft. wood-bar clamps with iron clamp screws sell for $ 0.85 apiece or $ 9 per doz. How much is saved in buying in dozen lots instead of single clamps? What is the per cent of saving?

10. A boy working with a dull plane can square up a board in 85 minutes, while a boy working with a sharp plane can do the same work in 40 minutes. What per cent of time is saved in working with a sharp plane?

Problems in Food Supplies

1. The weight of a live chicken is $4\frac{1}{2}$ lb. When dressed, it weighs only 3 lb. What per cent of live weight is waste?

2. The waste as follows is what per cent of the live weight; head and blood, 2 oz.; feathers, 4 oz.; feet, 2 oz.; entrails, 12 oz.?

3. The chicken described above, when boiled, weighed 1 lb. 12 oz. What per cent passed into broth?

4. Bones, skin, and surplus fat removed from cooked protein weigh 14 oz. What per cent of the live weight are they? What per cent of edible solid remains?

5. At 15 cents per pound, live weight, what is the cost? At 18 cents, dressed, what is the cost?

6. At 15¢ a pound live weight, how much was the cost per pound of the solid, edible part remaining when cooked?

7. A lady keeping a delicatessen shop bought 60 lb. of dressed fowl at 18 cents a pound. They were cooked, bones were removed, and then were made into a "pressed chicken" loaf. If the loss in weight in cooking and pressing was 65%, at what price per pound must the chicken be sold to make $33\frac{1}{3}$%? To make 25%? To make neither profit nor loss?

8. Potatoes contain 78% water; 2% proteid; 18% starch; 1% cellulose; 1% mineral salts.

In a bushel (weighing 60 lb.) what is the weight of each component part?

9. In a 2-pound package of corn meal, component parts weigh as follows:

Water, 4.128 oz.	Fat, 0.704 oz.
Starch, 24.032 oz.	Mineral matter, 0.288 oz.
Proteid, 2.848 oz.	

What per cent of the whole is each?

10. 2 level tablespoons of ground coffee make $\frac{1}{2}$ pt. of coffee. 16 level tablespoons $= \frac{1}{2}$ pt. liquid. What per cent of the coffee, when made, is the ground coffee?

11. If a pint of ground coffee weighs $\frac{1}{2}$ lb., how much (many ounces) will it require to make 4 gal. of coffee?

12. The best medium for deep frying is composed of $\frac{2}{3}$ lard, $\frac{1}{3}$ suet. What per cent of this is each?

13. French dressing is composed of oil and vinegar in the proportion of 3 tablespoons of oil to 1 of vinegar. How much of each will be required to make $\frac{1}{2}$ pt. (16 tablespoons) of dressing? What per cent of each is used?

14. There is 11% of cooked potatoes and 15% of cooked cabbage which is not utilized in the body. Which is cheaper, considering waste, potatoes at 5 cents a quart weighing 2 lb., or cabbage at 6 cents per head weighing 4 lb.?

15. Foodstuffs which cost $72.45 in 1897 cost $101.59 in 1902. What was the per cent of advance in five years?

16. Certain kinds of foodstuffs which cost 27¢ in 1905 increased to 45¢ in 1910. Find the per cent of increase during the five years.

17. Wheat sold for 64¢ a bushel in 1896 and for $1.23 in 1909. Find the per cent of increase in prices.

18. Whole beef carcasses sold for $5.50 per 100 lb. in 1896. In 1909 the price was $9. Find the per cent of increase in price.

19. New mess pork per barrel was \$8.25 in 1896. In 1909 it was \$25.75 per barrel. Find the per cent of increase.

Make a table of the per cent of increase or decrease from one period to another in the prices of foodstuffs given in the following table :

	Food	1896	1900	1901	1907	1908	1909
20.	Flour (bbl.)	\$3.25	\$3.40	\$3.45	\$3.35	\$4.10	\$5.40
21.	Eggs (doz.)	12½¢	19¢	14¢	29¢	34¢	38¢
22.	Bacon (100 lb.)	4.38	6.25	8.75	9.62	10.75	12.25
23.	Lard (100 lb.)	4.20	6.15	8.45	9.75	9.80	12.65
24.	Beans (bu.)	1.15	2.25	2.50	2.17	2.40	2.75
25.	Potatoes (bu.)	.25	.50	.79	.50	.71	.50

Industrial Problems

1. If burlap is 1 yd. wide, how many yards will be required for 20 oven holders, each 7 inches by 13 inches? What per cent of the burlap is wasted in cutting?

2. A class of 20 are to make book bags, cut 12 in. by 14 in. How many can be cut from 1 yd. of material 32 in. wide? What per cent is wasted in cutting?

3. Could a greater number be cut from 1 yd. of material 36 in. wide? What per cent of this would be wasted?

4. How many can be cut from 1 yd. of denim 30 in. wide, and what is the per cent of loss?

5. A child's dress skirt is 24 in. long and contains 3 breadths of gingham. How many yards shall be purchased, allowing 3½ in. for the hem?

6. Which would be the cheaper, to buy the gingham for the dress skirt by the yard, with no waste, at 9¢ a yard, or to make it from a remnant containing 2½ yd. at 20¢?

7. Following is the cost of each tool composing a boy's drawing set:

1 scale ruler	$0.08	1 drawing board	$0.65
4 thumb tacks	.02	1 T-square	.30
1 eraser	.05	1 45-degree triangle	.40
1 compass	.10	1 try-square	.40

What is the cost of the entire set? What would be the cost of equipping a shop for 24 boys with drawing outfits, if a discount of 12 % is allowed?

8. If one 2-foot folding rule costs 20 cents, what is the cost of 24, if a discount of 8 % is allowed?

9. What is the cost of 24 try-squares, if one costs 40 cents, and a discount of 5 % is given?

10. What is the cost of 24 1-inch chisels, if one costs 57 cents, and a discount of 10 % is given?

11. What is the cost of 12 $\frac{1}{2}$-inch chisels, if one costs 46 cents, and a discount of 12 % is given?

12. What is the cost of 12 $\frac{1}{4}$-inch chisels, if one costs 41 cents, and a discount of 8 % is allowed?

13. What is the cost of 24 jack planes, if one costs $2.09, and a discount of 15 % is allowed?

14. What is the cost of 12 smoothing planes, if one costs $1.47, and a discount of 10 % is allowed?

15. What is the cost of 12 marking gauges, if one costs $0.30, and a discount of 5 % is allowed?

16. How many square feet of material are used in making a taboret, if the pieces, when completed, are of the following sizes: top, $14'' \times 14''$; four legs, each $22'' \times 3''$; two braces, each $7'' \times 7''$. Add 7 % to the total area of the completed pieces to account for waste in making. Find the total cost at 7 cents per square foot.

17. How many square feet of material are used in making a bookrack having two ends $5\frac{1}{2}'' \times 5\frac{1}{2}''$; a bottom, $5\frac{1}{2}'' \times 14''$; and two braces, $1\frac{1}{2}'' \times 2''$? Add 4 % for the waste in making, and find the total cost at $3\frac{1}{2}$ cents per square foot.

18. How many square feet of material are used in making a footstool having two ends, $6\frac{1}{2}'' \times 6\frac{1}{2}''$; one top, $6\frac{1}{2}'' \times 14''$; and one brace $2'' \times 14''$? Add 7 % for waste in making, and find total cost at 6 cents per square foot.

19. A boy spends 9 periods, of $1\frac{1}{2}$ hours each, in drawing, and 31 periods in tool practice. What per cent of his time does he draw? What per cent of his time does he spend in tool practice?

20. If rip-saws cost $1.40 apiece, cross-cut saws cost $1.20 apiece, and back-saws cost $1.20 apiece, what is the cost of fitting out a shop with twelve of each?

21. When bought by the dozen, rip-saws cost $14.00 per dozen; cross-cut saws cost $12.50 per dozen; and back-saws cost $12.50 per dozen. What per cent is saved in buying the saws by the dozen lots?

Miscellaneous Problems

1. What is the capacity of a fruit jar $\frac{7}{12}$ of a foot high whose base has a diameter of $4\frac{5}{8}$ inches?

2. 27.7 cubic inches of water weigh one pound. What is the weight of water in a cylindrical jar whose diameter is 6 inches and which is $1\frac{3}{8}$ feet high?

3. Compare the areas of a square and a round bread board each 14 inches across.

4. What per cent of a square bread board is waste when making a round one from it with its diameter the same as the distance across the square one?

5. How many 4-inch tiles will be needed to finish the walls of a kitchen $12\frac{3}{4}$ ft. $\times$ $14\frac{1}{2}$ ft., building up 4 feet from the floor, allowing for two doors three feet wide?

6. If 85 % of the cubical contents of a Mason jar 4 inches in diameter and 7 inches high is preserved cherries, how many cubic inches are juice?

7. A circular teapot stand contains 42.25 square inches. What is the diameter of the base of the largest teapot that could be placed on the stand?

8. During a single month a merchant received goods amounting to $10,486. His average discount for cash was $4\frac{1}{2}$ %. How much did he save by paying cash?

9. How many cubic inches of earth will be required to fill a circular fern dish 8 inches in diameter and $3\frac{1}{2}$ inches high, if filled to within $\frac{5}{8}$ inch of the top?

10. At $5\frac{3}{4}$ cents per square inch, what would it cost to glaze the inside and outside of the above fern dish?

11. During the month of September 750 cubic feet of gas are consumed daily in a kitchen at 90 cents per thousand, and 10 % off for cash. What is due the gas company?

12. How many cubic feet of ice will fill a box $20\frac{1}{2}$ inches by $12\frac{3}{4}$ inches by $16\frac{3}{8}$ inches?

13. What is the total area of the inside of this box?

14. A circular pond 15 feet in diameter is covered with ice 2 feet thick. How many cubic feet of ice can be cut from this pond?

15. Estimate the cost per person for the following served to five people:

$3\frac{3}{4}$ lb. of roast beef at $16\frac{1}{2}$ cents per pound.
$\frac{1}{4}$ pk. of potatoes at 25 cents per peck.
$2\frac{3}{4}$ lb. of tomatoes at 8 cents per pound.

2 boxes of strawberries at 15 cents per box.

$\frac{1}{2}$ pt. of cream at 20 cents per pint.

16. A cubic foot of water weighs 62 lb. 8 oz. What is the weight of the water in a cylindrical jar whose base is 4 inches in diameter and which is $8\frac{1}{2}$ inches high?

17. A coal bin 16 feet long and 24 feet wide has a capacity of $362\frac{2}{3}$ cubic yards. How deep is the bin?

18. If a cubic foot of ice weighs 56 lb. 14 oz., what is the weight of a cake of ice 15 inches by 18 inches by 13 inches?

19. One cubic foot of water equals 7.48 gallons. How many gallons are there in a water boiler that measures $1\frac{3}{4}$ feet in diameter and is 5 feet high?

20. A cubic foot of coal weighs about 50 pounds. How many tons will a bin hold that is 24 ft. long, 28 ft. wide, and 5 ft. deep?

21. If a pound of American cheese contains 50% more nutriment than the same quantity of moderately lean beef, how much more expensive as a food is beef at 14 cents per pound than cheese at 18 cents per pound?

22. A standard dietary for a man at moderate muscular work consists of: 13 oz. round steak; 3 oz. butter; 6 oz. potatoes; and 22 oz. bread. What is the cost of food for this meal if steak costs 18 cents per pound, butter 36 cents per pound, potatoes 90 cents per bushel of 60 pounds, and bread 10 cents per loaf of $1\frac{3}{4}$ pounds?

23. The above dietary contains a total of 0.28 pounds of proteid. The steak contains 0.14 of a pound. What per cent of the whole is it?

24. If it requires $1\frac{3}{8}$ ounces of jam to spread a slice of bread, and $\frac{1}{3}$ of an ounce of butter to cover a slice of bread, would it cost more to use jam at 25 cents per pound, or butter at 36 cents per pound?

38. SIMPLE INTEREST

Money paid for the use of money is **interest.** (See p. 203). It is a certain *per cent* of the amount borrowed, called the **principal.** The **rate** quoted is for the use of the money for one year, even though the interest may be collected semi-annually or quarterly.

Oral Drill

1. At 5%, how much is the interest of $400 for 1 year? For 6 months? For 2 years? For 2 yr. 6 mo.?

2. At 6%, how much is the interest of $500 for 1 year? For 2 years? For $2\frac{1}{2}$ years? For 8 months, or $\frac{2}{3}$ year?

3. At 5%, how much is the interest of $800 for 1 year? For 6 months? For 3 months? For 9 months? For 1 yr. 3 mo.?

At 6%, what is the interest of:

4. $500 for 6 mo.?
5. $300 for 8 mo.?
6. $250 for 1 year?
7. $150 for $2\frac{1}{2}$ years?

At 5%, what is the interest of:

8. $800 for 1 yr. 6 mo.?
9. $200 for 2 yr. 6 mo.?
10. $300 for 2 yr. 8 mo.?
11. $400 for 2 yr. 3 mo.?
12. $800 for 1 yr. 9 mo.?
13. $700 for 4 years?

Exercises

1. At 5%, what is the interest of $540 for 2 yr. 7 mo.?

SOLUTION

$$\frac{31}{12} \times \frac{5}{100} \times \$540, \text{ or}$$

$$\frac{31 \times 5 \times \$540}{12 \times 100} = \$69.75$$

EXPLANATION. — $\frac{5}{100}$ of $540 = the interest for 1 yr. Now 2 yr. 7 mo. = $\frac{31}{12}$ yr. Hence the total interest = $\frac{31}{12} \times \frac{5}{100} \times \$540$.

Use cancellation when possible.

NOTE.—This is called the **general method** of finding interest.

Find the interest of:

2. \$5000 at 5 % for 1 yr. 3 mo.
3. \$6040 at 4 % for 1 yr. 8 mo.
4. \$7500 at 4 % for 2 yr. 2 mo.
5. \$1240 at 6 % for 1 yr. 9 mo.
6. \$4360 at 5 % for 2 yr. 8 mo.
7. \$2500 at 4 % for 1 yr. 4 mo.
8. \$6400 at 6 % for 3 yr. 10 mo.
9. \$3785 at 6 % for 4 yr. 6 mo.
10. \$7500 at 5 % for 2 yr. 9 mo.
11. \$4850 at 4 % for 3 yr. 1 mo.
12. \$1780 at 5 % for 1 yr. 11 mo.

Interest for Years, Months, and Days

In computing interest for days, 30 days are considered an interest month, and 360 days an interest year.

Find the interest of:

1. \$4500 at 6 % for 2 mo. 20 da.

SOLUTION

$\frac{80}{360} \times \frac{6}{100} \times \$4500 = \$60.$

EXPLANATION. — 2 mo. 20 da. = 80 da. 80 da. = $\frac{80}{360}$ year.

2. \$9460 at 6 % for 6 mo. 12 da.
3. \$1350 at 5 % for 8 mo. 20 da.
4. \$3460 at 4 % for 9 mo. 10 da.
5. \$4860 at 4 % for 90 da.
6. \$6500 at 5 % for 70 da.
7. \$1400 at 6 % for 110 da.
8. \$3050 at 6 % for 86 da.
9. \$5100 at 5 % for 2 mo. 5 da.
10. \$1950 at 4 % for 2 mo. 12 da.

11. $2500 at 6 % for 1 yr. 2 mo. 20 da.
12. $4675 at 4 % for 2 yr. 3 mo. 15 da.
13. $3700 at 5 % for 1 yr. 8 mo. 8 da.
14. $1280 at 6 % for 3 yr. 4 mo. 17 da.
15. $6400 at 5 % for 2 yr. 1 mo. 21 da.

39. PROMISSORY NOTES

One who borrows money from another gives a written promise, called a **promissory note**, to repay the money at a given time. The rate of interest to be paid is also stated unless the interest is paid in advance.

A *promissory note* is used also in buying as well as in borrowing. It is used, too, in paying for services rendered. Thus, a man may buy a piece of machinery and give a promissory note to pay for it at some future time, say in six months or a year. In general, then, a promissory note is a written promise to pay for "value received," whether it is money, merchandise, or service.

The following is the common form of a promissory note:

$450 *New York,* Nov. 16, *1910.*

Six months *after date* I *promise to pay to*

the order of R. M. Simpson

Four Hundred Fifty *Dollars.*

Value received. Interest 6%. Henry Williams.

Mr. Williams is called the **maker** of the note and Mr. Simpson the **payee**.

Exercises

1. Make out a promissory note in which you promise to pay William Robinson $1250 in 3 yr. at 5 %. Date it to-day.

2. Make out a promissory note for $2500 at 6 %, payable in 8 months, given by yourself to John Dowell. Date it to-day.

3. Make out such a promissory note as Richard Monroe would give you if you loaned him to-day $5000 at 6 %, to be repaid in 4 months.

4. Make out other notes, using whatever data you wish.

40. SECURITY

One who loans money or takes a note for property sold wishes, of course, to be reasonably secure from loss. That is, he wishes to be reasonably sure that the one owing the money will pay it when due.

Sometimes two or more sign the same note. Any one of those signing it thus becomes responsible for its payment. This method of securing a note is called **personal security.**

Sometimes the repayment is secured by the borrower making over certain *real estate* or *personal property* to the one of whom the money is borrowed. This is called a **mortgage**, and becomes void when the money is repaid.

When giving a note for property bought, a **lien**, or a form of mortgage, usually is given on the property.

Interest Exercises with Time to be Found

For method of finding the time between dates, see p. 273.

1. What is the interest of a note for $275, dated July 27, 1909, and paid May 12, 1910, bearing interest at 6 %?

2. What must I pay November 17, 1911, to take up my note for $387, given March 25, 1910? Interest at 5 %.

3. What is the value, on the 5th of January, 1910, of a 6 % note for $475, dated June 18, 1909?

4. What must I pay on the 8th of December, 1910, to redeem my note for $360, dated February 19, 1910? Interest at 6 %.

5. How much is due July 20, 1910, on a note for $725, given October 6, 1809, with interest at 5 %?

6. Find interest of $7685, April 25 to November 11, 1910, at 5 %.

7. What is the amount of the principal and interest of $2850 at 5 % from July 2, 1910, until December 25, 1910?

8. I took a 6 % mortgage of $6000 on February 28, 1910. How much interest shall I receive by November 30, 1911?

9. If you loaned your father $540 at 5 %, on March 12, 1911, and he paid it back to you on January 20, 1912, how much must he pay you in settlement?

NOTE. — Pupils may pretend that they borrow money from each other and loan money to each other, a note being given in each case, and the interest being computed by both the pupil who loans and the one who borrows.

Problems in Interest

1. A man bought a city lot for $2500, paying $1000 cash and giving his note, secured by a mortgage on the property, at 6 %, payable in one year, for the balance. How much must he pay at the end of the year to cancel the note and mortgage?

2. A man bought a house for $9500, paying $3500 cash, and giving his note (and mortgage) for the balance at 5 %. How much a year is his interest?

3. Instead of paying rent of $40 a month, a man buys a house for $6500. He pays $2500 cash and gives his note at 6 % for the balance. If his taxes are $70 a year, how much does he save annually?

4. A man who is paying \$50 per month rent has \$3000 in the savings bank. The bank pays him 4 % interest. Can he save money by buying a \$7000 house with the \$3000 and a 6 % note, if taxes are \$65 a year, and repairs and other expenses are \$80? How much?

SUGGESTION.—How much a year is he now paying out in excess of the interest from the bank? How much will he pay out yearly in interest, taxes, and repairs? Which is more, and how much?

5. A man who has \$4000 invested in shares of a building and loan association that pays him 5 % interest annually rents a flat for \$45 a month. Would he save money by buying a house for \$6500 with the \$4000 and a 6 % note, if all expenses on the house amounted to \$175 a year? How much?

6. A bank pays 3 % interest on money deposited in it, and loans this money at 6 %. How much does it make in 2 years on deposits amounting to \$45,000?

41. A SHORT METHOD OF FINDING INTEREST

Money is often borrowed for short periods, say 30, 60, or 90 days, especially at banks. 6 % is a very usual rate. In such cases, much time may be saved by observing that

> *The interest of any principal at* 6 %
> *For* 360 *days* = 0.06 *of the principal;*
> *For* 60 *days* = 0.01 *of the principal;*
> *For* 6 *days* = 0.001 *of the principal.*

Find the interest of \$720 at 6 % for 90 days.

$$\begin{array}{r} \$7.20 \\ 3.60 \\ \hline \$10.80 \end{array}$$

EXPLANATION. — Since $\frac{1}{100}$ of \$720, or \$7.20, is the interest for 60 days, \$3.60 must be the interest for 30 days, and their sum the required interest.

NOTE. — This is called the **banker's method** or the **60-day method** of computing interest.

Drill Exercises

At sight, give the interest at 6 % of:

1. \$540 for 60 da.
2. \$600 for 90 da.
3. \$840 for 30 da.
4. \$720 for 10 da.
5. \$900 for 120 da.
6. \$540 for 90 da.
7. \$120 for 45 da.
8. \$750 for 40 da.
9. \$630 for 120 da.
10. \$305 for 90 da.
11. \$1200 for 15 da.
12. \$1600 for 75 da.
13. \$2400 for 45 da.
14. \$3600 for 10 da.
15. \$4200 for 70 da.
16. \$5400 for 50 da.
17. \$6300 for 50 da.
18. \$2100 for 20 da.

Find the interest at 6 % of:

19. \$720 for 87 da.

\$ 7.20 =	int. for 60 da.
2.40 =	int. for 20 da.
0.60 =	int. for 5 da.
0.24 =	int. for 2 da.
\$10.44 =	int. for 87 da.

20. \$965 for 117 da.
21. \$1050 for 96 da.
22. \$1360 for 33 da.
23. \$1540 for 54 da.
24. \$1680 for 93 da.
25. \$2750 for 63 da.
26. \$742 for 72 da.
27. \$960 for 84 da.
28. \$1432 for 36 da.
29. \$1520 for 108 da.
30. \$4800 for 50 da.
31. \$1250 for 90 da.
32. \$600 for 90 da.
33. \$750 for 120 da.
34. \$1200 for 45 da.
35. \$1600 for 15 da.
36. \$360 for 20 da.
37. \$810 for 20 da.
38. \$1600 for 6 da.
39. \$2400 for 10 da.
40. \$3600 for 15 da.
41. \$8400 for 75 da.
42. \$2600 for 90 da.
43. \$7200 for 100 da.
44. \$960 for 110 da.
45. \$3200 for 75 da.

46. $4200 for 70 da.

47. $2800 for 45 da.

48. $1840 for 75 da.

49. $1280 for 115 da.

NOTE. — This method may be used to advantage with other rates. The following illustrates the way to use it.

At 5 %, find the interest of:

50. $840 for 75 da.

$$\begin{array}{rl} \$8.40 & = \text{int. at } 6\% \text{ for } 60 \text{ da.} \\ \underline{2.10} & = \text{int. at } 6\% \text{ for } 15 \text{ da.} \\ 6\overline{)10.50} & = \text{int. at } 6\% \text{ for } 75 \text{ da.} \\ \underline{1.75} & = \text{int. at } 1\% \text{ for } 75 \text{ da.} \\ \$8.75 & = \text{int. at } 5\% \text{ for } 75 \text{ da.} \end{array}$$

51. $820 for 70 da.

52. $950 for 63 da.

53. $720 for 93 da.

54. $875 for 72 da.

55. $970 for 85 da.

56. $450 for 20 da.

57. $120 for 50 da.

58. $830 for 45 da.

59. $650 for 45 da.

60. $810 for 75 da.

61. $960 for 70 da.

62. $875 for 110 da.

63. $760 for 96 da.

Problems in Interest

1. I needed the use of $800 for a short time, so I borrowed it for 90 days of a man who was in the business of loaning money. He charged 7 % interest. What was the amount of the interest?

2. A man who was building a house borrowed $3600 of a building and loan association at 6 %, and gave the association a lien on the house as security. At the end of 95 days he sold the house, and with part of the money paid off the note. How much did it take to pay off the note?

3. A man bought a building lot for $1200, paying one third cash, one third at the end of 90 days, with 5 % interest, and the rest at the end of 6 months, with 5 % interest. How much did the lot cost him?

PART TWO: EIGHTH YEAR

V. POWERS AND ROOTS

42. POWERS AND ROOTS

When a number is made up of two or more equal factors, it is called a **power** of one of the factors.

Thus, since $9 = 3 \times 3$, 9 is a power of 3; since $8 = 2 \times 2 \times 2$, 8 is a power of 2.

The power of a number is written in an abbreviated form by using an **exponent**.

Thus, $2 \times 2 \times 2$ is written 2^3. In 2^3, the 3 is the *exponent*. Similarly, $81 = 3 \times 3 \times 3 \times 3 = 3^4$. What is the exponent in 3^4? In 5^2?

If a number is the product of two equal factors, it is called the **square** of one of them; and if it is the product of three equal factors, it is called the **cube** of one of them. Thus, 5^2 is read "5 squared," or "the square of 5"; 4^3 is read "4 cubed," or "the cube of 4."

When a number is made up of two or more equal factors, one of these factors is called the **root** of the number. If a number is composed of *two* equal factors, one of these factors is the **square root** of the number. If composed of *three* equal factors, one of these factors is the **cube root** of the number.

Thus, since 7×7 or $7^2 = 49$, the square root of 49 is 7. This is written $\sqrt{49} = 7$. Since $5^2 = 25$, $\sqrt{25} = 5$.

The symbol $\sqrt{}$ is called the **radical sign.**

Exercises

Give the values of the following:

1. 2^2.	6. 10^2.	11. 3^3.	16. 10^3.
2. 4^2.	7. 12^2.	12. 4^3.	17. 12^3.
3. 6^2.	8. 16^2.	13. 5^3.	18. 20^3.
4. 8^2.	9. 15^2.	14. 6^3.	19. 25^2.
5. 9^2.	10. 20^2.	15. 8^3.	20. 100^2.

21. Learn thoroughly the squares of all the whole numbers from 1 to 16 so that you can give them as you would give any other table.

22. Find the squares of:

140; 234; 78; 521; 864; 970.

Give the roots of the following:

23. $\sqrt{9}$.	27. $\sqrt{81}$.	31. $\sqrt{900}$.
24. $\sqrt{25}$.	28. $\sqrt{121}$.	32. $\sqrt{1600}$.
25. $\sqrt{64}$.	29. $\sqrt{169}$.	33. $\sqrt{2500}$.
26. $\sqrt{100}$.	30. $\sqrt{400}$.	34. $\sqrt{3600}$.

35. What is the square root of 25×49 or 1225?

36. Find the square root of 11,025.

PROCESS

$$\begin{array}{r|r} 5 & 11{,}025 \\ \hline 5 & 2{,}205 \\ \hline 3 & 441 \\ \hline 3 & 147 \\ \hline 7 & 49 \\ \hline & 7 \end{array}$$

Since $5 \times 5 \times 3 \times 3 \times 7 \times 7 = 11{,}025$, therefore $5 \times 3 \times 7$, or 105, is the square root of 11,025. Why?

37. In the same way find the square root of 9216.

38. $\sqrt{396{,}900} = ?$	41. $\sqrt{176{,}409} = ?$
39. $\sqrt{194{,}481} = ?$	42. $\sqrt{169{,}744} = ?$
40. $\sqrt{117{,}649} = ?$	43. $\sqrt{331{,}776} = ?$

43. THE PROCESS OF EXTRACTING THE SQUARE ROOT OF ANY NUMBER

Separating a number into two equal factors is the reverse of squaring one of these equal factors. A careful analysis of the process of squaring will enable us to reverse the process and find the square root of a number when it cannot be readily found by factoring.

The Process of Squaring a Number

1. Square 47.

PROCESS

$$\begin{array}{r l}
47 & \\
\underline{47} & \\
329 & = 7^2 + \quad 7 \times 40 \\
\underline{1880} & = \quad\quad\quad 7 \times 40 + 40^2 \\
2209 & = 7^2 + 2 \times 7 \times 40 + 40^2
\end{array}$$

Observe that in the process of multiplication we first find 7×7 or 7^2; next, 7×40; next, 40×7; and finally, 40×40 or 40^2.

2. 83^2 in the same way is equal to $3^2 + 3 \times 80 + 80 \times 3 + 80^2 = ?$

3. Compare 3×80 with 80×3. Then $3 \times 80 + 80 \times 3 = 2 \times 3 \times 80$. Hence, $83^2 = 3^2 + 2 \times 3 \times 80 + 80^2$.

4. Square 64 by this method.

WORK

$$\begin{array}{r l}
64 & \\
\underline{64} & \\
16 & = 4^2 \\
480 & = 2 \times 4 \times 60 \\
\underline{3600} & = 60^2 \\
4096 & = 64^2
\end{array}$$

5. Which of the partial products is the largest? From which digit was it obtained?

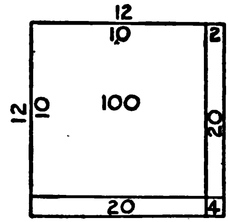
6. Which is the smallest of the partial products? From which digit was it obtained?

7. If 3600 were taken from the product, most of what remains is made from what factors?

8. State the rule for squaring numbers in this way.

Find by this method the squares of the following:

9. 35. 10. 62. 11. 84. 12. 58.



13. Show by this diagram that $12^2 = 2^2 + 2 \times 2 \times 10 + 10^2$.

14. Draw a similar diagram to show that $35^2 = 5^2 + 2 \times 5 \times 30 + 30^2$.

15. Show the squares of other numbers in the same way.

Comparing the Number of Figures in Roots and Powers

1. Give the squares of all the numbers from 1 to 9 inclusive.

2. How many figures in each of these squares?

3. Square the numbers 10, 20, 30, and so on to 100.

4. How many figures in the squares of numbers from 10 to 99 inclusive?

5. What is the square of 100? Of 200? Of 999?

6. How many figures in the squares of numbers from 100 to 999 inclusive?

7. If there are four figures in the square, how many in the root? How many in the root when five figures are in the square?

From Exercises 1–6 we see that the squares of the smallest and the largest integers composed of one, two, or three figures are as follows:

$1^2 = 1$	$10^2 = 100$	$100^2 = 10,000$
$9^2 = 81$	$99^2 = 9801$	$999^2 = 998,001$

8. Separate each of the squares shown above into periods of two figures each, beginning at the right. Thus, 98′ 01′; 1′ 00′ 00′; etc.

9. Compare the *number of periods* in each square with the *number of figures* in the corresponding root.

> *The number of periods of two figures each, beginning at ones, into which a whole number can be divided equals the number of figures in the square root.*

10. Give the number of figures in the square roots of:
9409 381 27,225 182,329 49,434,961.

11. Square 0.2; 0.02; 0.4; 0.12; 0.25; 0.03; 0.005.

12. Compare the number of decimal places in the square with the number in the root. Why can the square of a decimal never contain an odd number of decimal places?

13. State a principle for the number of periods in the decimal part corresponding to the principle given for integers.

Extracting the Square Root

Find the square root of 2809.

PROCESS

$$
\begin{array}{l}
2809 \quad (50+3) \\
\underline{2500} = 50^2 \\
\quad 309 = 2 \times 50 \times n + n^2 \text{ (why?)} \\
\quad \underline{300} = 2 \times 50 \times 3 \\
\qquad 9 = 3^3
\end{array}
$$

1. How many figures in this square?

2. Then how many in the root?

3. What is the square of 50? Of 60?

4. Between what two squares does 2809 come?

5. Then its root lies between what two numbers?

6. If the root lies between 50 and 60, the largest of the three partial products that make the square is what?

7. When 2500 is taken from 2809, what two partial products are contained in the 309 remaining? Most of the 309 is made from which of the partial products?

8. Then since 309 is more than 2×50 times the number that we are yet to find, about what must the number be?

10. When $2 \times 50 \times 3$ is taken from 309, what one of the three partial products remains?

11. Is 9 equal to 3^2? Then the second number must be 3, and the entire root is $50 + 3$ or 53. Prove by squaring.

12. Give the complete process of finding the root of a number containing three or four figures.

13. Find the square root of 8836.

SHORT PROCESS

$$\begin{array}{r|l}
 & 88'36(94 \\
 & 81 \\
2 \times 90 = 180 & 736 \\
4 & \\
\hline
184 & 736
\end{array}$$

We may omit the zeros in the square of 90, also the zero of the 90.

14. Compare $4 \times 180 + 4 \times 4$ with 4×184.

Thus we see that we may also save work by adding the 4 to 180 before multiplying by 4.

Solve by both processes and show what you save by the shorter:

15. $\sqrt{784}$.	**18.** $\sqrt{3136}$.	**21.** $\sqrt{5329}$.	**24.** $\sqrt{7569}$.
16. $\sqrt{3364}$.	**19.** $\sqrt{6889}$.	**22.** $\sqrt{4489}$.	**25.** $\sqrt{2809}$.
17. $\sqrt{8464}$.	**20.** $\sqrt{2704}$.	**23.** $\sqrt{9801}$.	**26.** $\sqrt{9409}$.

The process is the same for larger numbers. Study the following and describe the process.

27. Find the square root of 2,137,444.

PROCESS

$$\begin{array}{l|l}
 & 2'13'74'44(1462 \\
 & 1 \\
2 & 113 \\
24 & 96 \\
28 & 17\ 74 \\
286 & 17\ 16 \\
292 & 5844 \\
2922 & 5844
\end{array}$$

Find the square root of:

28. 283,024.	**34.** 529,984.
29. 299,209.	**35.** 484,416.
30. 404,496.	**36.** 638,401.
31. 556,516.	**37.** 725,904.
32. 755,161.	**38.** 295,156.
33. 6,017,209.	**39.** 1,739,761.

The Square Root of a Fraction

To find the square root of 0.501.

PROCESS

$$\begin{array}{l|l}
 & 0.501(0.707+ \\
\underline{0.7^2 =} & 0.49 \\
1.4 & 0.0110 \\
\underline{\ \ .007} & 0.009849 \\
1.407 & 0.001151
\end{array}$$

EXPLANATION. — As the square of *tenths* gives *hundredths*, to get the first root figure we take the first two figures at the right of the point or 0.50, the root of which is nearly 0.7. Each new quotient figure is determined by division, as in the case of intergers. Since the square of a decimal cannot give an odd number of figures, this decimal must be an *imperfect* power, and the root cannot be *exactly* determined. This is indicated by a + or − after the last root figure.

Find the square root of:

1. 0.5625.	4. 0.783.	7. 824.9.	10. 1932.4.
2. 0.9216.	5. 0.89.	8. 0.64.	11. 225.9009.
3. 42.225.	6. 19.467.	9. 0.064.	12. 0.8.

A. $\sqrt{\frac{320}{405}} = \sqrt{\frac{64}{81}} = \frac{8}{9}$.

B. $\sqrt{\frac{3}{8}} = \sqrt{0.375} = 0.612+$.

C. $\sqrt{7\frac{1}{9}} = \sqrt{\frac{64}{9}} = \frac{8}{3} = 2\frac{2}{3}$.

D. $\sqrt{4\frac{4}{9}} = \sqrt{\frac{40}{9}} = \frac{6.3245+}{3} = 2.108+$.

E. $\sqrt{6\frac{3}{8}} = \sqrt{6.375} = 2.52+$.

13. 234.7024.
14. 2,044,900.
15. 76.3876.
16. 0.8.

In finding the roots of fractions:

I. *First change them to simplest form, as in A or C.*

II. *Use the method in A or C when both terms are perfect powers.*

III. *Use B or E when both terms are imperfect powers.*

IV. *D may be used when the denominator is a square.*

17. $\sqrt{\frac{147}{432}}$.	19. $\sqrt{6\frac{19}{25}}$.	21. $\sqrt{82\frac{4}{5}}$.	23. $\sqrt{151\frac{7}{8}}$.
18. $\sqrt{\frac{5}{16}}$.	20. $\sqrt{5\frac{2}{3}}$.	22. $\sqrt{\frac{1905}{6480}}$.	24. $\sqrt{2\frac{3}{16}}$.

Some Applications of Square Root

1. Find the side of a square whose area is 178 sq. in. Carry the computation to tenths of an inch.

2. Find the side of a square whose area is 2764 sq. ft.

3. A square field contains 8 acres. How many rods long and wide is it?

4. A square board is to be cut that shall contain 92 sq. in. in its surface. How large a square must be made?

5. How many feet of fence does it take to go around a square lot containing 2 acres?

6. The area of a circle is 216 sq. in. Find its radius. Its diameter. Carry the computation to tenths of an inch.

7. The area of a circle is 27 sq. ft. Find its radius to tenths of a foot. Its diameter.

8. The cross section of a pipe is 7.5 sq. in. Find its diameter to tenths of an inch.

9. Two drain pipes in a house unite and discharge into one pipe with the same capacity as the two pipes together. The diameter of one small pipe is 2 in. and that of the other 3 in. Find the diameter of the large pipe into which they discharge.

10. Two sewer pipes, each 3 ft. in diameter, are to discharge into a large pipe whose capacity is equal to the two pipes together. How large must the large pipe be made?

11. What should be the diameter of the smokestack for a boiler which has 160 flues, each 2 in. in diameter?

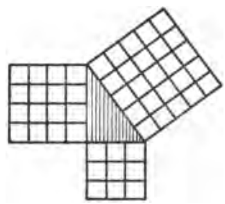
12. A water main is 18 in. in diameter. It is to be replaced by a pipe with 3 times the capacity. What must be the diameter of the new pipe? Compute to tenths of an inch.

VI. MENSURATION

44. THE PYTHAGOREAN THEOREM

1. Draw a right triangle with the sides which form the right angle, 3 inches and 4 inches respectively.

2. Measure the length of the other side, or **hypotenuse**.



3. Draw a square on each of the three sides as base.

4. Compare the square on the hypotenuse with the sum of the squares on the other sides.

Pythagoras proved about 500 B.C. that the fact that we find true here is true for *any* right triangle, viz. that

> *The square on the hypotenuse of a right triangle is equal to the sum of the squares on the other two sides.*

5. Carpenters make use of this fact in laying out the foundation for a building, when they want to form a right angle. A line 8 feet long is taken in one direction along which the foundation is to be made. Another line 6 feet long is fastened to one extremity of the first line and moved until a 10-foot rod will just reach the outer extremity of the two lines. Draw such a figure, and show that this gives a right triangle.

6. Use the test in Exercise 5, and find whether the walls of your schoolroom are perpendicular to the floor.

7. If the square on the hypotenuse is 100 sq. in. and on one of the sides 36 sq. in., what is the length of each side of the triangle?

Denoting the hypotenuse by H, the base by B, and the perpendicular by P, when these are abstract numbers representing the *number* of units in the dimensions, we may state from the above principle the following formulæ:

$$H = \sqrt{B^2 + P^2}$$
$$B = \sqrt{H^2 - P^2}$$
$$P = \sqrt{H^2 - B^2}$$

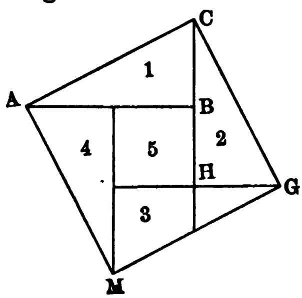
8. Explain the formulæ.

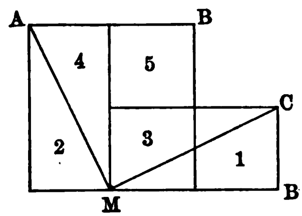
9. If $H = 15$ and $P = 14$, $B =$ what?

10. If $B = 15$ and $P = 16$, $H =$ what?

11. If $H = 25$ and $B = 20$, $P =$ what?

The truth of the Pythagorean theorem may be seen by drawing, or cutting from cardboard, figures like the following:





Let ABC be the right triangle. The square on the hypotenuse AC is equal to the four triangles, 1, 2, 3, and 4, and the small square, 5. Now put 1 and 2 in the position of the figure at the right, and the figure is equal to a square on AB and one on CB'.

Problems

1. The base of a right triangle is 48 feet and the perpendicular is 36 feet. What is the hypotenuse?

2. The hypotenuse is 85 feet and the perpendicular is 51 feet. What is the base?

3. The base is 76 feet and the hypotenuse is 95 feet. What is the perpendicular?

4. What is the diagonal of a rectangle 92 ft. long and 69 ft. wide?

5. What is the diagonal of a 30-ft. square?

6. What is the longest line that can be drawn on a sheet of paper 16 inches wide and 20 inches long?

7. What is the diameter of the largest wheel that can be got through a doorway measuring 7 feet by 5 feet?

8. What is the distance between the opposite corners of a field 200 rods long and half as wide?

9. If a window is 18 ft. from the ground, how long must a ladder be to reach to the window if the foot of the ladder is placed 6 ft. out from the building?

10. In decorating a room two ribbons are stretched, connecting the opposite corners. If the room is 30 ft. wide and 40 ft. long, how many yards of ribbon does it take?

11. A baseball diamond is 90 ft. square. How far is it from first to third base?

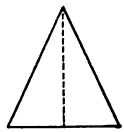
12. A derrick is 48 ft. high, and is supported by three steel cables, each reaching from the top of the derrick to a stake in the ground 45 ft. from the foot of the derrick. How much steel cable does it take, allowing 10 ft. for fastening all three cables?

13. The gable of a house is 24 ft. wide and 12 ft. high from the plate to the ridgepole. How long must the carpenters cut the rafters, if they are to project one foot over the eaves?

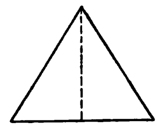
14. There are 16 steps to a stairway. The rise of each is 8 in. and the tread 10 in. How long must the timber be cut that runs from one floor to another to support the steps?

45. ISOSCELES AND EQUILATERAL TRIANGLES

A triangle having two equal sides is **isosceles**. One having all of the sides equal is an **equilateral** triangle.



ISOSCELES TRIANGLE



EQUILATERAL TRIANGLE

Prove by cutting or measuring that:

(1) *The altitude of an isosceles triangle divides the base into two equal parts.*

(2) *The perpendicular from any vertex of an equilateral triangle to the opposite side divides that side into two equal parts.*

Since an equilateral triangle is also isosceles whatever side is taken as base, (2) could have been inferred from (1).

1. If the base of an isosceles triangle is 12 in. and the equal sides 10 in., what is the altitude?

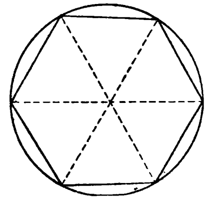
2. Find the altitude of a triangle 10 in. on each side.

3. Find the area of an isosceles triangle whose base is 10 in. and whose equal sides are each 12 in.

4. Find the area of an equilateral triangle each of whose sides is 14 in.

5. A *regular hexagon* is made up of six equilateral triangles. Study the figure and discover how to inscribe one in a circle.

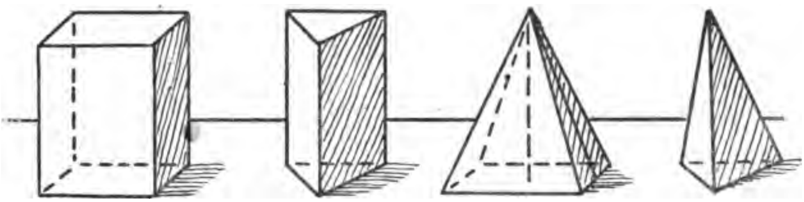
6. Find the area of a regular hexagon each of whose sides is 10 in.



A REGULAR HEXAGON

7. A bandstand in a park is in the form of a regular hexagon each of whose sides is 9 ft. How many square feet of lumber does it take to floor it?

46. PRISMS AND PYRAMIDS COMPARED



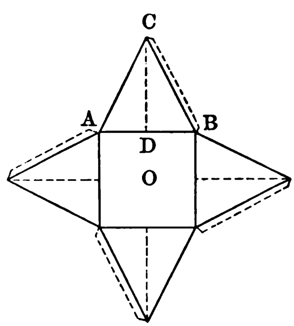
RECTANGULAR PRISM — TRIANGULAR PRISM — RECTANGULAR PYRAMID — TRIANGULAR PYRAMID

A **Pyramid** is a solid whose base is a polygon and whose sides or faces are triangles meeting at a common point called the **vertex** of the pyramid.

If the base is a regular polygon, as a square, or an equilateral triangle, and the sides are equal isosceles triangles, the pyramid is a **regular pyramid.**

The distance from the vertex to any side of the base of a regular pyramid is the **slant height.**

1. The distance from the vertex to the side is the altitude of the triangle, hence it divides the side into two equal parts. Why?



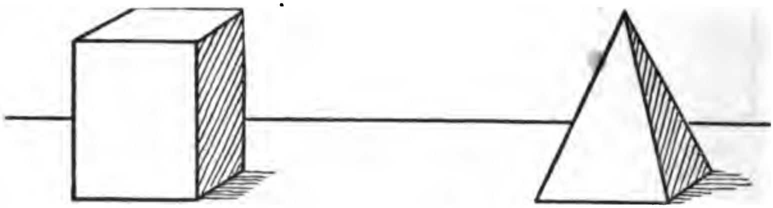
2. Construct from cardboard a pyramid whose base is a 4-inch square, and whose edges AC, etc., are 6 inches.

Draw a model, and leave lapels for pasting.

3. What is the length CD, or the slant height of this pyramid?

4. Having found CD, and knowing OD, observe the figure on preceding page, and find the height. (Observe that the altitude of a regular pyramid meets the base at its center.)

5. How could you have found AO and then the height from OA and AC? Find it.



6. Make a prism having base and altitude equal to those of the pyramid you have made.

Make an opening in the base of the pyramid, and fill with dry sand, and fill the prism from this as a measure.

7. What do you find true of their volumes?

8. Make of clay, or cut from a large potato, a prism and a pyramid with equal bases and altitudes. Weigh them, and compare their weights. Then what must be the relation of their volumes?

9. Make other prisms and pyramids as your teacher may direct, and test the accuracy of the following:

> *The volume of a pyramid is $\frac{1}{3}$ of that of a prism having an equal base and an equal altitude.*

10. Find the volume of a square pyramid whose base is 8 inches and its altitude 12 inches.

Problems

1. A prism has a base 4 ft. square, and is 7 ft. high. What is its volume?

2. A prism has an altitude of 16 in., and its base is a right triangle of which the sides forming the right triangle are 4 in. and 5 in. Find its volume.

3. A square pyramid is 12 ft. high and measures 3 ft. along one side of its base. What is its volume?

4. What part of a square prism is whittled away by a boy who is making the largest pyramid possible out of it?

5. The volume of a square prism is 36 cubic inches. What is the volume of a square pyramid of the same base and altitude?

6. Which is more easily measured, the slant height or the altitude of a pyramid? Which line of a triangle is the slant height of a regular pyramid?

7. The slant height of a square pyramid is 15 inches, and the side of the base 10 inches. Find its contents.

8. A square prism has a base 2 feet long and an altitude of 10 feet. It is made of granite weighing 165 pounds to the cubic foot. What is the weight of the prism?

9. How many surfaces has a square prism? What is the shape of each one?

10. A hexagonal pyramid is one having a hexagon as base. How many triangles make its visible surface? Of what kind?

11. What two lines in an isosceles triangle must be known in order to find its area?

12. What is meant by the slant height of a regular pyramid? The total area of all the faces of a prism is called its *lateral area* or *convex surface*.

13. Find the lateral area of a prism whose altitude is 4 ft. and base 3 ft. square. Find the whole area.

14. The altitude of a triangular prism is 12 in., and the base is a right triangle whose sides including the right angle are 6 in. and 8 in. Find its lateral area. Its total area.

15. A square pyramid has a slant height of 12 inches and a base of 4 inches. What is its convex surface?

16. An octagonal pyramid's slant height is 16 in., and the perimeter of its base is 48 in. What is its convex surface?

17. All four sides of a pyramid are equilateral triangles 6 inches long. Find thc slant height and the convex surface.

18. The altitude of a square pyramid is 15 in. and the side of its base is 12 in. Required its volume.

19. The wagon box most generally used in hauling dirt is 9 ft. by 3 ft. by 16 in. How many cubic feet will a load contain? How many cubic yards?

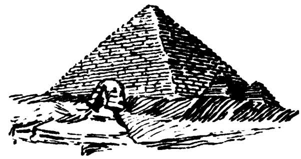
20. A freight car 36 ft. long and 8 ft. 6 in. wide, inside measurements, is filled with wheat to a depth of 5 ft. Allowing 0.8 bushel to a cubic foot, and 60 lb. to a bushel, find the weight of the load. If the capacity of the car is 60,000 lb., how much does it lack of being loaded to its full capacity?

21. The excavation for a house is 38 ft. long, 30 ft. wide, and 5 ft. deep. How many cubic yards of earth are removed in making the excavation?

22. How many cubic feet of water could a *V*-shaped gutter discharge (flowing full) in a day, if it is 8 in. deep, 16 in. wide at the top, and the water flows a foot a second?

23. How many cubic yards of stone are required to build a dam 300 ft. long, 15 ft. high, 10 ft. wide at the bottom, and 4 ft. wide at the top?

24. A cubic foot of iron weighs 450 lb. How many pounds must be weighed out to be melted in order to make a hollow casting with 1 in. walls, the outside dimensions of the casting being 28 in. × 16 in. × 12 in.?

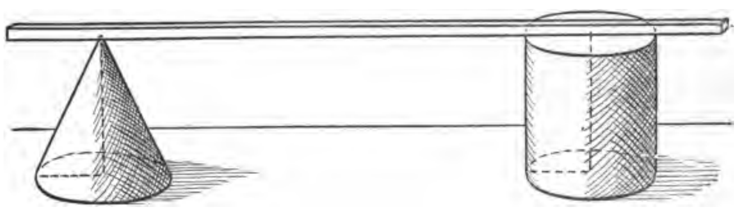


25. The Great Pyramid of Egypt, known as the pyramid of Cheops, is 764 ft. square, and its altitude is 480 ft. Find its volume. Allowing 170 lb. to the cubic foot, find its weight in tons, if it were solid.

26. How many acres of ground does this pyramid cover? Find the number of acres in its lateral area.

27. The Washington Monument at Washington, D.C., is crowned with a pyramid $34\frac{1}{2}$ ft. square and 25 ft. high. Allowing 170 lb. to the cubic foot, find the weight of it in tons.

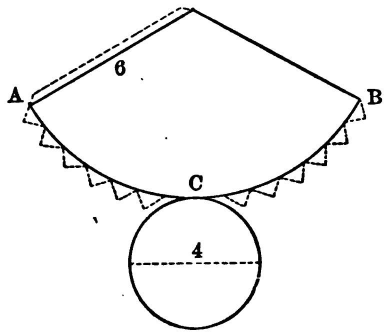
47. CYLINDERS AND CONES COMPARED



A solid having a circle for a base and tapering uniformly to a **vertex** is a **cone**. *AC* is the **slant height**. While there are other kinds of cones, we shall consider the kind described above. In this kind the altitude from the vertex passes through the center of the base. It is called a **right circular cone**.

1. How can you find the altitude when the slant height is known?

2. Make a cone whose base is a circle, whose radius is 2 inches, and whose slant height is 6 inches. Make a model as in the margin.



3. How long is the arc *ACB*? What is its radius?

4. What will be the height of the cone?

5. Make a model for a cylinder of the same dimensions.

6. What will be the size and shape of the convex surface?

7. Test the accuracy of your construction by measuring as shown in the lower figure on the preceding page.

8. Using the cone as a measure, fill the cylinder with dry sand.

9. How do their volumes compare?

10. Construct of clay, or cut from potatoes, a cylinder and a cone with equal bases and altitudes. Weigh them, and compare the weights. Hence, what is the relation between their volumes?

11. Make other sizes and show that:

> *The volume of a cone is equal to $\frac{1}{3}$ of that of a cylinder having an equal base and the same height.*

12. How do we find the volume of a cylinder?

13. Find the area of a conical steeple with a slant height of 32 feet, the diameter of the base of which is 4 feet.

Problems

1. Find the volume of a cylinder whose altitude is 10 in. and radius of base 4 in.

2. If a cylinder weighs 3 pounds, what will be the weight of a cone of the same material having the same base and altitude?

3. The largest possible cone is turned in a lathe out of a cylinder 6 inches long and 3 inches in diameter. What part of the cylinder goes into shavings? How many cubic inches in the cone?

4. The base of a cone is 6 square inches and its altitude is 12 inches. Find its volume.

5. The diameter of the base of a cone is 4 feet. Its altitude is 9 feet. Find its volume.

6. A cylinder of ebony weighs 1 lb. 8 oz. What will an ebony cone of the same base and altitude weigh?

7. What must be the height of a tomato can to hold a quart (57.75 cu. in.) if its diameter is 4 in.?

8. How many cubic yards of earth must be removed in digging a cistern 10 ft. in diameter and 10 ft. deep?

9. Allowing $7\frac{1}{2}$ gal. to a cubic foot, find how many gallons of water a cistern will hold that is 9 ft. in diameter and 10 ft. deep. Since $31\frac{1}{2}$ gal. $=1$ barrel, how many barrels will it hold?

10. How many barrels does a standpipe hold that is 18 ft. in diameter and 80 ft. high?

11. The cylindrical tank of the large sprinkling car used by a city street car company in sprinkling the tracks is 6 ft. in diameter and 26 ft. long. How many gallons does it hold?

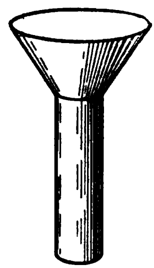
12. A cubic inch of brass weighs 0.303 lb. Find the weight of a brass tube 20 ft. long whose inner diameter is 1 in. and outer diameter $1\frac{1}{2}$ in.

13. How long a brass tube $\frac{1}{2}$ in. inner diameter and $\frac{7}{8}$ in. outer diameter can be run from 50 lb. of brass?

14. The piston of a pump is 8 in. in diameter, and makes a stroke of 18 in. How many gallons of water will it deliver in an hour, if it makes 30 strokes a minute? (231 cu. in. = 1 gal.)

15. A wash boiler 12 in. deep, 10 in. wide, and 20 in. long has round ends; *i.e.* each end is a half cylinder. How many gallons does it hold?

16. The figure represents a view of a "rain gauge," an instrument used for measuring the amount of rainfall. The opening at the top is 12 in. in diameter, and the cylindrical stem 4 in. in diameter. Suppose that in a rain the stem is filled to a depth of 4 in. What is the precipitation? (That is, what is the depth of the rainfall on level ground?)



17. Suppose that in a rain the stem in the gauge in Exercise 16 is filled to a depth of 5 in. What is the precipitation?

18. Find the length of a wire $\frac{1}{16}$ in. in diameter that can be drawn from a cubic foot of copper.

19. A boiler of an engine 4 ft. in diameter and 16 ft. long is traversed by 60 pipes, each 3 in. in diameter, which convey the heat through the water. How many gallons of water does the boiler hold?

20. Find the volume of a cone whose altitude is 6 in. and diameter of base 4 in.

21. Find the volume of a cone whose slant height is 5 in. and diameter of base 8 in.

22. A fruit raiser has a round pile of apples that is 6 ft. across at the bottom, and tapers to a point that is 4 ft. high at the middle. How many bushels in the pile? (Count 3 bu. to 4 cu. ft.)

23. A farmer has a pile of ear corn approximately in the form of a cone whose height is 8 ft. and width at the bottom 16 ft. How many bushels does it contain? (Count 2 bu. to 5 cu. ft.)

24. A farmer wishes to know how many tons of hay there are in a stack in the form of a cylinder 16 ft. in diameter and 7 ft. high, surmounted by a cone 8 ft. high. Allowing 512 cu. ft. to the ton, find the amount of hay in the stack.

25. Since the lateral area of a right cylinder is equal to that of a rectangle, how is it found? Find the lateral area of a right cylinder whose altitude is 8 ft. and diameter of base 6 ft.

26. How many square inches of tin does it take to make the lateral surface of a can $3\frac{1}{2}$ in. in diameter and 4 in. high, allowing $\frac{1}{4}$ in. for a seam?

27. In punching round holes through metal plates, the pressure exerted by the punch, in pounds, in the ordinary run of work, must be 60,000 times the area, in square inches, of the cylindrical surface sheared off. Find the pressure required to punch a hole $\frac{1}{2}$ in. in diameter through a steel plate $\frac{1}{2}$ in. thick.

28. Find the pressure required to punch a hole $\frac{3}{8}$ in. in diameter through a piece of boiler plate $\frac{7}{16}$ in. thick.

29. Find the heating surface of 120 flues of a boiler, each 3 in. in outside diameter and 14 ft. long.

30. In a steam engine 94 flues, or cylindrical pipes, each 2 in. in outside diameter and 12 ft. long, convey the heat from the fire box through the water. How much heating surface do they apply to the water?

31. A steel cylinder 3 ft. long and 1 ft. in diameter is reamed out on a lathe. The revolving surface moves at a speed of 10 in. a second. The edge of the cutting tool is $\frac{5}{16}$ in. wide. How long will it take to ream out the cylinder?

32. The figure formed by the arc of a circle and two radii is called a sector.

Show that the arc of a sector multiplied by $\frac{1}{2}$ of the radius will give the area of the sector. Compare the method with that of finding the area of a circle on page 288.

33. The lateral surface of a cone is a sector, the circumference of the base being the arc of the sector and the slant height of the cone its radius.

The circumference of the base is 8 in. and its slant height 10 in. Find the lateral area of the cone.

34. What is the lateral area of a cone whose slant height is 4 ft. and the diameter of whose base is 3 ft.?

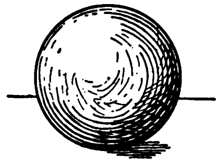
35. The radius of the base of a cone is $3\frac{1}{2}$ ft. The slant height is 5 ft. Find the entire area, *i.e.* the area of the base plus the lateral area.

36. How many yards of cloth will be required to make a conical tent 12 ft. in diameter and 15 ft. high? Add 5% for seams.

37. A square foot of brass, gauge 18, weighs 1.726 lb. What would be the weight of a lamp shade made of four pieces with two edges parallel, the altitude of each piece being 6 inches, and the top and bottom of each piece measuring 8 inches and 10 inches respectively?

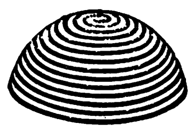
48. MEASUREMENT OF THE SPHERE

A **sphere** is a round solid bounded by a surface, all points of which are equally distant from a point within called the center. The distance from the center to any point of the surface is the **radius**, and the distance through the center of the sphere from surface to surface, which is twice the radius, is the **diameter**. The circle made by cutting a sphere by a plane through the center is called a *great circle of the sphere.*



1. Name some common objects that are spherical.

The curved surface of a hemisphere (a half-sphere) may be compared with its flat surface by placing a tack in the center of each and then carefully winding a hard cord completely over each surface as in the illustration.



Compare the lengths of these two cords and then show that the area of the curved surface of a hemisphere is just twice the area of a great circle of the sphere.

2. Then show that:

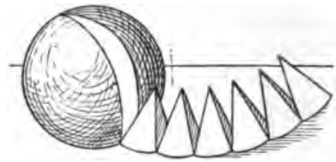
> *The area of the surface of a sphere is equal to* 4 *times the area of a great circle of the sphere.*

3. How is the area of a circle found? Show that the area of the surface of a sphere whose radius is R is $4 \times \pi \times R^2$.

4. If a sphere should be dissected as in the illustration, what solids would its parts most resemble?

5. What line in the sphere forms the altitude of each pyramid-like solid?

6. What forms the base of each?



7. Taken together, what will the bases of all the pyramid-like solids make?

8. If these were perfect pyramids, how would the volume of any one be found?

While these solids are not pyramids, for their bases are not plane figures, yet it is proved in geometry that:

> *The volume of a sphere is the same as that of a pyramid whose base is the surface of the sphere and whose height is the radius of the sphere.*
>
> *Hence, the volume of a sphere is equal to the area of its surface multiplied by one third of its radius.*

9. Since the area of the surface of a sphere is equal to 4 times the area of a great circle, or $4 \times \pi \times R^2$, show that:

The volume of a sphere $= \frac{1}{3} R \times 4 \times \pi \times R^2 = \frac{4}{3} \times \pi \times R^3$.

Exercises

1. Find the area of the surface of a sphere whose radius is 4 in.

2. Find the area of the surface of a sphere whose radius is 10 in.

3. Find the area of the surface of a sphere whose diameter is 6 ft.

4. Find the volume of a sphere whose radius is 3 ft. and the area of the surface 113 sq. ft.

5. Find the volume of a sphere whose radius is 9 ft.

6. Find the volume of a sphere whose diameter is 16 ft.

7. Taking the radius as 4000 mi., find the area of the entire surface of the earth.

8. The surface of a tiled dome, in the form of a hemispherical surface, whose diameter is 24 ft., is made of colored tiles each 1 in. square. How many tiles are required to make it?

9. Find the total area of a hemispherical bowl 1 in. thick whose external diameter is 12 in.

SUGGESTION. — We are to find the surface of two hemispheres and a ring (the rim).

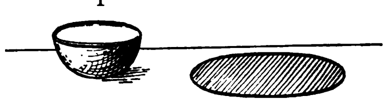
10. A gilded dome is in the form of a hemisphere whose diameter is 30 ft. How many square feet in its surface?

11. The diameter of a tennis ball is $2\frac{5}{8}$ in. How many square inches of material are required to cover 1000 tennis balls, no allowance being made for waste in cutting?

12. The dome of an astronomical observatory, which is in the form of a hemisphere, is 48 ft. in diameter. How many square feet of tin does it take to cover it?

13. A hemispherical skylight is 16 ft. in diameter. Not allowing for sash, how many square feet of glass are required to make it?

14. Vessels whose surfaces are spherical sometimes are made from sheet metal, by cutting out a circular "blank" from a flat sheet of



the metal and pressing it into the required form in a "die." The area of the circular blank must equal the area of the finished vessel. Find the diameter of the blank required to make a hemispherical brass bowl whose diameter is 8 in.

15. Hollow metal balls, used as ornaments, casters, anti-friction balls, etc., are made from circular blanks cut from sheet metal by use of a die. The area of the circular blank must equal the area of the finished ball. Find the diameter of the blank required to make a hollow metal ball $\frac{1}{2}$ in. in diameter.

16. Steel weighs 490 lb. to the cubic foot. Find the weight of a steel ball 10 in. in diameter.

17. A bowl in the form of a hemisphere is 6 in. in diameter. How many cubic inches does it hold?

18. A cubic foot of ivory weighs 114 lb. How many pounds of ivory does it take to make 1000 billiard balls, each 2 in. in diameter?

19. Find the weight of a cast-iron spherical shell one inch thick, with an outside diameter of 9 in., one cubic foot of cast iron weighing 450 lb.

20. A boiler is made in the form of a 4-foot cylinder 2 ft. in diameter, with hemispherical ends. How many gallons will it hold? (1 gal. = 231 cu. in.)

21. A haystack is approximately in the form of a cylinder 16 ft. in diameter and 8 ft. high, surmounted by a hemisphere. Allowing 512 cu. ft. to the ton, find the weight of the stack.

22. A hollow sphere of brass is found to weigh 50 lb. Its external diameter is 10 in. Find the thickness of the shell. (1 cu. in. of brass weighs 0.303 lb.)

23. If a cube 2 in. long weighs $1\frac{1}{2}$ lb., how much will one 4 in. long weigh?

24. Find the volume of a wire $\frac{1}{8}$ in. in diameter and 18 ft. long.

Miscellaneous Problems in Mensuration

1. The volume of a pyramid is what part of the volume of a prism having the same dimensions? What is the volume of a pyramid 10 ft. high, the area of whose base is 56 sq. ft.?

2. Compare the rule for finding the volume of a cone with that for finding the volume of a pyramid.

3. Compare the surface of a sphere with that of one of its great circles.

4. How is the volume of a sphere shown?

5. Explain the meaning of the following formulæ: (*a*) $V = \frac{1}{3} Ah$; (*b*) $V = \frac{1}{3} h\pi r^2$; (*c*) $S = 4\pi r^2$; (*d*) $V = \frac{4}{3}\pi r^3$.

6. If a square prism weighs 2 lb. 4 oz., what will be the weight of a square pyramid of the same base and altitude?

7. The contents of a square prism are 36 cu. in. How many cubic inches in a square pyramid of the same dimensions?

8. A bar of iron 12 in. by 2 in. by 3 in. will make how many pyramids of the same dimensions?

9. The largest possible cone is turned in a lathe out of a cylinder. What part goes to shavings?

10. If a sphere 3 inches in diameter is carefully turned out of a 3-inch cube, what part of the cube will go into shavings, and what part will remain in the sphere?

11. If a sphere is 0.5236 of a cube of the same diameter, what will be the contents of a sphere 4 inches in diameter?

12. If a cubic foot of iron weighs 450 pounds, what is the weight of an iron sphere 12 inches in diameter?

13. A cubic foot of ivory weighs 114 pounds. What is the weight of a set of 4 billiard balls 2 inches in diameter?

14. How many cubic miles in the moon, if we call its diameter 2000 miles?

15. If we call the diameter of the earth exactly 8000 miles, how many moons will be equal in volume to the earth? Shorten your work by cancellation.

16. Two 4-inch spheres are dropped into a pail even full of water and holding 864 cubic inches. How many cubic inches of water are displaced?

17. How does the curved surface of a hemisphere compare with its flat surface?

18. What is the surface of four great circles of a 5-inch sphere? What is the circumference of one of these circles?

19. Find the number of square inches in the surface of a 6-inch sphere.

20. A cubic foot of water weighs 1000 ounces, and gold is about 19 times as heavy. What would a sphere of gold 3 inches in diameter weigh?

21. How many square miles in the surface of the moon? Call its diameter 2000 miles.

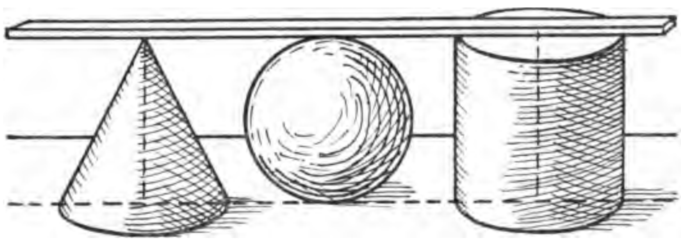
22. A hollow spherical steel shell is 1 inch thick, and its outside diameter 10 inches. Steel weighs 7.8 times as much as water, and a cubic foot of water weighs about 1000 ounces. Find the weight of the shell in pounds.

23. Two spheres of lead, of radii 2 inches and 4 inches, respectively, are melted and recast into a solid cylinder 6 inches in height. Show that the surface exposed is unchanged in amount.

24. Coal weighs a ton to 35 cu. ft. A man has 64 acres of coal land, the coal bed under it being 12 ft. thick. How many tons of coal in the bed? How much is it worth at $1.95 a ton in the ground?

25. A man has a town lot 50 ft. wide and 120 ft. long that is $4\frac{1}{2}$ ft. below the level of the street. How much will it cost him, at 50¢ a load, to have earth hauled with which to fill it to the level of the street?

26. Water is flowing into a cylindrical tank whose diameter is 16 ft. at a rate of 20 cu. ft. a minute. How long will it take to fill the tank to a depth of 6 ft.?



27. Compare the volumes of a cone, a cylinder, and a sphere, all having the same height and the same diameter. Suppose the diameter is 2 ft. and the height 2 ft.

Problems in Specific Gravity

SPECIFIC GRAVITIES, REFERRED TO WATER

Alcohol	0.79	Gold	19.3	Sea water	1.025
Brass	8.4	Granite	2.7	Silver	10.5
Cast iron	7.	Ice	0.92	Steel	7.8
Cork	0.24	Mercury	13.6	Sulphur	2.
Glass	2.5	Petroleum	0.7	Zinc	7.2

NOTE. — The *specific gravity* of a substance is the ratio of the weight of that substance to the weight of an equal volume of some substance taken as a standard. For the specific gravity of solids and liquids, distilled water is usually taken as the standard substance. For gases, either air or hydrogen is taken as the standard.

1 cu. ft. of water weighs 1000 *oz.*, or 62.5 *lb.*

1. Find the weight of a cubic foot of each of the following: cast iron; granite; glass; gold; silver.

2. A piece of zinc weighs 1125 lb. How many cubic feet are there?

3. How many cubic feet of glass weigh 1572.5 lb.?

4. Find the weight of 1 cu. ft. of cork; of sulphur.

5. Find the weight of 10 gal. of sea water.

6. Find the weight of one gallon of alcohol; of one gallon of petroleum.

7. What is the weight of a block of ice 12 in. thick, 18 in. wide, and 36 in. long?

8. How many tons of ice will an ice-house hold that is 24′ by 30′ by 20′, allowing $2\frac{1}{2}$ ft. on all sides and above and below for sawdust?

9. The specific gravity of good milk is 1.032. If 28 cu. in. of the milk taken from a can in a certain dairy wagon are found by the inspector to weigh just 1 lb., is the milk good?

10. One cubic foot of a certain kind of limestone is found to weigh 187.5 lb. What is the specific gravity?

11. One cubic foot of copper is found to weigh 556.25 lb. What is the specific gravity of copper?

12. A steel beam 16 ft. long, $2\frac{1}{4}$ in. wide, and 8 in. thick weighs how much?

13. A steel beam is 16 ft. long, $2\frac{2}{5}$ in. thick, and 14 in. wide. What is its weight?

14. An iceman delivered a block of ice 1 ft. long, $\frac{5}{6}$ of a foot wide, and $\frac{3}{4}$ of a foot thick, and charged for 50 lb. What was the shortage in weight?

15. What is the weight of a block of ice $6\frac{1}{2}$ ft. $\times$ $1\frac{1}{4}$ ft. $\times$ $4\frac{2}{3}$ ft.?

16. Find the weight of a cast-iron pipe 1.3 ft. long, 10-in. outside diameter, and $\frac{7}{8}$ in. thick.

17. What is the weight of a hollow steel pillar 10 ft. long, whose external diameter is 5 in., and internal diameter 4 in? What is the diameter of a solid pillar of the same weight and length?

18. A hollow sphere of brass is found to weigh 50 lb. Its external diameter is 10 in. What is its internal diameter?

19. A grindstone when new is 6 ft. in diameter and has a 14-in. face. What has it lost in weight when it is worn down 2 in.? (The specific gravity of sandstone is 2.42.)

20. What weight of brass is cut off in turning a cylinder 1 in. in diameter and 1 ft. long from a bar $1\frac{1}{4}$ in. square and 1 ft. long?

21. A tank car has a tank 7 ft. 9 in. inside diameter and 34 ft. long. What weight of petroleum will it carry?

22. Make other applied problems, based upon the above table of specific gravities, and let the class solve them.

Miscellaneous Problems in Measurement

1. How many laundry bags can be cut from a 12-yard bolt of unbleached muslin one yard wide, if the pattern is 18 inches wide and 32 inches long? If muslin sells at \$0.14 per yard, what would be the cost of each bag?

2. How many yards more would be required for a girl's dress that had four widths, each 23 inches long and 4 inches allowed on each width for hems, than another 19 inches long and 3 inches allowed for hems?

3. How much more would it cost at \$0.12$\frac{1}{2}$ per yard for goods for a boy's waist that required two widths 20 inches

long for the body and two widths 19 inches long for the sleeves than for a boy's waist that has two widths 18 inches long for the body and two widths $17\frac{1}{2}$ inches long for the sleeves?

4. How many square feet in an ironing board 6 feet long, $13\frac{1}{2}$ inches wide at one end, and $17\frac{3}{4}$ inches wide at the other?

5. Out of 16 square feet of leather at $0.38 a square foot, 7% was waste. What would be the cost of enough leather for a card case 5 inches by 8 inches, and two pieces for pockets 3 inches by 5 inches?

6. At $\frac{1}{2}$ cent per square inch, what would be the cost of glazing the inside and outside of a clay jar in the form of a cylinder whose diameter is $3\frac{1}{2}$ inches and which is $5\frac{1}{2}$ inches high and $\frac{1}{4}$ inch thick?

7. Bookbinder's cloth is 1 yard wide. How many science notebooks 9 inches by 11 inches can be covered from a 24-yard roll, allowing one inch on all sides for lapping?

8. The circular headband of a hat 6 inches in diameter is lined with lining silk $3\frac{1}{2}$ inches wide. What part of a yard of lining silk 19 inches wide is used?

9. The insertion in a dresser scarf $2\frac{1}{2}$ yards long and 18 inches wide extends diagonally across the scarf to the opposite corners. What would it cost to trim the scarf with the insertion, at $0.45 a yard, and the four edges with lace at $0.65 per yard?

10. How many yards of lace will be required to trim one dozen plate doilies 6 inches in diameter, one dozen glass doilies $2\frac{3}{4}$ inches in diameter, and one centerpiece 18 inches in diameter, adding $\frac{1}{2}$ in each case for fulling?

11. Which would cost more and how much, to trim a 21-inch square centerpiece or a circular centerpiece 21 inches in diameter with lace at $0.75 per yard?

12. It required $2\frac{1}{2}$ yards of lace, of which 14 inches were allowed for fulling, to trim a square handkerchief. How many yards would trim a circular one of the same diameter?

13. How many square feet in an asbestos mat for a circular table 54 inches wide?

14. At $0.03 per yard for hemming and $0.25 apiece for monograms, what would it cost to prepare one dozen 30-inch square napkins and a two-yard square tablecloth?

15. A room 14 feet long by $15\frac{1}{2}$ feet wide is covered with a 9×12 rug. What per cent of the floor will need to be polished?

16. A hall 24 feet long and 46 inches wide is covered with a runner 27 inches wide, and the same margin is allowed at both ends as at the sides. What per cent of the floor is uncovered?

17. If a one-pound box of floor wax at $0.25 a box will polish 100 square feet of flooring, what will the wax cost for a parlor floor $14\frac{5}{8}$ ft. by $16\frac{3}{4}$ ft.; a bedroom floor 9 ft. by $11\frac{1}{2}$ ft.; and a living room floor $13\frac{1}{2}$ ft. by $15\frac{3}{4}$ ft.?

18. From $1\frac{1}{2}$ yards of buckram 18 inches wide cut a circle 7 inches in diameter for the crown of a hat; cut a rectangular piece 4 inches wide to fit the circle and lap one inch; cut a circular brim 18 inches in diameter and 6 inches wide; cut a circular bandeau $7\frac{1}{2}$ inches in diameter and one inch wide. What per cent of the given material is waste? (Make a diagram.)

19. What part of a sheet of brass 3 feet long and 13 inches wide is used for a pen tray $\frac{5}{6}$ of a foot long and $\frac{1}{3}$ of a foot wide?

20. A square foot of copper, gauge 19, weighs 1.6258 lb. What would be the difference in weight of a square and a round tray each 14 inches across?

VII. APPLICATIONS OF PERCENTAGE

49. PROPERTY INSURANCE

In order that I may not have to bear the total loss in case my house is damaged or destroyed by fire, I pay an **insurance company** a certain per cent or **premium** for the **insurance** of the property.

Thus, the insurance company agrees to make good my loss to the extent of the sum named in the **policy**, which is their *agreement*, or *contract*, with me, in case my house accidentally is burned during the period specified in the policy.

1. I have my house insured for $5000 for 3 years. I have to pay the insurance company $25 for this. What is the face of the policy? What per cent of the face of the policy is the premium? This is what per cent a year?

2. If you insure your house for $6000 for five years at $1\frac{1}{4}\%$, how much is the premium? How much will you receive from the insurance company if your house is totally destroyed?

3. Is property usually insured for its full value? Give your reason.

4. I value my house at $8000. How much will it cost me to insure it for 3 years at half its value at $\frac{3}{4}\%$ of the face of the policy?

5. If I have my yacht insured for $3000, which is only $\frac{3}{4}$ of its value, what is my loss if it is totally destroyed and I have paid a premium of $1\frac{3}{8}\%$ of the face of the policy?

6. A merchant insured a stock of goods worth $40,000 at $\frac{3}{4}$ of their value at a premium of $\frac{1}{2}\%$. What was the cost of the insurance?

7. A hotel costing \$50,000 is insured for 3 years for half of its value at 1% a year. What is the total premium?

8. A church is insured for \$5000 in each of five different companies. The premium is \$500. What is the rate of insurance?

9. What is the premium on a \$12,000 policy at $1\frac{3}{4}$%?

10. Goods worth \$9000 are insured for $\frac{5}{8}$ of their value. What is the rate if the premium is \$75?

11. A factory worth \$60,000 is insured for $\frac{3}{4}$ of its value at $1\frac{7}{8}$%. The possible loss to the owner, including the premium, is how much?

12. A stock of goods costing \$250,000 is insured for $\frac{3}{5}$ of the cost. The rate is $1\frac{3}{4}$%. In case they are totally destroyed, what is the entire loss to the owner? How much does the insurance company lose?

13. A wooden tenement house valued at \$24,000 two miles from a fire engine, is insured for half of its value at $\frac{3}{4}$% a year. How much is the total premium for 5 years?

14. When the premium is \$36 on a \$4800 policy, what is the rate?

15. It is quite common to speak of the cost of insurance in terms of a certain number of dollars per thousand dollars. Thus, a man may insure his property for 1 year for \$20 per \$1000. What per cent is this?

16. How much will it cost me to insure my household goods 1 year for \$4000 at \$16 per \$1000?

17. A schoolhouse valued at \$250,000 is insured at the rate of \$12.50 per \$1000. What is the premium?

18. A factory worth \$162,500 is insured for $\frac{3}{4}$ of its value at \$16.50 per \$1000. What per cent is the rate? What is the premium?

19. A firm insures its building, valued at \$28,000, at \$12 per \$1000; the first floor contents, valued at \$12,000, at \$14 per \$1000; and the contents of the other floors, valued at \$35,000 at \$16 per \$1000. If the policies are written for $\frac{3}{4}$ of the value in each case, what is the premium on the building and the entire contents?

20. A vessel is insured for \$15,000 and its cargo for \$9000. What is the premium at $\frac{5}{8}\%$?

Give the premiums on the following policies at the rates named:

21. \$2400, $1\frac{1}{2}\%$.	**25.** \$12,000, $1\frac{1}{2}\%$.	**29.** \$26,000, $1\frac{3}{4}\%$.
22. \$5000, $1\frac{3}{4}\%$.	**26.** \$45,600, $1\frac{1}{4}\%$.	**30.** \$65,400, $1\frac{1}{2}\%$.
23. \$3600, $1\frac{1}{4}\%$.	**27.** \$84,000, $1\frac{1}{2}\%$.	**31.** \$18,500, $1\frac{3}{4}\%$.
24. \$6000, $2\frac{1}{8}\%$.	**28.** \$16,000, $1\frac{7}{8}\%$.	**32.** \$44,000, $1\frac{3}{4}\%$.

NOTE. — Other kinds of property insurance, in addition to insurance against fire, are insurance against loss by tornado, by theft, by disaster at sea (called marine insurance), etc. In these the premium is computed as in fire insurance.

50. TAXES

Most of the expenses of towns, cities, counties, and states are met by **taxes** levied by the proper officers upon the property of the town, city, county, or state.

Property is divided into two classes for taxation:

I. **Real Estate**, regarded as immovable, as land and buildings, including mines, quarries, forests, railroads, etc.; and

II. **Personal Property**, which is movable.

Besides the property tax, in some states all male citizens over 21 years old are required to pay a *poll tax*.

The rate of taxation is often stated as a certain number of mills on \$1, or as a certain number of cents on \$100. Sometimes it is stated as a per cent of the assessed value of the property.

1. Name some things for which the state needs money. Some things for which the county needs money. Name some things for which cities and villages need money.

2. If taxes are 16 mills on $1, what rate per cent are the taxes? How much are the taxes on a valuation of $16,000?

3. At $1.25 on $100 what is the per cent of tax rate? What is the amount of taxes upon property assessed at $16,000?

4. How large a tax on property is assessed in your city or town?

Suppose a town has to raise a certain sum of money for the expenses of the coming year. Officers, called **assessors,** first estimate the value of the property to be taxed, and then **assess** each owner in proportion to what he has.

5. If the amount to be raised is $20,000, and the poll tax amounts to $1600, how large a property tax must be assessed?

6. The assessed valuation of the property of a certain town is $200,000. The tax to be raised is $4000. What is the *rate of taxation?*

7. What shall Mr. Smith pay, who owns $5000 worth of property in the town?

8. My property is assessed at $3000. The tax rate is $1\frac{1}{2}\%$. What is my tax if I pay a poll tax of $2?

9. The tax to be raised is $15,000. There are 250 polls, that is, male citizens over 21. What must be raised by a property tax if the poll tax is $2 each?

10. If the tax is $16 on every thousand dollars of the valuation of my property, how large will my tax be if I am assessed for $5000?

Find the tax to be raised on the property under the following conditions:

	Total Tax	No. of Polls	Single Poll Tax		Total Tax	No. of Polls	Single Poll Tax
11.	$28,000	218	$1.50	**14.**	$740,000	9,287	$2.00
12.	$27,500	960	$1.75	**15.**	$752,000	10,426	$2.50
13.	$76,400	2580	$2.00	**16.**	$427,000	7,315	$2.15

In the following express the tax rate as (*a*) mills on $1; (*b*) cents on $100; (*c*) dollars on $1000; (*d*) as a per cent:

	Assessed Valuation	Tax to be Raised on Property		Assessed Valuation	Tax to be Raised on Property
17.	$48,000	$1,200	**21.**	$51,000,000	$750,000
18.	$650,000	$13,000	**22.**	$49,000,000	$630,000
19.	$1,650,000	$110,000	**23.**	$135,000,000	$900,000
20.	$2,470,000	$190,000	**24.**	$215,000	$2,700

25. The valuation of property in a certain town is $2,306,000. The tax to be raised on the property is $39,202. What must the rate be?

26. In 1908, the total value of all property in Chicago was $2,385,951,995. Property was taxed on $\frac{1}{5}$ of the full value. How much was this?

For What Expense	Amt. Taxes	Rate
State	$2,385,952	
County	$3,865,242	
City, including library .	$10,261,521	
Schools	$12,407,872	
Sanitary district . . .	$3,244,895	
Parks	$4,154,874	
Totals		

The preceding table gives the amount of taxes, to the nearest dollar, levied in Chicago in 1908, for different purposes:

27. Using the $\frac{1}{5}$ valuation of property found in Problem 26, compute the rate per cent of taxation required for each of the expenses, state, county, etc. Find also the total amount of money required for all taxes and the total rate for all purposes. This rate is what per cent of the full valuation?

51. NATIONAL REVENUES

The expenses of our national government in a recent year were $736,717,582. In the same year the income of the national government was $762,386,905, which came from customs, internal revenues (tobacco, etc.), sale of public lands, and other minor sources. The people are not taxed directly for the support of the national government as they are for state, county, and local governments. The chief source of income are the *customs*, *tariff*, or *duties*, levied on foreign goods that are imported into the United States.

Some goods are on the *free list*, *i.e.*, not subject to duty.

Some goods are subject to an *ad valorem* duty, which is a per cent of the value of the goods at the place of purchase.

Some goods are subject to *specific* duty, which is a certain amount per pound, per bushel, etc.

And some goods are subject to both *ad valorem* and *specific* duties.

In 1909 a new tariff law, known as the Payne-Aldrich tariff law, fixed new duties on imports.

Problems

1. Ink and ink powders are subject to an *ad valorem* duty of 25 %. Find the duty at the New York customs port of ink costing $4000 in a foreign country.

2. Olive oil is subject to a specific duty of 40¢ a gallon when bought in large holders. What is the duty on 850 gal. of olive oil?

3. Perfumery, if containing alcohol, is subject to an *ad valorem* duty of 50% and a specific duty of 60¢ a pound. What is the duty on a shipment of perfumery weighing 250 lb. and purchased abroad for $590?

4. The duty on decorated china-ware is 60%. Find the duty on a set of china dishes costing $18.50 in Germany. If you allow the merchant $15 for expenses, profits, etc., how much must you pay him for these dishes?

5. The duty on imported automobiles is 45%. What is the duty on a machine costing $1600 in Paris?

6. The duty on hooks and eyes is $4\frac{1}{2}$¢ per pound and 15%. Find the duty on 45 lb. of hooks and eyes, valued at $20.

7. The duty on watch movements with more than 17 jewels is $3 each and 25%. Find the duty on a watch movement costing $8.50 in Europe.

8. The duty on cane sugar in its natural state is 20%. Find the duty on $1875 worth of cane sugar.

9. The duty on imported fish in tin packages is 30%. Find the duty on a shipment of $950 worth of sardines.

10. Chocolate valued at between 15¢ and 24¢ per pound is subject to a duty of $2\frac{1}{2}$¢ a pound and 10%. Find the duty on 670 lb. of chocolate valued at 18¢ a pound.

11. Ready-made clothing composed of cotton or vegetable fiber is subject to a duty of 50%. Find the duty on $750 worth of boys' suits.

12. Find the duty on a fur coat costing $65 at a rate of 50%.

13. The duty on stockings, hose, and half-hose, valued at between $2 and $3 per dozen pairs, is $1.20 per dozen pairs and 15%. Find the duty on 50 doz. pairs of boys' half-hose valued at $2.50 a dozen pairs.

52. TRADE DISCOUNT

1. Show the difference between grower or producer and importer; wholesaler and retailer.

2. Do you buy from wholesalers or retailers?

3. With whom do wholesalers have to deal?

It is the general custom of wholesale dealers, manufacturers, and publishers to fix a price, called the **list price**, on their goods and then allow a certain per cent **discount** from this price to "the trade," as we have seen before.

4. A dealer bought a bill of hardware listed at $360, but got a trade discount of $33\frac{1}{3}$% from this price. How much did the hardware cost him?

5. A dealer in chinaware paid $450 for a bill of dishes listed at $600. What was the rate of discount?

6. A dinner set listed at $48 was sold to a dealer at a discount of 40%. How much did it cost him?

7. Find the net price of a piano listed at $450, but sold at a discount of 20%.

Give the discount:

	List Price	Discount		List Price	Discount		List Price	Discount
8.	$600	20%	**13.**	$720	$8\frac{1}{3}$%	**18.**	$1450	20%
9.	$750	$33\frac{1}{3}$%	**14.**	$900	$66\frac{2}{3}$%	**19.**	$1700	40%
10.	$840	$12\frac{1}{2}$%	**15.**	$1200	$33\frac{1}{3}$%	**20.**	$1800	25%
11.	$960	25%	**16.**	$1600	25%	**21.**	$900	30%
12.	$1200	10%	**17.**	$900	10%	**22.**	$800	40%

Give the rate of discount:

	List Price	Discount		List Price	Discount		List Price	Discount
23.	$800	$200	**28.**	$1800	$450	**33.**	$8000	$1000
24.	$900	$300	**29.**	$2000	$500	**34.**	$4800	$1600
25.	$1000	$250	**30.**	$2400	$600	**35.**	$5400	$900
26.	$1200	$400	**31.**	$3500	$700	**36.**	$6400	$800
27.	$1500	$1000	**32.**	$4200	$700	**37.**	$7200	$900

Copy and fill the following table:

	List Price	Rate of Discount	Discount	Net Price		List Price	Rate of Discount	Discount	Net Price
38.	$465	20%			**45.**	$950		$96	
39.	$970	$33\frac{1}{3}$%			**46.**	$1050		$175	
40.	$1640	40%			**47.**	$1260			$1050
41.	$1830	5%			**48.**	$940		$23.50	
42.	$1960	$2\frac{1}{2}$%			**49.**	$630			$598.50
43.	$3470	5%			**50.**	$126			$108
44.	$1698	$16\frac{2}{3}$%			**51.**			$236.25	$708.75

53. SUCCESSIVE DISCOUNTS

When wholesale houses wish to increase the discount already given, they usually add another discount to be taken from the former discounted, or *net* price, rather than give a new *single* discount.

Thus, if goods are quoted at $600, less 25% and 10%, it means that $600 is to be discounted 25%, and the remainder, $450, is then to be discounted 10%, leaving a net cost of $405.

Secure, if possible, the catalogue of some dealer, together with his discount sheet, and examine both carefully.

Problems

1. A bill of goods quoted at $400 is sold at 25 % and 10 % off. What is the net cost?

2. After receiving discounts of 20 % and 10 % from goods listed at $1000, how much will they cost me?

By inspection give the net price:

	List Price	Discounts	Net Price		List Price	Discounts	Net Price
3.	$600	20 %, 10 %		8.	$2400	25 %, 10 %	
4.	$900	33^1 %, 10 %		9.	$3600	$33\frac{1}{3}$ %, $12\frac{1}{2}$ %	
5.	$1000	40 %, 15 %		10.	$1600	25 %, 5 %	
6.	$1200	$16\frac{2}{3}$ %, 20 %		11.	$3500	$14\frac{2}{7}$ %, 2 %	
7.	$1800	$33\frac{1}{3}$ %, 20 %		12.	$6000	10 %, 10 %	

13. A bill of hardware was listed at $96, 40 % and 10 % off. Find the net price.

14. Find the net cost of a bill of $85, 40 % and 5 % off.

15. Find the net cost of 8 doz. drip pans listed at $4.45 per dozen, and 15 coal hods listed at $2.10 per dozen; discounts 60 % and 10 %.

16. Find the net cost of 24 doz. basting spoons at $3 per dozen, and $\frac{1}{4}$ gross galvanized buckets at $58 per gross; discounts 75 % and 10 %.

17. A dealer received a bill of window glass listed at $730, but the discounts were 90 % and 15 %. Find the net cost.

18. Find the net cost of a bill of $850, 50 % and 15 % off.

19. A bill of chinaware listed at $736 had discounts of $66\frac{2}{3}$ % and 10 %. Find the net price, including $8.36 for boxing, freight, and drayage.

20. One third of the gross amount of a bill of silverware amounting to $846 was discounted at 40 %, 10 %, and 10 %, and the remainder at 40 % and 15 %. Find the net amount of the bill. If the dealer retails the entire bill at an average of 90 % of the list price, what does he make? What per cent of the net cost is this?

21. A dealer receives a bill the gross amount of which is $334. The discounts are 40 % and 10 %. The freight, drayage, and sundry expenses amount to $12.50. If the dealer receives an average of 85 % of the list price, what per cent does he make on the net cost?

22. A dealer received the following invoice of wagons: 3 listed at $79 each; 2 listed at $81 each; 4 listed at $103 each; and one listed at $85. The discounts were 40 % and 5 %. Find the net invoice. If a further discount of 5 % for cash is given, what will he save by paying cash?

23. A department store bought a lot of toys amounting to $850, discount 20 %, 10 %, and 5 %. What was the net cost?

24. A publishing house sold a lot of books to a bookstore amounting to $1240, with 15 % off, and an additional 2 % for cash. How large a check paid the bill?

25. A school bought of J. L. Hammett, Boston, the following supplies for number work:

6 tin thermometers, 12-inch	@	$0.15
4 clock faces, cardboard	@	0.25
2 sets dry measures	@	1.75
2 sets liquid measures	@	1.25
1 set sphere, cone, cylinder	@	0.75
36 boxes toy money	@	0.15
22 boxes inch cubes	@	0.50

A discount of 20 % and an additional discount of 2 % for cash were allowed. How much was the bill?

26. The following number games were bought at "The Fair":

1 Tiddlywinks	@	\$0.25
1 Ring My Nose	@	0.48
1 Lotto	@	0.25
1 Tumble In	@	0.98
1 Crazy Traveler	@	0.65

A 10% discount and a further discount of 2% for cash were allowed. What was the amount of the bill?

27. Find the amount of the following bill for school supplies:

12 doz. Higgins's inks	@	\$3.75
12 doz. water colors	@	0.35
4 doz. scissors	@	1.50
5 lb. Boston erasers	@	0.80
3 doz. Dennison's paste	@	0.85

Discounts, 10%, 15%, 5%.

54. A SINGLE DISCOUNT EQUIVALENT TO TWO DISCOUNTS

A merchant may wish to know the single discount equivalent to two or more discounts. For example, he may wish to know which is better, 40% and 10%, or 45%.

SOLUTION: 40% + 10% of 60% = 46%.

EXPLANATION. — A single discount of 40% leaves the cost 60% of the list price. A further discount of 10% of 60% of the list price is 6% of the list price. Hence the total discount is 40% + 6%, or 46%.

Problems

1. Which is better, discounts of 40% and 20%, or of 50% and 10%?

SOLUTION: 40% + 20% of 60% = 52%.
50% + 10% of 50% = 55%.
Hence the second is better.

2. One dealer offers me 60 % and 10 % off. Another offers me 50 % and 25 % off. Another offers me a single discount of 65 %. Which is the best offer?

By inspection, give the single discount equivalent to:

3. 10 % and 10 %.
4. 20 % and 10 %.
5. $33\frac{1}{3}$ % and 10 %.
6. 40 % and 25 %.
7. 40 % and $33\frac{1}{3}$ %.
8. 50 % and 10 %.
9. $66\frac{2}{3}$ % and 10 %.
10. 20 % and 25 %.
11. 40 % and 20 %.
12. 60 % and 20 %.
13. 60 % and 25 %.
14. 30 % and 10 %.
15. 40 % and $16\frac{2}{3}$ %.
16. 20 % and $12\frac{1}{2}$ %.
17. 40 % and 15 %.
18. 50 % and 5 %.
19. 50 % and 25 %.
20. 50 % and 40 %.
21. 60 % and $12\frac{1}{2}$ %.
22. 60 % and 25 %.
23. 55 % and $33\frac{1}{3}$ %.
24. 40 % and 30 %.
25. 25 % and $33\frac{1}{3}$ %.
26. 36 % and $12\frac{1}{2}$ %.
27. 52 % and $8\frac{1}{3}$ %.
28. 28 % and $16\frac{2}{3}$ %.

55. GIVING DISCOUNTS ON GOODS BOUGHT AT A DISCOUNT

A merchant may wish to know what discount he can give from the *list price* from which he bought at a discount, and still make a profit.

For example, a merchant may buy goods at a discount of 40 % and wish to know what discount he can give from the same list price and still make 25 %.

Solution and Analysis

The cost = 60 % of the list price.
The gain = 25 %, or $\frac{1}{4}$ of 60 % of the list price, or 15 % of the list price.
The selling price = 60 % + 15 %, or 75 % of the list price.
Hence, he can give a discount of 25 % (100 % − 75 %) of the list price.

Problems

1. If I can buy an article at a discount of 50%, what discount can I give and still make 20%?

2. I wish to make $33\frac{1}{3}$% on goods bought at 40% below the list price. What discount can I give?

Find the discounts that can be given:

	Discount	Gain		Discount	Gain		Discount	Gain
3.	60%	25%	7.	40%	$8\frac{1}{3}$%	11.	30%	20%
4.	40%	20%	8.	80%	50%	12.	$33\frac{1}{3}$%	10%
5.	70%	10%	9.	60%	50%	13.	$66\frac{2}{3}$%	40%
6.	20%	$12\frac{1}{2}$%	10.	50%	40%	14.	75%	50%

A Game: Buying, Selling, and Canceling Indebtedness

To make more concrete the work of discounts, etc., as well as of methods of sending money to pay for goods bought, let the pupils play "Going into business." Take the class to a local bank. Study the method of depositing money in a bank. Get samples of a *deposit slip* and a page from a *bank book.*

Get also some *checks* and show the method of checking out money from a bank.

Also discuss other forms of paying off indebtedness, as *drafts*, *postal money orders*, *express money orders*, etc. Get blanks of each of these, and discuss the methods of *indorsement*, which becomes a receipt for the money received.

Let pupils represent large wholesale houses in some large city. Let others be retail merchants buying from them. The wholesale merchants will send bills, with discounts. The retailers will check up the bills and pay by draft, check, or money order.

Problems

NOTE.—The study of this set of problems is to follow a trip to a bank suggested on p. 397.

1. Suppose you bought of E. L. Holmes & Co., St. Louis, Mo., a bill of hardware as follows:

26 doz. axes at \$4.80; 32 doz. files at \$1.20; 36 doz. saws at \$3.40. Discounts, 40% and 10%. Terms: 2% discount for cash in 10 days; net 60 days.

Make out a bill such as E. L. Holmes & Co. would send you, showing net price.

Send a draft in payment. What is the face of the draft? Where will you get it? To whom will you have it made payable? If made payable to yourself, how will you indorse it before sending it?

2. Suppose you bought of M. E. Dawson & Co., New York, the following:

4 kitchen cabinets at \$13.25; 3 kitchen cabinets at \$15.25; and 1 kitchen cabinet at \$17.75. Terms: 2% off for cash in 10 days; net 30 days.

Make out the proper bill. Find the net cash price. Send them your check for the payment. What will Dawson & Co. do with this check when they receive it? What will a postal money order to settle the bill cost?

3. F. R. Smith & Sons, Peru, Ind., bought of J. L. Morris & Co., Chicago, the following:

2 safes at \$12.50; 3 armchairs at \$9; 2 rockers, leather, at \$25; 1 couch at \$8; 3 couches at \$6.50; 2 rockers at \$9. Terms: net 60 days.

Make out the proper bill, finding the price in 60 days. Pay by check.

4. A. L. Morgan of Salem, Ill., bought of Reed, Murdock, & Co., Chicago, the following:

86 lb. tea at 43 ct.; 280 lb. of coffee at 28 ct.; 250 lb. prunes at $11\frac{1}{2}$ ct.; 116 lb. raisins at 14 ct.; 144 cans salmon at 15 ct.; 86 lb. apricots at 18 ct. Terms: 60 days net; 2 % off for cash in 10 days.

Make out the proper bill. Make out the proper form of draft. What would Reed, Murdock, & Co. do with this draft?

5. I. C. Carpenter bought of the Van Cleve Glass Co. the following:

Three boxes 7×9, single, at \$26.75; 5 boxes 10×14, single, at \$28.25; 2 boxes 16×20, single, at \$30; 5 boxes 12×20, single, at \$28; 6 boxes 18×24, single, at \$31.75. Discounts 85 % and 10 %. Terms: 60 days net or 2 % off in 10 days.

Write out a check for the cash payment.

6. R. L. Stevens & Sons sold to J. H. Boyce the following:

Two rockers at \$13.50; 1 rocker at \$8.25; 2 Morris chairs at \$15; 3 chairs at \$3.75; 1 hall tree at \$11.50; 1 hall tree at \$13.75. Terms: net 90 days.

Make out the proper bill. The proper draft. The proper check.

56. SIMPLE INTEREST

General Method of finding Interest

1. If I pay 6 ¢ for a year's use of a borrowed dollar, what is the rate of interest?

2. What does the expression "6 per cent interest" mean?

3. At 6 %, what is a year's interest of \$300?

4. What part of a year is 2 months? If the interest for 1 year is \$18, what should it be for 2 months?

5. What is the interest of \$400 for 1 year and 6 months at 5%?

6. At 7%, what is the interest of \$200 for 2 years, 6 months?

7. What is the first step in finding the interest of any principal? The second step?

8. At 5%, what is the interest of \$720 for 2 yr. 5 mo.?

SOLUTION

$\frac{29}{12} \times \frac{5}{100} \times \$720 = \$87.$

EXPLANATION. — $\frac{5}{100}$ of \$720 = int. for 1 yr. 2 yr. 5 mo. = $\frac{29}{12}$ yr. Hence the total interest = $\frac{29}{12} \times \frac{5}{100} \times \$720$.

Find the interest of:

9. \$6000 at 5% for 1 yr. 4 mo.

10. \$4800 at 6% for 2 yr. 8 mo.

11. \$7500 at 5% for 3 yr. 7 mo.

12. \$9640 at 6% for 2 yr. 9 mo.

13. \$4575 at 6% for 4 yr. 2 mo.

14. \$5280 at 4% for 1 yr. 11 mo.

15. \$6800 at 6% for 2 mo. 12 da. ($\frac{72}{360}$ yr.)

16. \$12,000 at 5% for 1 mo. 27 da.

17. \$28,400 at 5% for 2 mo. 10 da.

18. \$1890 at 6% for 3 mo. 14 da.

19. \$7630 at 5% for 4 mo. 24 da.

20. \$1200 at 6% for 90 da.

21. \$3500 at 6% for 35 da.

22. \$4850 at 5% for 75 da.

23. \$9600 at 4% for 67 da.

24. \$1450 at 6% for 24 da.

25. \$5260 at 5% for 45 da.

The Bankers' Method of finding Interest

At 6 % the interest of any principal for:

12 *months* =	6 % *of it.*
2 *months* =	1 % *of it.* (Why?)
20 *months* =	10 % *of it.* (Why?)
200 *months* =	100 % *of it, or the principal itself.*

1. Find the interest at 6 % of \$380 for 2 yr. 7 mo. (31 mo.)

PROCESS

Interest for 20 months =	\$38.00	**Observe** *that the time was so separated as to avoid multiplying by anything except* 10.
Interest for 10 months =	19.00	
Interest for 1 month =	1.90	
Interest for 31 months =	\$58.90	

2. Into what convenient parts would you separate the time if it were 26 mo.? 37 mo.? 3 yr. 7 mo.? 5 yr. 8 mo.? 3 yr. 11 mo.? 1 yr. 7 mo.? 8 yr. 4 mo.?

Explain the following process of finding the interest at 6 %:

3. Of \$725 for 3 yr. 11 mo.

Interest for 47 mo. of \$725.

$$\text{Int. for}\begin{cases}20 \text{ mo.} = \$72.50\\ 20 \text{ mo.} = 72.50\\ 5 \text{ mo.} = 18.125\\ \underline{2} \text{ mo.} = \underline{7.25}\\ 47 \text{ mo.} = \$170.375\end{cases}$$

4. Of \$278 for 1 yr. 7 mo.

Interest for 19 mo. of \$278.

$$\text{Int. for}\begin{cases}20 \text{ mo.} = \$27.80\\ \underline{1} \text{ mo.} = \underline{1.39}\\ 19 \text{ mo.} = \$26.41\end{cases}$$

Find the interest at 6 %:

5. Of \$280 for 2 yr. 8 mo.
6. Of \$640 for 3 yr. 7 mo.
7. Of \$95 for 4 yr. 11 mo.
8. Of \$73.50 for 1 yr. 4 mo.
9. Of \$649 for 7 yr. 8 mo.
10. Of \$750 for 8 yr. 4 mo.
11. Of \$295.75 for 5 yr. 11 mo.
12. Of \$641.86 for 3 yr. 3 mo.

13. How many days in an interest month? In an interest year?

14. 60 days is what part of a year?

15. Since at 6 % the interest for a year is 6 % of the principal, the interest for 60 days is what per cent of the principal?

16. The interest for 6 days is what part of the interest for 60 days?

> *At* 6 % *the interest of any principal for:*
> 60 *days* = 1 %, *or* $\frac{1}{100}$ *of it.*
> 6 *days* = 0.1 %, *or* $\frac{1}{1000}$ *of it.*

17. Find the interest at 6 % of \$720 for 75 days.

PROCESS

Interest for 60 days =	\$7.20
Interest for 15 days =	1.80
Interest for 75 days =	\$9.00

18. How was 15 days' interest found from 60 days' interest being known?

Explain the process of finding the interest at 6 % :

19. Of \$196 for 115 days.

Int. for 115 da. of \$196.

$$\text{Int. for}\begin{cases}60\text{ da.} = \$1.96\\ 30\text{ da.} = 0.98\\ 20\text{ da.} = 0.6533+\\ 5\text{ da.} = 0.1633+\\ \overline{115\text{ da.}} = \$3.7566+\end{cases}$$

20. Of \$119 for 89 days.

Int. for 89 da. of \$119.

$$\text{Int. for}\begin{cases}60\text{ da.} = \$1.19\\ 20\text{ da.} = 0.3966+\\ 6\text{ da.} = 0.1190\\ 3\text{ da.} = 0.0595\\ \overline{89\text{ da.}} = \$1.7651\end{cases}$$

Find the interest at 6 % :

21. \$780 for 67 da.

22. \$640 for 93 da.

23. \$920 for 3 mo. 12 da.

24. \$875 for 117 da.

25. \$706 for 2 mo. 17 da.

26. \$940 for 200 da.

27. \$762 for 5 mo. 14 da.

28. \$815 for 86 da.

29. \$924 for 8 mo. 11 da.

30. \$785 for 17 da.

At 5 % find the interest of:

31. $840 for 75 da.

$8.40 = int. at 6 % for 60 da.
2.10 = int. at 6 % for 15 da.
6)10.50 = int. at 6 % for 75 da.
1.75 = int. at 1 % for 75 da.
$8.75 = int. at 5 % for 75 da.

32. $820 for 70 da.

33. $950 for 63 da.

34. $720 for 93 da.

35. $875 for 72 da.

36. $970 for 85 da.

37. $450 for 20 da.

38. $120 for 50 da.

39. $830 for 45 da.

40. $650 for 45 da.

41. $810 for 75 da.

42. $960 for 70 da.

Interest on Notes

1. What is a promissory note?

2. In computing the interest on a note, how is the time between dates found? Find the time between March 6, 1908, and September 2, 1909.

> *$ 1500 Chicago, Illinois, August 15, 1910.*
> *Sixty days after date I promise to pay to the*
> *order of Thomas Simpson*
> *Fifteen Hundred Dollars*
> *Interest 6 %.*
> *Value received. Gordon Bradford.*

3. Find the interest on the above note when due. What is the amount that Gordon Bradford must pay in settlement?

4. A note given May 3, 1910, for $850, at 5 %, was paid off July 18, 1910. How much was paid in settlement?

5. A note given August 25, 1909, for $2400, at 6 %, was paid off June 16, 1910. How much was the interest?

charged by the bank is *bank discount*, being computed on the value of the note at maturity at a certain rate per cent for the time from the date when the note is discounted until it is due. The maturity value of the note less the discount is the *proceeds*.

NOTE. — Banks usually compute the time in the exact number of days in discounting a note. In some parts of the country, both the day that the note is discounted and the day that it is due are included. The pupil should ascertain and follow the local custom.

Problems

1. A note for $800, at 5%, dated March 6, 1910, and due June 6, 1910, was discounted at a bank April 15, 1910, at 6%. How much did the bank pay for the note, *i.e.* what were the proceeds?

SOLUTION

Face =	$800.00
Int. 3 mo., 5%	10.00
Maturity value	$810.00
Discount, 52 da., 6%	7.02
Proceeds	$802.98

EXPLANATION. — From March 6 to June 6 = 3 mo. From April 15 to June 6 = 52 da. The interest for 3 mo. at 5% is added to the principal to get the maturity value. The discount on the maturity value is found for 52 da. at 6%.

NOTE. — Local customs vary. Visit a bank in your city and find how the cashier would discount this note.

2. A note for $1200, without interest, due September 1, is discounted at 6% July 6. What are the proceeds?

3. A note for $450, without interest, due May 10, is discounted at 6% April 1. Find the proceeds.

4. A merchant sold $350 worth of goods, and took the purchaser's note for the amount at 5%, due in 60 days. Needing the cash, he took the note 10 days later to a bank and sold it at a discount of 6%. How much did he receive for it?

5. A dealer bought \$1250 worth of goods, and gave his note on February 5 for 90 days, at 6%. On February 12 the holder of the note discounted it at a bank at 6%. What were the proceeds?

6. If you were to buy an automobile for \$1600 on April 20, and give your note for the amount at 5%, due in 90 days, and the automobile company discounted the note at a bank at 6% the day it was made, how much cash would the company receive for the machine?

Find proceeds of notes under these conditions:

	Face	Rate	Date of Note	Runs	Discounted
7.	\$450	4%	May 10	60 da.	28 da. after date.
8.	\$720	5%	Aug. 15	90 da.	48 da. before maturity.
9.	\$850	6%	June 1	90 da.	15 da. after date.

10. Find the proceeds of a note for \$958, dated October 12, 1910, and having 45 days to run, if discounted at date at 7 per cent.

11. A note of \$600, dated May 3, 1910, interest 6%, to run 4 months, was discounted May 20 at 6%. Find the proceeds.

12. A note of \$560, to run 3 months, dated August 4, 1910, bearing 5% interest, was discounted at 6% on September 19. Find the discount and the proceeds.

13. A man sold his farm for \$6500, taking a note due in 6 months, interest 5%. He at once sold the note to a bank, discount 6%. What did he get in cash for his farm?

14. A certain wholesale house sells goods on 90 days' time, taking in payment a 90-day note without interest. What does the house get for an invoice of \$820 discounted at 6% 5 days after the date of the note?

15. Some firms date their bills for certain kinds of goods two or three months ahead of the actual date of making out the invoice. A bill of wall paper amounting to $540 was billed December 15, 1909, but dated March 1, 1910. If this bill is paid before March 1, a discount equal to 6% interest may be deducted. What will settle the bill December 15? What will settle it January 1, 1910? What February 1?

59. COMPUTING INTEREST BY TABLES

We have found that bank discount usually is computed for the exact number of days that a note has to run from the date that it is discounted. This time is quickly found by the use of the following table:

THIS TABLE SHOWS THE NUMBER OF DAYS FROM ANY DAY OF ANY MONTH TO THE SAME DAY OF ANY MONTH NOT MORE THAN ONE YEAR LATER

FROM	To JAN.	FEB.	MAR.	APR.	MAY	JUNE	JULY	AUG.	SEPT.	OCT.	NOV.	DEC.
Jan.	365	31	59	90	120	151	181	212	243	273	304	334
Feb.	334	365	28	59	89	120	150	181	212	242	273	303
March	306	337	365	31	61	92	122	153	184	214	245	275
April	275	306	334	365	30	61	91	122	153	183	214	244
May	245	276	304	335	365	30	61	92	123	153	184	214
June	214	245	273	304	334	365	30	61	92	122	153	183
July	184	215	243	274	304	335	365	31	62	92	123	153
Aug.	153	184	212	243	273	304	334	365	31	61	92	122
Sept.	122	153	181	212	242	273	303	334	365	30	61	91
Oct.	92	123	151	182	212	243	273	304	335	365	31	61
Nov.	61	92	120	151	181	212	242	273	304	334	365	30
Dec.	31	62	90	121	151	182	212	243	274	304	335	365

Thus, the number of days from June 13 to November 27 of the same year is found as follows:

By the table, from June 13 to November 13 = 153 days.
From Nov. 13 to November 27 = 14 days.
Total = 167 days.

INTEREST TABLE. INTEREST AT 6%

DA.	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1000	DA.
1	0.017	0.033	0.050	0.067	0.083	0.100	0.117	0.133	0.150	0.167	1
2	0.033	0.067	0.100	0.133	0.167	0.200	0.233	0.267	0.300	0.333	2
3	0.050	0.100	0.150	0.200	0.250	0.300	0.350	0.400	0.450	0.500	3
4	0.067	0.133	0.200	0.267	0.333	0.400	0.467	0.533	0.600	0.667	4
5	0.083	0.167	0.250	0.333	0.417	0.500	0.583	0.667	0.750	0.833	5
6	0.100	0.200	0.300	0.400	0.500	0.600	0.700	0.800	0.900	1.000	6
7	0.117	0.233	0.350	0.467	0.583	0.700	0.817	0.933	1.050	1.167	7
8	0.133	0.267	0.400	0.533	0.667	0.800	0.933	1.067	1.200	1.333	8
9	0.150	0.300	0.450	0.600	0.750	0.900	1.050	1.200	1.350	1.500	9
10	0.167	0.333	0.500	0.667	0.833	1.000	1.167	1.333	1.500	1.667	10
11	0.183	0.367	0.550	0.733	0.917	1.100	1.283	1.467	1.650	1.833	11
12	0.200	0.400	0.600	0.800	1.000	1.200	1.400	1.600	1.800	2.000	12
13	0.217	0.433	0.650	0.867	1.083	1.300	1.517	1.733	1.950	2.167	13
14	0.233	0.467	0.700	0.933	1.167	1.400	1.633	1.867	2.100	2.333	14
15	0.250	0.500	0.750	1.000	1.250	1.500	1.750	2.000	2.250	2.500	15
16	0.267	0.533	0.800	1.067	1.333	1.600	1.867	2.133	2.400	2.667	16
17	0.283	0.567	0.850	1.133	1.417	1.700	1.983	2.267	2.550	2.833	17
18	0.300	0.600	0.900	1.200	1.500	1.800	2.100	2.400	2.700	3.000	18
19	0.317	0.633	0.950	1.267	1.583	1.900	2.217	2.533	2.850	3.167	19
20	0.333	0.667	1.000	1.333	1.667	2.000	2.333	2.667	3.000	3.333	20
21	0.350	0.700	1.050	1.400	1.750	2.100	2.450	2.800	3.150	3.500	21
22	0.367	0.733	1.100	1.467	1.833	2.200	2.567	2.933	3.300	3.667	22
23	0.383	0.767	1.150	1.533	1.917	2.300	2.683	3.067	3.450	3.833	23
24	0.400	0.800	1.200	1.600	2.000	2.400	2.800	3.200	3.600	4.000	24
25	0.417	0.833	1.250	1.667	2.083	2.500	2.917	3.333	3.750	4.167	25
26	0.433	0.867	1.300	1.733	2.167	2.600	3.033	3.467	3.900	4.333	26
27	0.450	0.900	1.350	1.800	2.250	2.700	3.150	3.600	4.050	4.500	27
28	0.467	0.933	1.400	1.867	2.333	2.800	3.267	3.733	4.200	4.667	28
29	0.483	0.967	1.450	1.933	2.417	2.900	3.384	3.867	4.350	4.833	29
Mo.	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1000	Mo.
1	0.500	1.000	1.500	2.000	2.500	3.000	3.500	4.000	4.500	5.000	1
2	1.000	2.000	3.000	4.000	5.000	6.000	7.000	8.000	9.000	10.000	2
3	1.500	3.000	4.500	6.000	7.500	9.000	10.500	12.000	13.500	15.000	3
4	2.000	4.000	6.000	8.000	10.000	12.000	14.000	16.000	18.000	20.000	4
5	2.500	5.000	7.500	10.000	12.500	15.000	17.500	20.000	22.500	25.000	5
6	3.000	6.000	9.000	12.000	15.000	18.000	21.000	24.000	27.000	30.000	6
7	3.500	7.000	10.500	14.000	17.500	21.000	24.500	28.000	31.500	35.000	7
8	4.000	8.000	12.000	16.000	20.000	24.000	28.000	32.000	36.000	40.000	8
9	4.500	9.000	13.500	18.000	22.500	27.000	31.500	36.000	40.500	45.000	9
10	5.000	10.000	15.000	20.000	25.000	30.000	35.000	40.000	45.000	50.000	10
11	5.500	11.000	16.500	22.000	27.500	33.000	38.500	44.000	49.500	55.000	11
YR.	$100	$200	$300	$400	$500	$600	$700	$800	$900	$1000	YR.
1	6.00	12.00	18.00	24.00	30.00	36.00	42.00	48.00	54.00	60.00	1
2	12.00	24.00	36.00	48.00	60.00	72.00	84.00	96.00	108.00	120.00	2
3	18.00	36.00	54.00	72.00	90.00	108.00	126.00	144.00	162.00	180.00	3
4	24.00	48.00	72.00	96.00	120.00	144.00	168.00	192.00	216.00	240.00	4
5	30.00	60.00	90.00	120.00	150.00	180.00	210.00	240.00	270.00	300.00	5

When men have much computing of interest or bank discounts to do, time is saved by using an interest table like the one on the preceding page.

Exercises in using Interest Tables

1. Find the bank discount on $240, at 6%, from March 12, 1910, to July 25, 1910.

By the tables, from March 12 to July 12	=	122 days.
From July 12 to July 25	=	13 days.
Total	=	135 days, or 4 mo. 15 days.

By the tables, Int. of $200 for 15 days	=	$0.500
Int. of 40 for 15 days	=	0.100
Int. of 200 for 4 mo.	=	4.000
Int. of 40 for 4 mo.	=	0.800
		$5.40

By the tables, find the interest, at 6%, of:

2. $850 for 18 da.	11. $7840 for 69 da.
3. $970 for 33 da.	12. $645 for 27 da.
4. $680 for 42 da.	13. $837 for 39 da.
5. $910 for 47 da.	14. $972 for 41 da.
6. $780 for 68 da.	15. $895 for 111 da.
7. $3400 for 71 da.	16. $908 for 117 da.
8. $5600 for 83 da.	17. $675 for 175 da.
9. $7200 for 91 da.	18. $861 for 134 da.
10. $8600 for 113 da.	19. $276 for 142 da.

Find the bank discount and proceeds of:

20. $450 from August 6, 1909, to November 15, 1909, at 6%.

21. $960 from January 3, 1910, to June 18, 1910, at 6%.

22. $4300 from March 18, 1910, to June 3, 1910, at 6%.

23. $1850 from October 1, 1909, to January 19, 1910, at 6%.

24. $3800 from May 30, 1910, to July 3, 1910, at 6%.

60. PARTIAL PAYMENTS ON A NOTE

In general, modern business custom will not allow an advance payment upon a note unless it is so stipulated in the note. Such stipulations are usually to the effect that such payments are to be made at the end of interest-paying periods. There was a time, however, when borrowing among individuals was more common than now, and when the payments of interest and any other payments were more irregular. To govern the final settlements in such transactions, the United States Supreme Court decreed that —

Partial payments of notes must first be used to cancel the interest due. Any balance remaining may be used to lessen the principal. If, however, the payment is too small to pay the interest due, the unpaid interest must not be used to increase the principal, which must never represent more than the money actually and previously due.

NOTE. — It is customary in partial payment notes to indorse the payments on the back of the note when made.

Problems

1. What is due January 16, 1908, on a note of $1600 drawing 6% interest, given May 3, 1904, the following payments having been made: May 3, 1905, $200; December 18, 1905, $80; June 25, 1906, $300; April 16, 1907, $450?

2. What is due July 23, 1908, on a note of $2400 drawing 5% interest, given January 4, 1903, the following payments having been made: September 4, 1903, $300; July 19, 1904, $50; January 4, 1905, $300; April 16, 1906, $250; December 26, 1907, $500?

NOTE. — Since the payment on July 19, 1904, was insufficient to pay the interest then due, the principal left on September 4, 1903, will have to be used again. Work will be saved by finding the amount from September 4, 1903, to January 4, 1905, and then deducting $350, or both payments.

3. A note of $3500, dated August 15, 1905, interest 6%, has the following indorsements: August 15, 1906, $500; February 15, 1907, $100; July 10, 1907, $400; December 15, 1907, $1500. What is due August 15, 1908?

4. A note of $7500, dated May 3, 1905, interest 5%, has the following indorsements: May 3, 1906, $2500; August 3, 1906, $3500; February 18, 1907, $50; May 3, 1907, $500. What is due May 3, 1908?

5. I paid a note that had been running just 5 years. It drew 6 per cent interest, and its face was $900. It was given Dec. 11, 1907, and $300 had been paid on it 2 yr. 8 mo. after date. What sum cancelled it?

61. COMPOUND INTEREST

When interest due at the end of any interest period is added to the principal, and thus draws interest for the next interest period, and so on, we have **compound interest.** Thus, in a savings bank, the interest due at any interest-paying date is credited to one's account, and thus draws interest, giving *compound interest.*

In most states the collection of compound interest on a note is illegal. In modern practice, then, the subject is merely of use to large investors, as building and loan associations, life insurance companies, banking corporations, etc., who wish to compute the final incomes from reinvesting all interest as it falls due. Such computations are made by the use of compound interest tables.

A SECTION OF A COMPOUND INTEREST TABLE

Periods	1 per cent.	1½ per cent.	2 per cent.	2½ per cent.	3 per cent.	Periods
1	1.010000	1.015000	1.020000	1.025000	1.030000	1
2	1.020100	1.030225	1.040400	1.050625	1.060900	2
3	1.030301	1.045678	1.061208	1.076891	1.092727	3
4	1.040604	1.061364	1.082432	1.103813	1.125509	4
5	1.051010	1.077284	1.104081	1.131408	1.159274	5
6	1.061520	1.093443	1.126162	1.159693	1.194052	6
7	1.072135	1.109845	1.148686	1.188686	1.229874	7
8	1.082857	1.126493	1.171660	1.218403	1.266770	8
9	1.093685	1.143390	1.195093	1.248863	1.304773	9
10	1.104622	1.160541	1.218994	1.280085	1.343916	10
11	1.115668	1.177949	1.243374	1.312087	1.384234	11
12	1.126825	1.195618	1.268242	1.344889	1.425761	12
13	1.138093	1.213552	1.293607	1.378511	1.468534	13
14	1.149474	1.231756	1.319479	1.412974	1.512590	14
15	1.160969	1.250232	1.345868	1.448298	1.557967	15
16	1.172579	1.268986	1.372786	1.484506	1.604706	16
17	1.184304	1.288020	1.400241	1.521618	1.652847	17
18	1.196147	1.307341	1.428246	1.559659	1.702433	18
19	1.208109	1.326951	1.456811	1.598650	1.753506	19
20	1.220190	1.346855	1.485947	1.638616	1.806111	20

Periods	3½ per cent.	4 per cent.	4½ per cent.	5 per cent.	6 per cent.	Periods.
1	1.035000	1.040000	1.045000	1.050000	1.060000	1
2	1.071225	1.081600	1.092025	1.102500	1.123600	2
3	1.108718	1.124864	1.141166	1.157625	1.191016	3
4	1.147523	1.169859	1.192518	1.215506	1.262477	4
5	1.187686	1.216653	1.246181	1.276281	1.338226	5
6	1.229255	1.265319	1.302260	1.340096	1.418519	6
7	1.272279	1.315932	1.360861	1.407100	1.503630	7
8	1.316809	1.368569	1.422100	1.477455	1.593848	8
9	1.362900	1.423312	1.486095	1.551328	1.689479	9
10	1.410600	1.480244	1.552969	1.628895	1.790848	10
11	1.459970	1.539454	1.622853	1.710339	1.898299	11
12	1.511069	1.601032	1.695881	1.795856	2.012197	12
13	1.563956	1.665074	1.772196	1.885649	2.132928	13
14	1.618695	1.731676	1.851945	1.979931	2.260904	14
15	1.675348	1.800944	1.935282	2.078928	2.396558	15
16	1.733986	1.872981	2.022370	2.182875	2.540351	16
17	1.794676	1.947901	2.113376	2.292018	2.692773	17
18	1.857489	2.025817	2.208478	2.406619	2.854339	18
19	1.922501	2.106849	2.307860	2.526950	3.025600	19
20	1.989789	2.191123	2.411714	2.653298	3.207136	20

Problems in the Use of Compound Interest Table

1. If a man deposits $500 semiannually in a savings bank, what amount will he have to his credit at the end of 5 years, interest 3%, payable semiannually?

SOLUTION

$$
\begin{array}{r}
\$1.015000 \times 500 \\
1.030225 \times 500 \\
1.045678 \times 500 \\
1.061364 \times 500 \\
1.077284 \times 500 \\
1.093443 \times 500 \\
1.109845 \times 500 \\
1.126493 \times 500 \\
1.143390 \times 500 \\
1.160541 \times 500 \\
\hline
\$10.863263 \times 500 = \$5431.63
\end{array}
$$

EXPLANATION. — This rate is 1½% for 10 periods. The last deposit draws interest for but one period of 6 months; the next for 2 periods, and so on to 10 periods. Work is saved by adding before multiplying by 500.

2. How much can one accumulate in 10 years by depositing $800 annually, interest 4 %, compounded annually?

3. To what will an annual payment of $564 for 10 years amount, interest 3½ %, compounded annually?

4. What may one draw at the end of 10 years if he regularly deposits $400 semiannually in a savings bank paying 4 %, payable semiannually?

5. If one can invest $600 yearly from his salary where it will yield him 6 % yearly, and can keep all the interest invested at the same rate as fast as it accrues, how much can he accumulate in 20 years?

6. On his sixth birthday and on each succeeding birthday, including his 21st, a man gave his son $100, which, together with the interest, was kept in a savings bank paying 4 % interest annually. How much had the young man to his credit from this source on his 21st birthday?